# Effect of Size of Barrier on Reflection of Love Waves

Pushpander Kadian [#1], Jagdish Singh [*2]

[#1] Maharaja Surajmal Institute, New Delhi, India

[1] pushpanderkadian@gmail.com

[*2] M. D. University, Rohtak, India

[2] js_nandal_2k5@yahoo.com

*Abstract*— **The problem of reflection of Love waves at a rigid barrier is studied in this paper by taking the barriers of different sizes. The barrier is present in the homogeneous, isotropic and slightly dissipative surface layer. The reflected waves are obtained by Wiener – Hopf technique and Fourier transformations. Numerical computation has been done and conclusion has been drawn from the graphs of amplitudes versus wave number of the reflected Love waves. The amplitude of the reflected waves decreases rapidly with the increase in wave number and then it decreases at slower rate and ultimately becomes saturated which shows that love waves take a long time to dissipate and go on moving around earth surface for a long time. The comparison of graphs also shows that the barriers of large sizes result in reflected Love waves with larger amplitudes.**

***Keywords:*** **Fourier transforms, free surface, reflection, rigid barrier, Wiener – Hopf technique**

## I. MATERIAL AND METHODS

Love waves are the surface seismic waves that cause horizontal shifting of earth during an earthquake. The particle motion of Love waves forms a horizontal line perpendicular to direction of propagation. These waves take a long time to dissipate due to large amount of energy they contain [6, 11]. Here we discuss the reflection of Love waves at a rigid barrier $-H \le z \le -h$, $x = 0$, present in the layer $-H \le z \le 0$ superimposed on a solid half space $z \ge 0$ in zx – plane. This paper deals with the reflection of Love waves due to irregularities like rocks in the surface layer. The problem is based on a paper by Sato [8] who studied the problem of reflection and transmission of Love wave at a vertical discontinuity in a surface layer. Zaman et al. [4, 9] have solved the problem of diffraction of Love waves normally incident on two parallel perfectly weak half planes lying in surface layer. Deshwal and Mudgal [7] have solved the problem of scattering of love waves due to surface irregularity in form of barrier of finite depth. Tomar and Kaur [10] have studied the problem of reflection and transmission of a plane SH-wave at a corrugated interface between a dry sandy half space and an anisotropic elastic half space. They used the Rayleigh's method of approximation for studying the effect of sandiness, the anisotropy, the frequency and the angle of incidence on the reflection and transmission coefficients. Chattopadhyay et. al. [1] have studied the reflection of shear waves in visco-elastic medium at parabolic irregularity. They found that amplitude of reflected wave decreases with increasing length of notch and Increases with increasing depth of irregularity. Here we discuss the propagation of Love waves through irregularity in form of rigid barrier and the results have been discussed by taking different sizes of barrier.The present paper is based on reflection of Love waves at a rigid barrier in a solid layer of thickness H, the upper surface of which is a free surface. The incident love wave propagates parallel to x-axis in vertical zx-plane. The geometry of the problem is given in figure 1. Let the incident love wave is given by Sato [8].

$$v_{0,1} = A\ \cos\theta_{2N} H\, e^{-(\theta_{1N}z\ -\ ik_{1N}x)}\ ; \quad z \ \ge\ 0, \tag{1}$$

$$v_{0,2} = A\ \cos\theta_{2N}\,(z+H)\, e^{-(\,ik_{1N}x)}; \quad -H\ \le\ z\ \le\ 0, \tag{2}$$

where

$$\theta_{2N} = \sqrt{k_2^2 - k_{1N}^2}\,,\ \theta_{1N} = \sqrt{k_{1N}^2 -\ k_1^2}\ \text{ and } |\,k_1| < |\,k_2\,| \tag{3}$$

and $k_{1N}$ is a root of equation

$$\tan\theta_{2N} H = \gamma\frac{\theta_{1N}}{\theta_{2N}}\ ,\ \ \gamma = \frac{\mu_1}{\mu_2}, \tag{4}$$

where, $\mu_1$ *and* $\mu_2$ are rigidities of shear waves in the half space and in the layer respectively

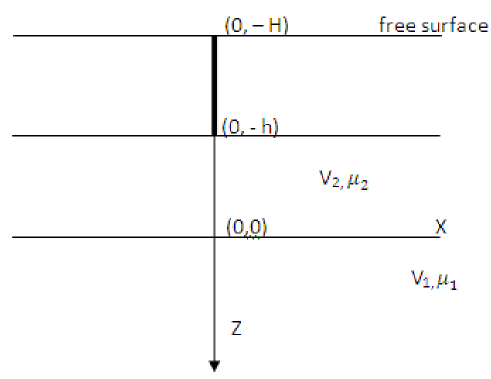


Figure 1: Geometry of the problem

The problem is being solved by the Wiener – Hopf technique [2] and Fourier transforms [5].

The wave equation is given by

$$(\nabla^2 + k_j^2)\, v_j = 0; \quad j = 1, 2 \tag{5}$$

where

$$k_j = \sqrt{\frac{\omega^2 + i\varepsilon\omega}{V_j^2}} = k_j' + k_j'', \tag{6}$$

where $V_1$ and $V_2$ are velocities of shear waves in half space and in the layer $-H \le z \le 0$ respectively.

Let the total displacement is given by

$$v = v_{0,1} + v_1 \quad ; \quad z \ge 0, \tag{7}$$

$$= v_{0,2} + v_2 \quad ; \quad -H \le z \le 0, \tag{8}$$

with the boundary conditions

$$v_{0,2} + v_2 = 0; \quad -H \le z \le -h, \; x = 0, \tag{9}$$

$$\frac{\partial}{\partial z}(v_{0,2} + v_2) = 0; \; x \ge 0, x \le 0, z = -H, \tag{10}$$

$$v_1 = v_2, \; \mu_1 \frac{\partial v_1}{\partial z} = \mu_2 \frac{\partial v_2}{\partial z}, \; z = 0, -\infty < x < \infty \tag{11}$$

Taking Fourier transforms of equation (5) and considering the reflection of incident Love waves at the rigid barrier $-H \le z \le -h$, we obtain

$$\frac{d^2 \bar{v}_2(p,z)}{dz^2} - \theta_2^2\, \bar{v}_2(p,z) = 0, \tag{12}$$

where $\theta_2 = \pm\sqrt{p^2 - k_2^2}$. (13)

Solving equation (12) we get

$$\bar{v}_2(p,z) = -\frac{\theta_2 \cosh\theta_2 z - \gamma\,\theta_1 \sinh\theta_2 z}{\theta_2(\theta_2 \sinh\theta_2 h + \gamma\,\theta_1 \cosh\theta_2 h)}\, \bar{v}'_2(p). \tag{14}$$

The displacement $v_2(x,z)$ is obtained by taking inverse Fourier transform of the above equation

$$v_2(x,z) = \frac{1}{2\pi}\int_{-\infty+i\beta}^{\infty+i\beta} \bar{v}_2(p,z) e^{-ipx} dp =$$

$$\frac{1}{2\pi}\int_{-\infty+i\beta}^{\infty+i\beta} \frac{-1}{\theta_2}\left[\frac{\theta_2 \cosh\theta_2 z - \gamma\theta_1 \sinh\theta_2 z}{\theta_2 \sinh\theta_2 h + \gamma\theta_1 \cosh\theta_2 h}\right]\left[\bar{v}'_{2^+}(p) + \bar{v}'_{2^-}(p)\right] e^{-ipx} dp\,, \tag{15}$$

where

$$\begin{aligned}\bar{v}'_{2^+}(p) &= \left[\frac{T(p^2 - k_2^2)}{p + k_{1N}} + \frac{\bar{v}'_{2^+}(k_2) P_+(k_2)(p + k_2)}{2k_2} \right.\\ &+ \frac{i\bar{v}'_{2^-}(-p_{2m}) k_+(p_{2m})(p^2 - k_2^2)\delta k_+(p)}{P_+(p_{2m})} \\ &- 2iA\delta\theta_{2N}\sin\theta_{2N}(p^2 - k_2^2).\sum_{n=1}^{\infty}\frac{1}{(k_{1N}^2 + p_n^2)(p + ip_n)P_+(ip_n)} \\ &- \frac{\bar{v}'_{2^+}(k_2)(p - k_2)}{2k_2 P_+(k_2)} + \sum_{n=1}^{\infty}\frac{i\bar{v}'_{2^+}(ip_n)(p^2 - k_2^2)}{p_n(p + ip_n)P_+(ip_n)} \\ &\left. + \frac{iA\sin\theta_{2N}\delta(p - k_2)}{\theta_{2N} P_+(k_2)}\right]\frac{1}{P_+(p)},\end{aligned} \tag{16}$$

where $\delta = H - h$ and

$$\begin{aligned}T &= \left[A\theta_{2N}\sin\theta_{2N}\delta.\sum_{n=1}^{\infty}\frac{1}{p_n(k_{1N} - ip_n)} \right.\\ &+ iA\theta_{2N}\sin\theta_{2N}\delta f_+(k_{1N})\delta - \frac{iA\theta_{2N}\sin\theta_{2N}\delta}{2k_2(k_2 - k_{1N})} \\ &\left. + iA\delta\cos\theta_{2N}\delta\right]\frac{1}{P_+(k_{1N})}\end{aligned} \tag{17}$$

Similarly we find out $\bar{v}'_{2^-}(p)$.

The reflected Love wave [3] is given by

$$v_2(x,z) = A\cos\theta_{2N}(Z + H) e^{-ik_{1N}x}\;; \quad x < 0, \; -H \le z \le -h, \tag{18}$$

where

$$\begin{aligned}A = &\left[\frac{iP_+(k_2)\bar{v}'_{2^-}(-k_2)(k_{1N} - k_2)}{2k_2} - \frac{i\bar{v}'_{2^+}(p_{2m})k_-(k_{1N})k_+(p_{2m}))}{P_+(p_{2m})}\theta_{2N}^2\delta - \frac{i\bar{v}'_{2^-}(-k_2)(k_2 + k_{1N})}{2k_2 P_+(k_2)} - \right.\\ &2A\theta_{2N}^3\sin\theta_{2N}\delta.\sum_{n=1}^{\infty}\frac{1}{(k_{1N}^2 + p_n^2)(k_{1N} - ip_n)P_+(ip_n)} + \sum_{n=1}^{\infty}\frac{\bar{v}'_{2^-}(-ip_n)\theta_{2n}^2}{p_n(k_{1N} - ip_n)P_+(ip_n)} - \\ &\left.\frac{A\sin\theta_{2N}\delta(k_2 + k_{1N})}{\theta_{2N}P_+(k_2)}\right]\frac{\sin\theta_{2N}\delta P_+(k_{1N})}{\theta_{2N}\delta\cos\theta_{2N}H\frac{d}{dp}[f_1(p)]_{p=k_{1N}}}\end{aligned} \tag{19}$$

where

$$\frac{d}{dp}[f_1(p)]_{p=k_{1N}} = \left[\frac{\theta_{1N}H + \gamma}{\theta_{1N}}\cos\theta_{2N}H + \frac{1 + \gamma\theta_{1N}H}{\theta_{2N}}\sin\theta_{2N}H\right], \tag{20}$$

and amplitude of reflected Love waves is given by taking modulus of A.

## II MATHEMATICAL COMPUTATION AND GRAPHS

For finding the variation of amplitude with respect to wave number of reflected Love waves , mathematical computation has been done and graph have been drawn by taking barriers of different sizes. For numerical computation, take h = 0.49 km. $\gamma = 2$ , $V_2/V_1 = 3/4$ , $k_{1N} = k_2$, Z = - H and $k_2\delta$ very small. The graphs showing the variation of amplitude versus wave number of reflected Love waves are being plotted by taking the barriers of different sizes and is shown in figures from fig. 2 to fig. 7. Fig. 8 shows the comparison of all the graphs.

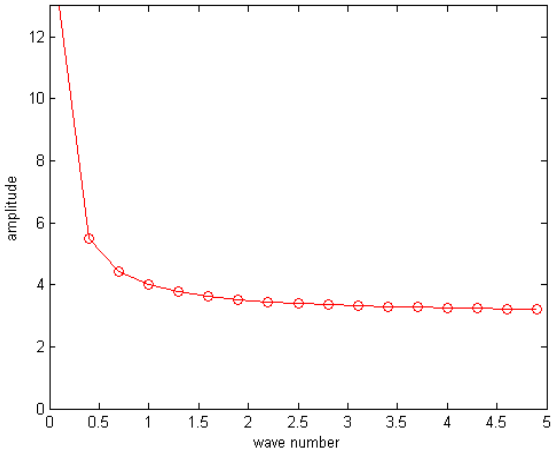


Fig.2. Variation of amplitude versus wave number for $\delta$ = 0.01 km.

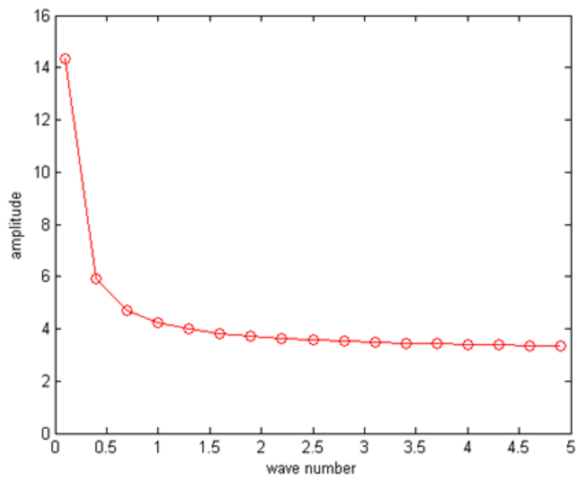


Fig.3. Variation of amplitude versus wave number for $\delta$ = 0.06 km

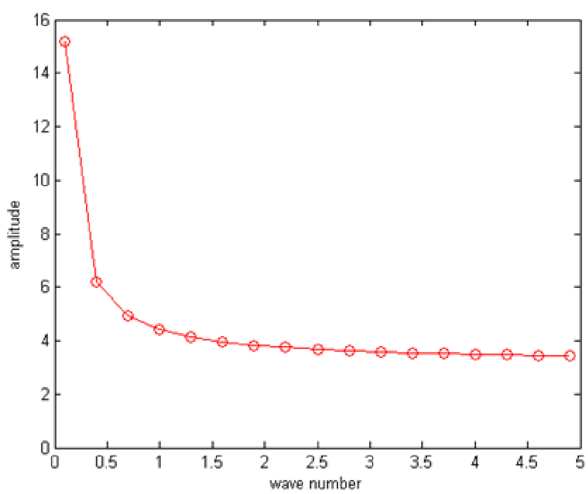


Fig.4. Variation of amplitude versus wave number for $\delta$ = 0.11 km

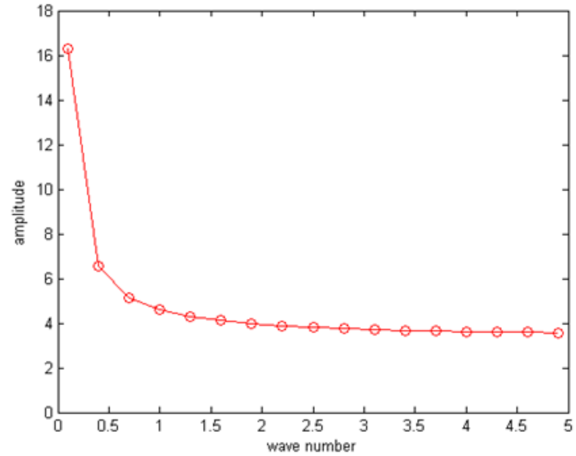


Fig.5. Variation of amplitude versus wave number for $\delta$ = 0.16 km

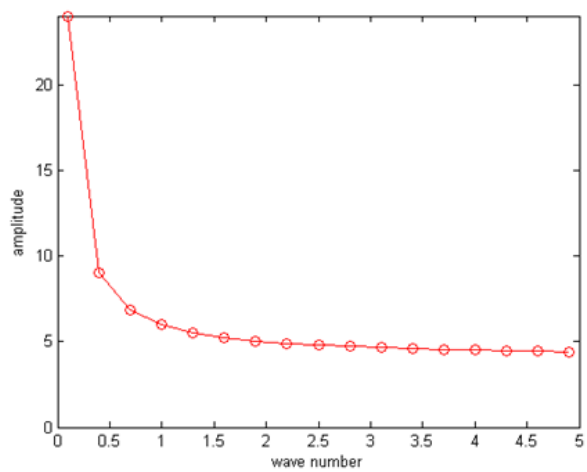


Fig 6. Variation of amplitude versus wave number for $\delta$ = 0.51 km

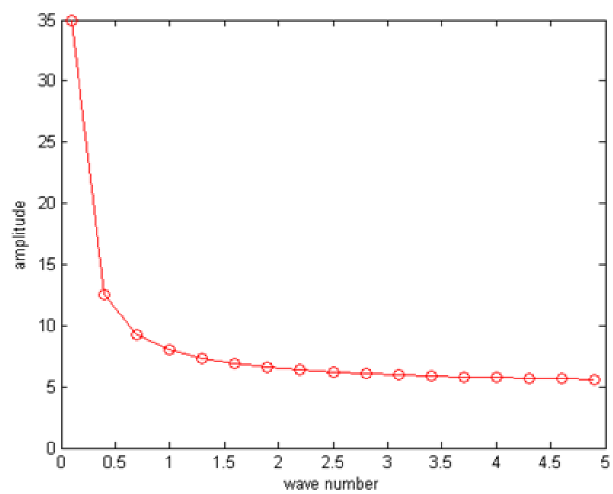


Fig.7. Variation of amplitude versus wave number for $\delta$ = 1.01 km

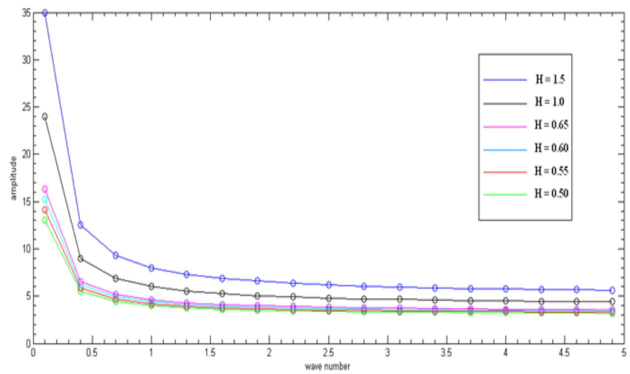


Fig.8. Variation of amplitude versus wave number for different values of δ

## III. Discussion and Conclusions

The amplitude of reflected Love wave is given by (19) by taking its modulus value. The variation of amplitude with wave number of reflected Love waves in the surface layer $-H \leq Z \leq 0$, has been shown by graphs. For computation and graphical purpose we have fixed h = 0.49 km. and graphs have been plotted by taking H = 0.50, 0.55, 0.60, 0.65, 1.00, 1.50 km. From all the graphs plotted we conclude that amplitude of reflected Love waves falls of rapidly as the wave number increases and then it decreases at a very slow rate with the increase in wave number and becomes stable at a particular value showing that the reflected Love waves take a very long time to dissipate making these wave more destructive to human life and buildings. It is also clear from comparison of graphs that amplitude of the reflected Love waves depends upon size of barrier. Taking H = 0.50 km., the amplitude attains saturation at 3.0010 (fig. 2) and for H= 0.55 , it fixes at 3.0030 (fig. 3). If we take H = 1.5, the amplitude becomes approximately stable at 5.0032 (fig.7). The variation of amplitude with wave number by taking barriers of different sizes shows that the behavior of reflected Love waves depends on the size of irregularity. In particular, the theory presented in this paper shows that larger the size of barrier, larger is the amplitude of reflected wave resulting into more energetic reflected Love wave. This explains why the regions with more irregularities in earth surface face frequent earthquake with high intensity.

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