Analysis of Dynamic Model of a Structure with Nonlinear Damped Behavior

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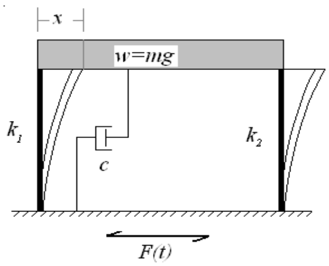
***Abstract*- In this work, it has been attempted to analytically treat the nonlinear behavior of structures. Since analysing nonlinear problems is of great difficulty, different numerical methods and software are advised to treat such problems. Despite the increasing expenses of building structures to maintain their linear behavior, nonlinearity has been inevitable, and therefore, nonlinear analysis has been of great importance to the scientists in the field. As structures confront lateral forces and intense earthquakes especially near fault regions, a part of the structure remains linear, but some part of it behaves nonlinearly for example dampers, columns and beams. This is simulated by a damped in nonlinear oscillator. In this paper, the nonlinear equation of oscillator with damping which has nonlinear behavior is representative of the dynamic behavior of a structure has been solved analytically. In the end, the obtained results are compared with numerical ones and shown in graphs and in tables; analytical solutions are in good agreement with those of the numerical method.**

***Keywords**: Nonlinear Oscillator Equation; nonlinear Damping; Nonlinearly Dynamic Structure*

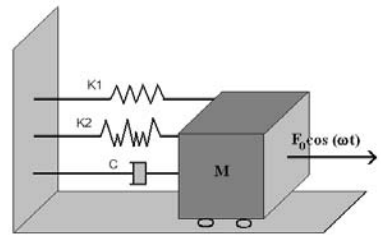
1. Introduction

Until recently, different numerical methods have been implemented to solve the problem of a nonlinear oscillator (which represents the nonlinear behavior of a structure under dynamics loads). But in this project, it has been attempted to propose an analytic solution for such problem, which is much simpler for engineers to interpret and to use in their designs. This is because an equation is obtained rather than only some data.

In the dynamic model of this problem, the earthquake force has been modelled with a harmonic force and the columns with nonlinear behavior are modelled with the spring k_2 and the columns with linear behavior are modelled with the spring k_1. The coefficient β_1, β_2 represents the nonlinear behavior of damping which roots in the joints, the material and other parameters [1-5].To fully demonstrate the problem, let us consider a structure whose columns are under the harmonic load (e.g. earthquake). This load results in a nonlinear behavior in a part of the structure, while another part still behaves linearly. The important point is the analysis of such system to obtain the displacement equation, which is extremely useful to study the structure.



(a)



(b)

Figure 1. (a) Schematic view of a structure under harmonic load.
(b) The dynamic model of a structure under harmonic load

The analytical method which is used in this article is Homotopy Perturbation method (HPM) [6-16]. This method has been implemented to many engineering problems by many other scientists in different fields [17-29]. This method is capable of solving highly nonlinear problems while the constant coefficients are parametrically inserted into the equation. Therefore, the obtained results can be graphically shown and analysed for different cases, and by inserting different values for these parameters regarding each single case of study.

In the end, a comparative study is conducted to verify the accuracy of the analytical method as compared with the numerical solution. This is also shown in graphs and tables.

2. Dynamic and mathematical model of the problem [30-34]

The general equation of an oscillator with a nonlinear spring, a linear spring and a nonlinear damper under a harmonic load is as follows [34]:

$$m\ddot{x} + (\beta_1 + \beta_2 x^2)\dot{x} + k_1 x + k_2 x^3 = F_0 \cos(\omega t) \qquad (2.1)$$

Subject to the following initial conditions:

$$x(0) = A, \qquad \dot{x}(0) = 0 \qquad (2.2)$$

Where m is the mass, β_1 and β_2 are damping coefficients, k_1 is a linear stiffness coefficient, and k_2 is a nonlinear stiffness coefficient. The harmonic excitation force is characterized by the force amplitude, F_0 with excitation frequency of ω. A is the initial amplitude of displacement.

As in [31], ω can be found easily by doing the same procedure and having the parameters, A, m, β_1, β_2, k_1, k_2 and F_0.

Figure 2 shows how the stiffness coefficients of nonlinear and linear springs behave, where $f(x)$ is the spring force and x is the displacement:

In the following section the basic concepts of the analytical and numerical methods are discussed and later applied to the nonlinear equation, above.

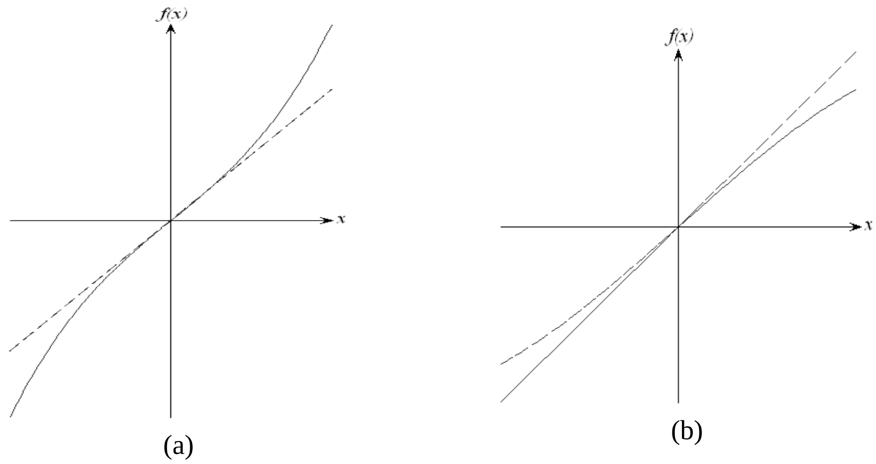


Figure 2. (a) Hard spring stiffness nonlinear behavior.
(b) Soft spring stiffness nonlinear behavior.

3. The basic concept of the solutions

In this section, the basic of the utilized methods are explained for the better understanding of the reader.

3.1. HPM

To illustrate the basic ideas of this method, we consider the following equation:

$$A(x) - f(r) = 0 \quad r \in \Omega \tag{3.1}$$

with the boundary condition of:

$$B\left(x, \frac{\partial x}{\partial t}\right) = 0 \quad r \in \Gamma \tag{3.2}$$

Where A is a general differential operator, B a boundary operator, $f(r)$ a known analytical function and Γ is the boundary of the domain Ω. A can be divided into two parts of L and N, where L is linear and N is nonlinear. Eq. (3.1) can therefore be rewritten as follows:

$$L(x) + N(x) - f(r) = 0 \quad r \in \Omega \tag{3.3}$$

Homotopy perturbation structure is shown as follows:

$$H(v, p) = (1-p)\left[L(v) - L(x_0)\right] + p\left[A(v) - f(r)\right] = 0 \tag{3.4}$$

Where,

$$v(r,p): \ \Omega \times [0,1] \to R \tag{3.5}$$

In Eq. (3.4), $p \in [0, 1]$ is an embedding parameter and x_0 is the first approximation that satisfies the boundary condition. We can assume that the solution of Eq. (3.1) can be written as a power series in p, as following:

$$v = v_0 + p v_1 + p^2 v_2 + \cdots = \sum_{i=0}^{n} v_i p^i \tag{3.6}$$

And the best approximation for the solution is:

$$x = \lim_{p \to 1} v = v_0 + v_1 + v_2 + \cdots \tag{3.7}$$

3.2. Runge-Kutta

For the numerical approach to verify the analytic solution, the fourth RK (Runge-Kutta) method has been used. This iterative algorithm is written in the form of the following formulae for the second-order differential equation:

$$\dot{x}_{i+1} = \dot{x}_i + \frac{\Delta t}{6}(h_1 + 2h_2 + 2h_3 + k_4)$$
$$x_{i+1} = x_i + \Delta t(\dot{x}_i + \frac{\Delta t}{6}(h_1 + h_2 + k_3)) \qquad (3.8)$$

Where, Δt is the increment of the time and h_1, h_2, h_3, and h_4 are determined from the following formulae:

$$h_1 = f(\dot{x}, x_i, \dot{x}_i)k,$$
$$h_2 = f(t_i + \frac{\Delta t}{2}, x_i + \frac{\Delta t}{2}\dot{x}_i, \dot{x}_i + \frac{\Delta t}{2}h_1),$$
$$h_3 = f(t_i + \frac{\Delta t}{2}, x_i + \frac{\Delta t}{2}\dot{x}_i, \frac{1}{4}\Delta t^2 h_1, \dot{x}_i + \frac{\Delta t}{2}h_2), \qquad (3.9)$$
$$h_4 = f(t_i + \Delta t, x_i + \Delta t\dot{x}_i, \frac{1}{2}\Delta t^2 h_2, \dot{x}_i + \Delta t h_3).$$

The numerical solution starts from the boundary at the initial time, where the first value of the displacement function and its first-order derivative are determined from initial condition. Then, with a small time increment Δt, the displacement function and its first-order derivative at the new position can be obtained using Eq. (3.8). This process continues to the end of the time limit.

4. The Solutions

In this section the applications of the two methods to the nonlinear equation of oscillator are discussed.

4.1. HPM (Analytic)

As the HPM was applied to the nonlinear equation of (2.1), we have:

$$(1-p)(m\ddot{x} + \beta_1\dot{x} + k_1 x) + p(m\ddot{x} + (\beta_1 + \beta_2 x^2)\dot{x} + k_1 x + k_2 x^3 - F_0\cos(\omega t)) = 0 \qquad (4.1)$$

After expanding the equation and collecting it based on the coefficients of p -terms, we have:

$$\begin{cases} p^0 : m\ddot{x}_0 + \beta_1\dot{x}_0 + k_1 x_0 = 0 \\ p^1 : m\ddot{x}_1 + (\beta_1 + \beta_2 x_0^2)\dot{x}_1 + k_1 x_1 + k_2 x_0^3 - F_0\cos(\omega t) = 0 \\ P^2 : m\ddot{x}_2 + \beta_1\dot{x}_2 + \beta_2 x_0^2\dot{x}_1 + 2\beta_2 x_0 x_1 \dot{x}_0 + k_1 x_2 + 3k_2 x_0^2 x_1 = 0 \\ P^3 : .... \end{cases} \qquad (4.2)$$

One can now try to obtain the solution of different iterations (4.2),in the form of :

$$x_0(t) = \frac{1}{2}\frac{(-\beta_1^2 + 4k_1 m - \beta_1\sqrt{\beta_1^2 - 4k_1 m})e^{\frac{1}{2}\frac{(-\beta_1 + \sqrt{\beta_1^2 - 4k_1 m})t}{m}}}{-\beta_1^2 + 4\omega^2 m}$$
$$+ \frac{1}{2}\frac{(-\beta_1^2 + 4k_1 m + \beta_1\sqrt{\beta_1^2 - 4k_1 m})e^{-\frac{1}{2}\frac{(-\beta_1 + \sqrt{\beta_1^2 - 4k_1 m})t}{m}}}{-\beta_1^2 + 4k_1 m} \qquad (4.3)$$

The obtained iteration is used to generate the equation for the next iteration, and therefore the second and third iterations are obtained. Since the two other ones and therefore the general solution are too long to be written in this article, we have shown them in graphs.

In table 1, the numerical values for x and $\dot{x}$ for different points of time and for $f = 1,\ A = 0.04, \beta_1 = 7, \beta_2 = 0.02, \omega = 3.536163732$.

Table 1. The numerical values for x and $\dot{x}$ for eleven different points of time (Analytic), for $f=1,\ A=0.04, \beta_1=7, \beta_2=0.02, \omega=3.536163732$.

t	x	$\dot{x}$
0	0.04	0
1	-0.0047900421	-0.0146147286
2	0.0251075641	0.1111212726
3	-0.0343846581	-0.0715226752
4	0.0396777396	0.0187245327
5	-0.0386351750	0.0365470429
6	0.0316988911	-0.0862762009
7	-0.0198832285	0.1227331312
8	0.0050134361	-0.1403312942
9	0.0106270740	0.1363633189
10	-0.0246343983	-0.1114395304

4.2. Runge-Kutta (Numerical)

In this section, the Maple Package has been utilized for the numerical analysis of the problem, in which the Rkf45 is used to solve ODEs. The solution for the displacement and the velocity for eleven different points of time are shown in table 2.

Table 2. The numerical values for x and $\dot{x}$ for eleven different points of time (Numerical), for $f=1,\ A=0.04, \beta_1=7, \beta_2=0.02, \omega=3.536163732$.

t	x	$\dot{x}$
0	0.04	0
1	-0.0047905671	-0.1461467333
2	0.0251075801	0.1111212105
3	-0.0343847766	-0.0715221141
4	0.0396777432	0.0187237632
5	-0.0386350967	0.0365478116
6	0.0316987491	-0.0862767370
7	-0.0198830505	0.1227332923
8	0.0050132652	-0.1403312934
9	0.0106272300	0.1363632356
10	-0.0246345296	-0.1114393022

5. Results and discussions

In this section, the results for displacement and the velocity for different times are shown in tables 3 and 4, for different f 's and A 's, in order to evaluate the accuracy of the analytic solution.

As it is obviously seen, the results of the analytic and numerical approaches have shown excellent compatibility. In order to have a better scheme of the problem, displacement x is shown in figure 3 based on time, for ten seconds (different f 's and A 's are assumed).

In the figure 4, the velocity of each position is drawn versus its position; therefore, the velocity of any specific point x can be easily read. This can only be done using the analytic method; since the equation of displacement is readily given by this method, the first and second differentiations can be simply done by differentiating with respect to t.

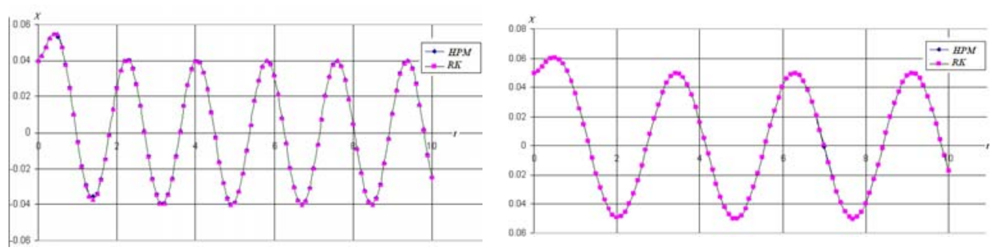
Also using figure 5, the acceleration of any specific point x can be easily read. As mentioned earlier, this can only be done using the analytic approach.

The important point which cannot be seen on the figures of $(\dot{x} - x)$ and $(\ddot{x} - x)$ is that the starting part of these diagrams which refers to the times between t 's from 0 to 1, is not drawn. The reason is that in this period of time, the behavior of the displacement equation has not yet become harmonic, and therefore, the velocity and acceleration is not in the rage of the above diagrams.

6. Conclusions

As structures are exposed to lateral harmonic forces and intense earthquakes, parts of the structure remains linear, but some parts of it inevitably behave nonlinearly; this is simulated by a nonlinear damped in nonlinear oscillator. In this work, HPM which is a new analytical method has been applied to the nonlinear equation of an oscillator with nonlinear damped, and the results have been compared with that of the numerical solution. The results have shown good agreement with the numerical ones. Having obtained the displacement equation, one is able to determine the velocity and acceleration equations. The target of the present work was to determine the displacement, velocity and acceleration equations of the structure under the specified harmonic load, which gives a better viewpoint for engineering design to scientist in the field. The obtained displacement equation can be used by designers to minimize displacements.

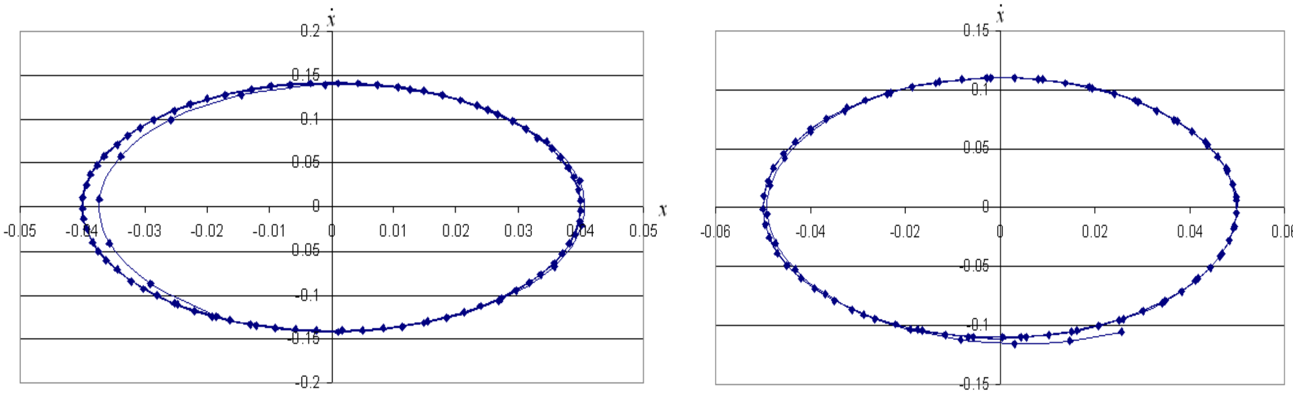
The main advantage of applying HPM is that the results are readily obtained and a few iterations are used. The significant merit of the analytic approach is to provide scientists with the general parametric relation between the dependent and independent variables, namely, displacement and time, respectively. Therefore, the related equations can be simply obtained, giving one the opportunity for further studies, for different cases and thereby different parameters.



(a) (b)

Figure 3. Displacement x based on time t for

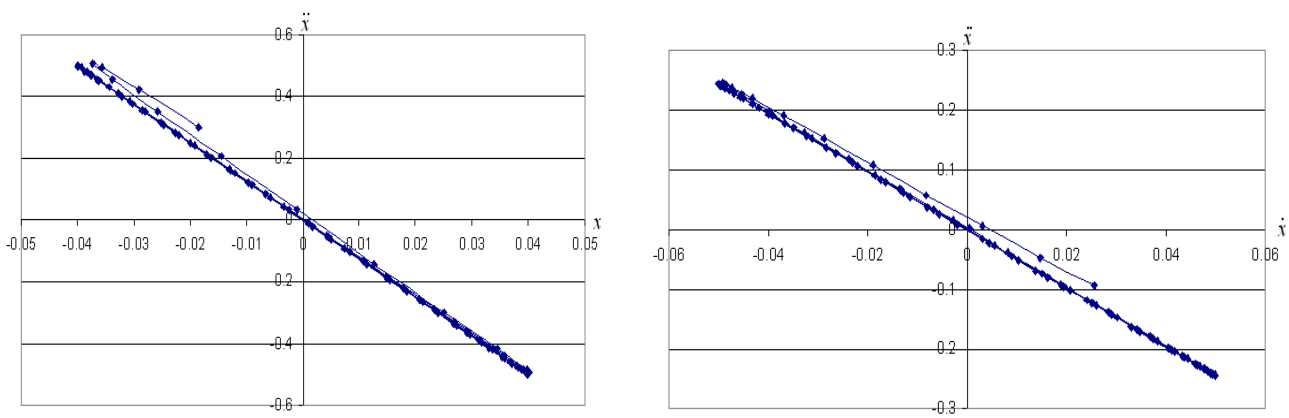
(a) $f = 1,\ A = 0.04,\ \omega = 3.536163732$ and (b) $f = 0.8,\ A = 0.05,\ \omega = 2.208420786$.



(a) (b)

Figure 4. Velocity $\dot{x}$ based on displacement x for

(a) $f=1,\ A=0.04,\ \omega=3.536163732$ and (b) $f=0.8,\ A=0.05,\ \omega=2.208420786$.



(a) (b)

Figure 5. Acceleration $\ddot{x}$ based on displacement x for

(a) $f=1,\ A=0.04,\ \omega=3.536163732$), (b) $f=0.8,\ A=0.05,\ \omega=2.208420786$.

Table 3. A comparative table for error detection of the analytic method, for $f = 1,\ A = 0.04,\ \omega = 3.536163732$.

	x		$\dot{x}$	
t	HPM	RKf45	HPM	RKf45
0	0.04	0.04	0	0
1	-0.0047900421	-0.0047900421	-0.0146147286	-0.1461467333
2	0.0251075641	0.0251075641	0.1111212726	0.1111212105
3	-0.0343846581	-0.0343846581	-0.0715226752	-0.0715221141
4	0.0396777396	0.0396777396	0.0187245327	0.0187237632
5	-0.0386351750	-0.0386351750	0.0365470429	0.0365478116
6	0.0316988911	0.0316988911	-0.0862762009	-0.0862767370
7	-0.0198832285	-0.0198832285	0.1227331312	0.1227332923
8	0.00501343617	0.00501343617	-0.1403312942	-0.1403312934
9	0.0106270740	0.0106270740	0.1363633189	0.1363632356
10	-0.0246343983	-0.0246343983	-0.1114395304	-0.1114393022

Table 4. A comparative table for error detection of the analytic method, for $f = 0.8,\ A = 0.05,\ \omega = 2.208420786$.

	x		$\dot{x}$	
t	HPM	RKf45	HPM	RKf45
0	0.05	0.05	0.00	0.00
1	0.0357889238	0.0357877175	-0.0944705259	-0.0944701811
2	-0.0491327293	-0.0491327017	-0.0052798774	-0.0052781411
3	0.0284987344	0.0284989920	0.0907084539	0.0907078760
4	0.0162529086	0.0162523396	-0.1044988428	-0.1044984371
5	-0.0476567523	-0.0476563249	0.0333790354	0.0333803038
6	0.0405213404	0.0405215086	0.0646987668	0.0646977378
7	-0.0005805570	-0.0005810284	-0.1104188261	-0.1104182780
8	-0.0398289599	-0.0398283828	0.0667613105	0.0667618788
9	0.0480002055	0.0480001785	0.0309339902	0.0309325437
10	-0.0173189259	-0.0173193016	-0.1035906579	-0.1035901309

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