Comparism of Computer Based Yield Line Theory with Elastic Theory and Finite Element Methods for Solid Slabs

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***Abstract*— The complexity and conservative nature of the Yield Line Theory and its being an upper bound theory have made many design engineers to jettison the use of the analytical method in the analysis of slabs. Before now, the method has basically been a manual or hand method which some engineers did not see a need for its use since there are many computer based packages in the analysis and design of slabs and other civil engineering structures. This paper presents a computer program that has adopted the yield line theory in the analysis of solid slabs. Two rectangular slabs of the same depth but different dimensions were investigated. The Yield Line Theory was compared with two other analytical methods namely, Finite Element Method and Elastic Theory Method.**

The results obtained for a two-way spanning slab showed that the yield line theory is truly conservative, but increasing the result by 25% caused the moment obtained to be very close to the results of the other two methods. Although it was still conservative, the check for deflections showed that it is reliable and economical in terms of reinforcement provision.

For a one way spanning slab the results without any increment falls in between the two other methods with the Elastic method giving a conservative results. The paper concludes that the introduction of a computer-based yield line theory program will make the analytical method acceptable to design engineers in the developing countries of the world.

Keywords: Analytical method, Moments, Solid slabs, Yield line theory

I. INTRODUCTION

The application of the yield line theory is gradually becoming popular in some developing nations of the world such as Nigeria, although with a lot of reservations from some of the users due mainly to the conservative nature of the analytical method. This method is already well accepted in the Scandinavian and some European countries, but the apprehension in the developing nation is because the advantages of the method have not been well understood by many structural engineers and the seemingly complexity of the method.

This paper has tried to simplify the method by introducing a computer-based yield line theory program. Yield line theory investigates failure mechanism at the ultimate limit state. It does not deal with serviceability issues such as deflection *per se.* Nonetheless, deflection can be dealt with by simple formulae based on yield line moment (Kennedy and Goodchild, 2004). The basic assumption of the yield line theory is that a reinforced concrete slab, similar to a continuous beam or frame of a perfectly plastic material will develop yield line hinges under overload, but will not collapse until a mechanism is formed (Dunham,1964). The theory also permits the prediction of the ultimate load of a slab system by postulating a collapse mechanism which is compatible with the boundary conditions (Buyukozturk, 2004). Yield-line analysis is seen as a useful technique to determine the collapse load of slabs (Johansen, 1963). The band in which yielding has occurred are referred to as yield lines which divide the slab into a series of elastic plates.

The Finite Element Method (FEM) is based on the division of the structures into small pieces (elements) whose behaviours are formulated to capture the local behaviour of the structure. Each element's definition is based on its material properties, geometry, location in the structure, and relationship with surrounding elements. These elements can be in the form of line elements, two dimensional elements and three-dimensional elements to represent the structure. The intersection between the elements are called nodal points in one dimensional problems, while in two and three dimensional problems, they are called nodal line and nodal planes respectively (Maher, 2007).

Analyses based on Elastic theory give much more detailed and more precise information about the state of stress, strain, and deformation at any point within the body of a structure. In particular, the theory is excellent for investigating the state of stress and deformation in the immediate vicinity of small holes, notches, and cuts in an elastic body. Also the theory permits a much more detailed treatment of boundary conditions, for example the boundary condition can be examine at every point throughout the depth of a plate in which solid slab falls into. Moreover the deflection or slope can be specified. In general, the external applied forces may be regarded as continuations of the internal stresses as determined by elasticity theory. That is, on surface elements of the body, the stresses must be in equilibrium with the applied external forces, and this is very similar to the principle of virtual work method that is applied in the Yield line theory.

According to Wang *et al* (2003), when concrete is under triaxial compressive loading, both its strength and ductility will have a significant increase as a result of resistance to the compressive force by the concrete materials (molecules). Initially, at service load, the response of a slab is elastic with maximum steel stress and deflection occurring at the center of the slab. At this stage, it is possible that some hairline cracking will occur on the suffix where the flexural tensile capacity of the concrete has been exceeded at mid span. Increasing the load hastens the formation of these hairline cracks. Further increment of the load will increase the size of the cracks and induce yield of the reinforcement, initiating the formation of large cracks emanating from the point of maximum deflection (Kennedy and Goodchild, 2004). This portion acts like a plastic hinge. On increasing the load further, the hinging region rotates plastically and the moments due to additional loads are redistributed to adjacent sections, the concrete section at the position of a yield line is incapable of carrying any further load, causing them to collapse (Thompson and Haywood, 1986; Macgregor, 1997).

A. Load Path Designation in Solid Slabs

Reinforced concrete is very unique in it behavior, and this has made it popular as construction material. In solid slabs; at flexural failure, concrete slabs develop hinge lines. A hinge line mobilizes much of the reinforcement passing through it to resist the moment along its length, contributing to the safety of the slab (Aalami, 2005).

Prior to the calculation of the design moments and shears, the first thing that must be considered is to anticipate the load path, which set the orientation and position of the reinforcement. Since the major work of the longitudinal reinforcement is to provide flexural strength for the concrete slab (Sivagamasundari and Kumara, 2008). For example, in a solid two-way slab, the function of the distribution bar is to distribute the load from the slab to the bottom or main bar, while the bottom bar will distribute the load to the supports at the edges of the slab. Both the distribution and main bar are designed for in this type of slab. The amount of bending moment in each direction will depend on the ratio of the two spans and the condition of restraint at each support (Mosley and Bungey, 1990). While in one-way slab it is only the main bar that is design, although appropriate provision is made for distribution bar in this type of slab. Top (torsion) reinforcement is provided at the supports or edges of slabs to prevent cracks as concrete is known to be weak in tension (BS8110, 1997).

II. SAMPLE PROPERTIES

In order to carry out investigation and arrive at a reasonable value, the dimensions for a given two-way floor slab of 6x4 m was considered and another 5x2 m for a one-way slab. Keeping the residential and office floors in mind, the general thickness adopted was 200 mm; larger spans would require higher floor thickness.

A. Floor loading

Only gravity loading on the floor was considered. The live load adopted was 3 kN/m^2, floor finish load was taken as 1.4 kN/m^2, safety factors for both dead and live loads were applied, and the density of reinforced concrete as 24 kN/m^2.

III. METHOD OF ANALYSIS

A. Analytical method

Over time, there have been different analytical methods that have been used in the analysis of slabs. Among these methods are: the Orthotropic plate theory, the Finite element method, Simple frame method, Equivalent frame method, the BS 8110 slab coefficient method which is based on the Elastic method, and the Yield line theory. This paper has developed a computer-based program that has simplified some of the complex equations in the Yield line theory. The work method which was simplified to standard formulae by Kenedy and Goodchild (2004) was adopted in the program. Some of the adopted formulae are shown below. Equation 1 is for a two-way slab supported on all four sides while Equation 2 is for one-way spanning slab. Equation 1 was modified in this paper to Equation 3 in order to take care of the variability in the results. All the equations are for isotropic slabs only; these are slabs with the same amount of reinforcements in both ways.

$$m = \frac{n \times a_r \times b_r}{8\left(1 + \frac{b_r}{a_r} + \frac{a_r}{b_r}\right)} \qquad 1$$

$$m = \frac{nL^2}{2\left(\sqrt{1+i_1} + \sqrt{1+i_2}\right)^2} \qquad 2$$

$$m = \frac{n \times a_r \times b_r}{8\left(1 + \frac{b_r}{a_r} + \frac{a_r}{b_r}\right)}(k + v) \qquad 3$$

B. Computer program

There have been different computer programs that were developed by various researchers and engineers for the analysis of slabs of different shapes and configurations. It has been discovered that most of these programs, except very few, adopted the Finite elements method (FEM) of analysis of structures. One of these programs (PROKON) was compared with the computer based Yield line theory program and another Elastic method programme by the Reinforced Concrete Council (RCC) which is based on BS 8110 1997.

IV. RESULTS AND DISCUSSIONS

A. Floor plans 6x4 m and 5x2 m

The results of the analytical study on 6x4 m and 5x2 m rectangular slabs under a live load intensity of 3 kN/m^2 are presented in Table 1 and Table 4 respectively, while the area of reinforcement required and provided is in Table 2 and Table 3 for the long and short span of 6x4 m slab respectively.

It is clear from the results shown in Table 1 that yield line theory is conservative for all the support conditions. Kennedy and Goodchild (2004) introduced the 10% rule which was stated that “a 10% margin on the design moments should be added when using the work method or formulae for two way-slabs to allow for the method being upper bound and to allow for corner levers.” When this rule was applied, it was observed that it gave conservative results which may still bring some doubt as per the reliability of the theory. Hence the design moment was increased to 15%, 20%, 25%, and 30% in this work (Figure 1). But the 25% was adopted because, the results for a slab that is supported on all four sides was 7.65 kNm which is a little below the results of RCC and Prokon that gave 8.20 kNm and 7.80 kNm respectively. Results for continuous supports on three, two, and one sides were marginally small, although, they were conservative, it is more reliable than the 10% which is too small or the 30% which is high and will defeat the conservative nature of Yield line theory. When the result was compared with elastic and finite element methods, the difference in the results are minimal and manageable. All these results are for the short span of the slab (Table 2). When the long span was considered, it was observed that the Finite element method gave conservative results when compared with both the elastic method and yield line theory, (Table 3). This has allowed the yield line theory to make up for its conservative results in the short span.

In the other two methods (Finite element and Elastic methods), the results are for orthotropic slabs while that of yield line is for isotropic slabs. From Table 2, it can be observed that the reinforcements provided in the span of slabs that are continuous on 4, 3 and 2 edges are the same for the three analytical methods. This is as a result of the closeness of the required reinforcements. However, for the span of the slabs on one continuous edge and the slab with all free edges (simply supported), yield line showed a very high conservative results unlike the elastic and finite element methods. The elastic method has the highest flexural moments and hence a higher area of reinforcement is required in the short span. The reinforcement provided in both elastic and finite element methods are orthotropic because of the differences between the short and long span moments, and this led to the provision of different reinforcement in the two directions.

The results of the provisional reinforcement for all the analytical methods did not deflect under the design load when it was checked. The reinforcements required and provided for, in the free and one edge continuous slabs for yield line theory has confirmed that the analytical method is economical under these conditions.

The bending moment generated on the edges of the slabs are determined in Yield line theory by adopting the same bending moment or twice the bending moment in the span, but it must not be less than the mid span moments. It is thus highly dependent on the engineering judgment of the designer. Unlike the Elastic theory where there are developed slab coefficients in the analysis of the slab edges (BS 8110, 1997), the Finite elements method determined the moments based on the behavior of the elements in that particular area during analysis.

For the 5x2 m one-way slab, the result from Table 4 showed that the flexural moment of the Elastic method is more conservative when compared with the results of both Yield line theory and the Finite element method, with the FEM having the highest value of 43 kNm for a simply supported condition. The results of the Yield line falls between the two analytical methods. This should be an advantage for the method because it is not too high or too low, hence it is safe. In all the continuous edges for both slabs, it was assumed that the adjoining bays are of similar spans.

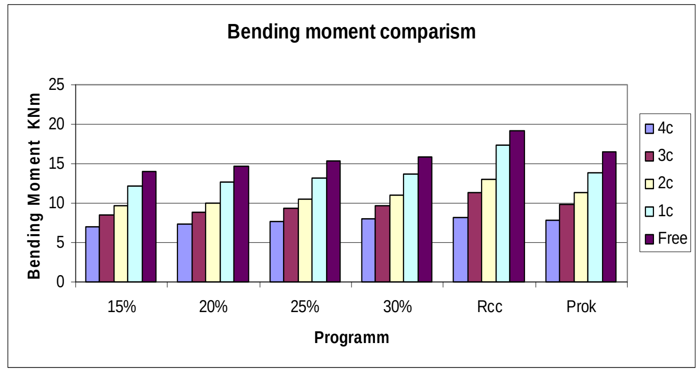

Figure 1. Compared analytical result for 6 x 4 m slab

Table 1: Analytical results for 6x4m slab

Continuous edges	Yield line theory (kNm) Normal	25%	RCC (kNm)	Prokon (kNm)
4	6.12	7.65	8.20	7.80
3	7.42	9.27	11.30	9.90
2	8.40	10.50	12.93	11.40
1	10.54	13.17	17.38	13.90
Free	12.24	15.30	19.10	16.50

Table 2: Area of reinforcement for 6x4m slab (Short span)

Continuous edges	Yield line 25% (mm^2)		RCC (mm^2)		Prokon (mm^2)	
	R	P	R	P	R	P
4	117	262	134	262	176	262
3	143	262	185	262	224	262
2	161	262	199	262	256	262
1	202	262	285	314	314	314
Free	235	262	312	314	372	393

R= Required reinforcement
P= Provided reinforcement

Table 3: Area of reinforcement for 6x4m slab (Long span)

Continuous edges	Yield line 25% (mm^2)		RCC (mm^2)		Prokon (mm^2)	
	R	P	R	P	R	P
4	117	262	88	262	90	262
3	143	262	101	262	137	262
2	161	262	125	262	194	262
1	202	262	159	262	258	262
Free	235	262	201	262	274	314

R= Required reinforcement
P= Provided reinforcement

Table 3: Analytical results for 5x2m one-way slab

Continuous edges	**Yield line(kNm) Normal**	**RCC(kNm)**	**Prokon (kNm)**
2	20.19	20.40	22.40
1	27.71	21.30	32.40
FREE	40.33	34.00	43.00

V. CONCLUSION

The yield line theory has been enhanced by comparing it with the two most accepted analytical methods of slabs (Elastic and Finite elements methods), and it has been found to be safe and economical. Before now, the yield line theory has been known to be a manual or hand calculated method, this has made it unpopular and made the acceptability low in some countries, but the development of the theory to a computer package with some variation in the analytical results will make the method acceptable to design engineers.

Notations

m = the ultimate design moment (kNm/m)
n = the ultimate uniformly distributed load (kN/m^2)
a_r = reduced short span dimension
b_r = reduced long span dimension
i = the fixity ratio at the supports, i.e, i_1,i_2.
a = short span dimension
b =long span dimension
If:
$i_1 = i_2 = 1$, then that support is a continuous support
$i_1 = i_2 = 0$, then that support is a simple support
k = is a constant = 1
v = is a variable = 0 < 0.5

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