Order Reduction of Linear Interval Systems Using Genetic Algorithm

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Abstract— **This paper presents an algorithm for order reduction of higher order linear interval system into stable lower order linear interval system by means of Genetic algorithm. In this algorithm the numerator and denominator polynomials are determined by minimizing the Integral square error (ISE) using genetic algorithm (GA). The algorithm is simple, rugged and computer oriented. It is shown that the algorithm has several advantages, e.g. the reduced order models retain the steady-state value and stability of the original system. A numerical example illustrates the proposed algorithm.**

I. INTRODUCTION

The analysis and design of practical control systems become complex when the order of the system increases. Therefore, to analyze such systems, it is necessary to reduce it to a lower order system, which is a sufficient representation of the higher order system. In recent decades, much effort has been made in the field of model order reduction for linear dynamic systems and several methods like: Aggregation method [1], Pade approximation [2], Routh approximation [3], Moment matching technique [4], Routh stability technique [5], and L^∞ optimization technique [6], have been proposed. Among them Routh approximation and Pade technique has been recognized as the powerful method. But the serious disadvantage of Pade approximation is that sometimes it leads to an unstable reduced order system for a stable original system. Further, numerous methods of order reduction are also available in the literature [7-9], which are based on minimization of the ISE criterion.

In general, the practical systems have uncertainties about its parameters. Thus practical systems will have coefficients that may vary and it is represented by interval. Interval arithmetic such as addition, subtraction, multiplication and division are discussed in [11]. In [13, 14] model reduction technique for higher order uncertain system were presented using advantage of Routh and Pade approximation methods. The limitations of above method are discussed in [15,16]. A generalized method for constructing the Routh table of interval polynomial is proposed in [15] which overcome some of the limitations of [13, 14].

In recent years, one of the most interesting research fields has been "Evolutionary Techniques", an area utilizing analogies with nature or social systems. Evolutionary techniques are finding popularity within research community as design tools and problem solvers because of their versatility and ability to optimize in complex multimodal search spaces applied to non-differentiable objective functions. Recently, Genetic Algorithm (GA) technique appeared as a promising algorithm for handling the optimization problems. GA can be viewed as a general-purpose search method, an optimization method, or a learning mechanism, based loosely on Darwinian principles of biological evolution, reproduction and ''the survival of the fittest'' [10]. GA maintains a set of candidate solutions called population and repeatedly modifies them. At each step, the GA selects individuals from the current population to be parents and uses them to produce the children for the next generation. In general, the fittest individuals of any population tend to reproduce and survive to the next generation, thus improving successive generations. However, inferior individuals can, by chance, survive and also reproduce. GA is well suited and has been extensively applied to solve complex design optimization problems because it can handle both discrete and continuous variables, non-linear objective and constrained functions without requiring gradient information

In the present work, the paper presents AN algorithm for order reduction of linear interval systems based on minimization of the ISE by genetic algorithm (GA). Algorithm guaranteed the stability of reduced order system provided that original one is stable.

II. REDUCTION ALGORITHM

Consider a high order linear SISO interval system represented by the transfer function as

$$G(s) = \frac{N(s)}{D(s)} \tag{1}$$

$$G(s) = \frac{[c_0^-,c_0^+]+[c_1^-,c_1^+]s+[c_2^-,c_2^+]s^2+\cdots+[c_{n-1}^-,c_{n-1}^+]s^{n-1}}{[d_0^-,d_0^+]+[d_1^-,d_1^+]s+[d_2^-,d_2^+]s^2+\cdots+[d_n^-,d_n^+]s^n} \tag{2}$$

Where $[c_i^-, c_i^+]$, $i = 0,1,2,\cdots,n-1$ and $[d_i^-, d_i^+], i = 0,1,2,\cdots,n$ are the interval coefficients of higher order numerator and denominator polynomials respectively.

The objective is find a r^{th} order reduced interval system.

Let corresponding r^{th} order reduced model is

$$R(s) = \frac{[a_0^-,a_0^+]+[a_1^-,a_1^+]s+[a_2^-,a_2^+]s^2+\cdots+[a_{r-1}^-,a_{r-1}^+]s^{r-1}}{[b_0^-,b_0^+]+[b_1^-,b_1^+]s+[b_2^-,b_2^+]s^2+\cdots+[b_r^-,b_r^+]s^r} \tag{3}$$

Where $[a_i^-, a_i^+]$, $i = 0,1,2,\cdots,r-1$ *and* $[b_i^-, b_i^+], i = 0,1,2,\cdots,r$ are the interval coefficient of lower order numerator and lower order denominator polynomial respectively.

The numerator and denominator coefficients of the reduced order model is determined by minimizing Integral square error between the transient part of step response of original system and reduced system using genetic algorithm.

The deviation of the lower order system from the original system response is given by the error index 'ISE' known as the Integral square error, which is given as follow:

$$ISE = \int_0^\infty [g(t) - r(t)]^2 dt \qquad (4)$$

Where g(t) and r(t) are the unit step response of the original and reduced order systems, respectively.

In this method, GA is employed to minimize the objective function 'ISE' as given in Eq. (4), and the parameter to be determined are the coefficients of the numerator and denominator of the lower order system.

For the purpose of minimization of Eq. (4), routines from GA optimization toolbox are used. For different problems, it is possible that the same parameters for GA do not give the best solution and so these can be changed according to the situation. In Table 1, the typical parameters for GA optimization routines, used in the present study are given. The description of GA operators and their properties can be found in [12].

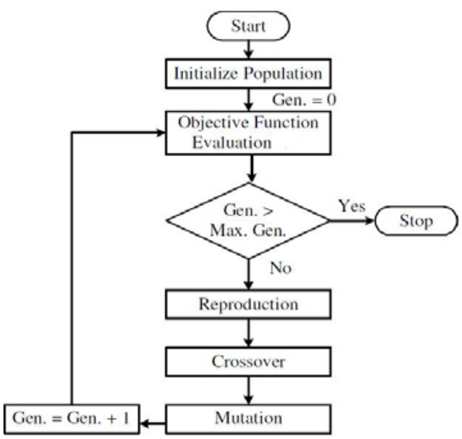

Fig. 1 Flowchart of Genetic algorithm

One more important point that affects the optimal solution more or less is the range for unknowns. For the very first execution of the program, wider solution space can be given and after getting the solution one can shorten the solution space nearer to the values obtained in the previous iteration. The computational flow chart of the proposed algorithm is shown in Fig. 1.

TABLE1
TYPICAL PARAMETERS USED BY THE GENETIC ALGORITHM

Name	Value(type)
Number of generations	200
Population size	100
Type of selection	uniform
Type of crossover	Arithmetic
Type of mutation	uniform
Termination method	Maximum generation

III. NUMERICAL EXAMPLE

Consider a 7th order Interval system transfer function

$$G(s) = \frac{N(s)}{D(s)}$$

Where

$N(s) = [1.9,2.1]s^6 + [24.7,27.3]s^5 + [157.7,174.3]s^4 + [541.975,599.025]s^3 + [929.955,1027.845]s^2 + [721.81,797.79]s + [187.055,206.745]$.

And

$D(s) = [.95,1.05]s^7 + [8.779,9.703]s^6 + [52.231,57.729]s^5 + [182.875,202.125]s^4 + [429.02,474.18]s^3 + [572.47,632.73]s^2 + [325.28,359.52]s + [57.352,63.389]$

By using proposed algorithm, the following reduced 2nd order model is obtained:

$$R_2(s) = \frac{[364.7\ 429.7]s + [271.7,293.2]}{[61.5\ 68.99]s^2 + [255.7\ 347.1]s + [83.8\ 87.67]}$$

The 2nd order reduced model by B. Bandyopadhyay [13] method is also determined-

The γ table for D(s) formed by the algorithm proposed in [13]

[57.35,63.69]	[527.47,632.75]	[182.88,202.13]	[8.78,7.03]
[325.28,359.52]	[429.02,474.18]	[52.23,57.73]	[0.95,1.05]
[434,623.69]	[155.28,214.2]	[7.759,10.56]	
[175.3,564.55]	[30.29,77.08]	[0.662,1.51]	
[-36.94,614.78]	[0.741,32.37]		

The 2nd order system obtained by method [13] is

$$R_{2B}(s) = \frac{[1.16\ 1.84]s + [.27\ .53]}{s^2 + [.52\ ..83]s + [.08\ .16]}$$

It is noted that the lower bound of the interval entry $[d_{51}^{-}, d_{51}^{+}]$ of the γ table is negative, thus restricting the completion of the table. Hence reduced order interval polynomials of degree four or greater cannot be obtained by [13].

IV. Results

A. Checking Robust Hurwitz stability of reduced interval system

Kharitonov [18] stated that an interval family of polynomials $D(s)$ is robustly stable if, and only if, the following Kharitonov polynomials are stable.

$$D^{++}(s) = a_0^{+} + a_1^{+}s + a_2^{-}s^2 + a_3^{-}s^3 + a_4^{+}s^4 + a_5^{+}s^5 + \cdots$$

$$D^{+-}(s) = a_0^{+} + a_1^{-}s + a_2^{-}s^2 + a_3^{+}s^3 + a_4^{+}s^4 + a_5^{-}s^5 + \cdots$$

$$D^{-+}(s) = a_0^{-} + a_1^{+}s + a_2^{+}s^2 + a_3^{-}s^3 + a_4^{-}s^4 + a_5^{+}s^5 + \cdots$$

$$D^{--}(s) = a_0^{-} + a_1^{-}s + a_2^{+}s^2 + a_3^{+}s^3 + a_4^{-}s^4 + a_5^{-}s^5 + \cdots$$

After Anderson and Jury modified this, they stated that

The testing set for an interval polynomial of invariant degree is

$D^{+-}(s)$	for n=3
$D^{+-}(s), D^{++}(s)$	for n=4
$D^{+-}(s), D^{++}(s), D^{-+}(s)$	for n=5
$D^{+-}(s), D^{++}(s), D^{-+}(s), D^{--}(s)$	for n>5

For n=1 and n=2, a necessary and sufficient condition for robust stability is positive lower bounds on the coefficients.

The denominator polynomial of the reduced lower order system is

$$D_2(s) = [61.5\ 68.99]s^2 + [255.7\ 347.1]s + [83.8\ 87.67]$$

n=2, therefore a necessary and sufficient condition for robust stability is positive lower bounds on the coefficients.

$$D_2(s) = 61.5s^2 + 255.7s + 83.8$$

It is clear that $D_2(s)$ is stable. Thus the proposed method guarantees the robust stability of reduced order systems.

B. Simulation Result

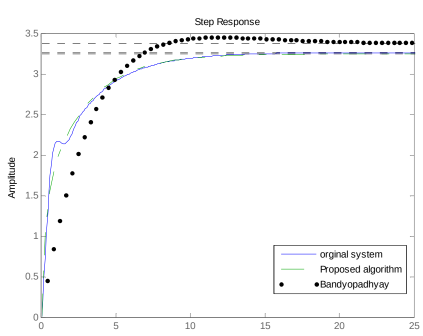

Fig. 2 Comparison of step response for lower limit

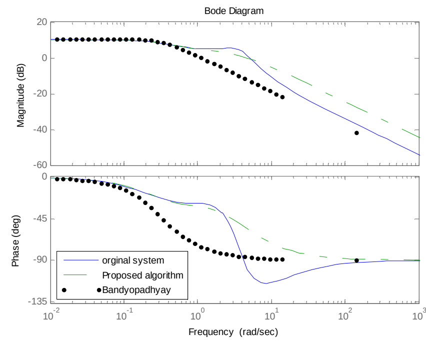

Fig. 3 Comparison of frequency response for lower limit

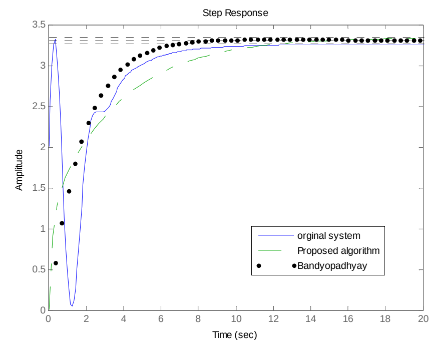

Fig. 4 Comparison of step response for upper limit

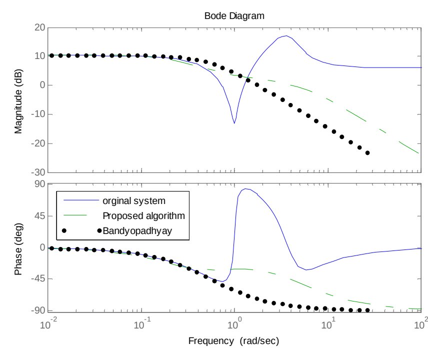

Fig. 5 Comparison of frequency response for upper limit

C. Comparison of Error

TABLE 2
Comparison of Errors

Method of order reduction	Reduced Models $R_2(s)$	ISE for lower limit	ISE for upper limit
Proposed GA Algorithm	$\frac{[364.7\ 429.7]s+[271.7,293.2]}{[61.5\ 68.99]s^2+[255.7\ 347.1]s+[83.8\ 87}$	.062	4.927
B. Bandyopadhyay [13]	$\frac{[1.16\ 1.84]s+[.27\ .53]}{s^2+[.52\ ..83]s+[.08\ .16]}$	2.2599	5.954

V. CONCLUSION

In this paper, an evolutionary method using Genetic algorithm for reducing a high order large scale linear interval system into a lower order interval system has been proposed. Genetic algorithm method based evolutionary optimization technique is employed for the order reduction of Interval Systems where the numerator and denominator polynomials are determined by minimizing an Integral Squared Error (ISE) criterion. The proposed algorithm guarantees stability for a stable higher order linear Interval system and thus any lower order Interval model can be derived with good accuracy. The reduction of seventh order interval system to second order interval system gives better step as well as frequency responses than the B.Bandyopadhyay [13].

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