Supply Chain with Several Competing Retailers and One Manufacturer

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Abstract - In this paper, we use algebraic method to find the optimal solution for a supply chain that will help practitioners without the knowledge of calculus to learn inventory models. Through our approach, we find three necessary conditions to ensure the positivity of the maximum value.

Keywords —Supply chain, Competing retailers, Demand disruption

I. INTRODUCTION

Xiao and Qi (2008) published a paper related to a supply chain with one manufacturer and two competing retailers for price competition, cost and demand disruptions and coordination. Their significant article has been cited for 221 times. We just list some of them for the last few years. Guohua (2013) worked on research on the fresh agricultural product supply chain coordination with supply disruptions. Malik et al. (2013) investigated real world practices in electric fan manufacturing industry of Pakistan. MacKenzie et al. (2014) modeled a severe supply chain disruption and post-disaster decision making with application to the Japanese earthquake and tsunami. Mahmoodi and Eshghi (2014) examined price competition in duopoly supply chains with stochastic demand. Chen et al. (2015) developed robust supply chain strategies for recovering from unanticipated disasters. Hafezalkotob and Ghezavati (2015) considered distribution network design of a decentralized supply chain with fuzzy committed distributors. Mahdiraji et al. (2015) studied game theoretic approach for coordinating unlimited multi echelon supply chains. Konur and Geunes (2016) examined decisions for price for a supplier within a region for localized retail stores to seek for the optimal strategy under a decentralization environment. Mohammadzadeh and Zegordi (2016) examined a two-level supply chain under an unreliable supplier with a manufacturer and three suppliers. They constructed two competitive bi-level models to study a cooperative approach among manufacturer and reliable suppliers. Sluis and De Giovanni (2016) tried to look for the key drivers to decide a supply chain to examine their performance. They constructed logistic regression models to obtain relation among parameters. Soleimani et al. (2016) investigated pricing management supply chain problems in both centralized and decentralized conditions. They showed that production cost and demand disruptions are dependent with each other. MacKenzie and Apte (2017) considered a fresh produce supply chain with disruptions. They constructed a mathematical model to decide the optimal safety stock and then examined by a practical application for coli spinach contamination reveals. Mardan et al. (2017) solved a multi-period, multi-product inventory system under a fuzzy environment for a risk-averse decision maker to study emergency ordering and product substitution. Adediran and Al-Bazi (2018) examined manufacturing flow-shops under unpredictable disruptions with recovery strategy to develop a heuristic optimization system to present practical information for decision-makers. Ali et al. (2018) studied Supply chain management to consider disruptions. They derived both a centralized and a decentralized supply chain structure to solve a real-life problem. Feyzian-Tary et al. (2018) formulated supply chain structures to examine market share and competition to maximize the future revenue under Nash equilibrium conditions. They derived qualitative properties of the equilibrium pattern. Chernonog and Avinadav (2019) developed a two-echelon supply chain with a manufacturer and a retailer under the condition, that the retailer have the knowledge of the demand distribution. Sacco and De Giovanni (2019) constructed two models to study coordination among manufacturer and retailers. They compared their findings with classical models. Rahmani and Yavari (2019) studied decision policy under disruption for a dual-channel green supply chain. Their supply chain contains green products with one manufacturer and one retailer. The above brief review, this research issue is a prevailing topic. Therefore, we will follow this tendency to study this supply chain problem with algebraic method to locate the maximum solution without using differential calculus. Consequently, some researchers did not have the background of calculus and then still can absorb this imperative inventory model.

II. REVIEW OF XIAO AND QI (2008)

Xiao and Qi (2008) wanted to maximize

$$\pi(p_1, p_2) = (p_1 - c_1 - c_0)(a - p_1 + dp_2) + (p_2 - c_2 - c_0)(a - p_2 + dp_1) \quad (1)$$

by analytic approach. In this paper, we will apply algebraic method to locate the optimal solution of p_1 and p_2 with the maximum value.

For completeness, we provide an analytic approach for researchers.

$$\frac{\partial \pi(p_1, p_2)}{\partial p_1} = -(p_1 - c_1 - c_0) + (a - p_1 + dp_2) + d(p_2 - c_2 - c_0), \quad (2)$$

and

$$\frac{\partial \pi(p_1, p_2)}{\partial p_2} = d(p_1 - c_1 - c_0) - (p_2 - c_2 - c_0) + (a - p_2 + dp_1). \quad (3)$$

To solve the system of $\frac{\partial \pi(p_1, p_2)}{\partial p_1} = 0$ and $\frac{\partial \pi(p_1, p_2)}{\partial p_2} = 0$, we are facing the following problem:

$$-2p_1 + 2dp_2 = d(c_2 + c_0) - c_1 - c_0 - a, \quad (4)$$

and

$$2dp_1 - 2p_2 = d(c_1 + c_0) - c_2 - c_0 - a. \quad (5)$$

Based on Equations (4) and (5), we can imply that

$$-p_1 + dp_2 = \frac{d(c_2 + c_0)}{2} - \frac{c_1 + c_0 + a}{2}, \quad (6)$$

and

$$p_1 - \frac{p_2}{d} = \frac{(c_1 + c_0)}{2} - \frac{c_2 + c_0 + a}{2d}. \quad (7)$$

We add Equation (6) with Equation (7) to obtain that

$$\frac{d^2 - 1}{d} p_2 = (d^2 - 1)\frac{(c_2 + c_0)}{2d} - \frac{(d+1)a}{2d}. \quad (8)$$

We further simply Equation (8) to obtain that

$$p_2 = \frac{(c_2 + c_0)}{2} + \frac{a}{2(1-d)}. \quad (9)$$

By the similar method, we can also rewrite Equations (4) and (5) as follows,

$$-\frac{p_1}{d} + p_2 = \frac{(c_2 + c_0)}{2} - \frac{c_1 + c_0 + a}{2d}, \quad (10)$$

and

$$dp_1 - p_2 = \frac{d(c_1 + c_0)}{2} - \frac{c_2 + c_0 + a}{2}. \quad (11)$$

We add Equation (10) with Equation (11) to obtain that

$$\frac{d^2 - 1}{d} p_1 = (d^2 - 1)\frac{(c_1 + c_0)}{2d} - \frac{(d+1)a}{2d}. \quad (12)$$

We further simply Equation (12) to derive that

$$p_1 = \frac{(c_1 + c_0)}{2} + \frac{a}{2(1-d)}. \qquad (13)$$

After finding the optimal solutions: $p_1^* = \frac{(c_1 + c_0)}{2} + \frac{a}{2(1-d)}$, of Equation (13) and $p_2^* = \frac{(c_2 + c_0)}{2} + \frac{a}{2(1-d)}$ of Equation (9), researchers should begin to compute the optimal value, $\pi(p_1^*, p_2^*)$ as follows,

$$\pi(p_1^*, p_2^*) = \left(\frac{a}{2(1-d)} - \frac{c_1 + c_0}{2}\right)$$
$$+\left(\frac{a}{2} - \frac{c_1 + c_0}{2} + \frac{d(c_2 + c_0)}{2}\right)$$
$$+\left(\frac{a}{2(1-d)} - \frac{c_2 + c_0}{2}\right)$$
$$\left(\frac{a}{2} - \frac{c_2 + c_0}{2} + \frac{d(c_1 + c_0)}{2}\right). \qquad (14)$$

The above expression containing negative sign, such that the positivity is not guaranteed. We must directly point out that Xiao and Qi (2008) did not discuss $\pi(p_1^*, p_2^*)$. In this paper, we show that to guarantee the result of Equation (14) being a positive value for the maximum profit, then there are three additional conditions that should be required.

III. OUR ALGEBRA APPROACH

We rewrite Equation (1) in the descending order of p_1 to find that

$$\pi(p_1, p_2) = -p_1^2 +$$
$$[(a + c_1 + c_0) + d(2p_2 - c_2 - c_0)]p_1$$
$$-(c_1 + c_0)(a + dp_2)$$
$$+(p_2 - c_2 - c_0)(a - p_2). \qquad (15)$$

We complete the square for p_1 to imply that

$$\pi(p_1, p_2) =$$
$$-\left[p_1 - \frac{a + c_1 + c_0 + d(2p_2 - c_2 - c_0)}{2}\right]^2$$
$$+\frac{[(a + c_1 + c_0) + d(2p_2 - c_2 - c_0)]^2}{4}$$
$$-(c_1 + c_0)(a + dp_2)$$
$$+(p_2 - c_2 - c_0)(a - p_2). \qquad (16)$$

We observe that the coefficient being -1 of $\left[p_1 - \frac{a + c_1 + c_0 + d(2p_2 - c_2 - c_0)}{2}\right]^2$ which is less than zero. To attain the maximum value, we should have

$$p_1 = \frac{a + c_1 + c_0 - d(c_2 + c_0)}{2} + dp_2. \qquad (17)$$

We plug the results of Equation (17) into Equation (16), then we arrange it in the descending order of p_2, to obtain that

$$\pi(p_2) = -(1-d^2)p_2^2 + \left[a + c_2 + c_0 + da - d^2(c_2 + c_0)\right]p_2 + \frac{1}{4}\left[a + c_1 + c_0 - d(c_2 + c_0)\right]^2 - a(c_2 + c_1 - 2c_0). \quad (18)$$

We complete the square of p_2 to derive that

$$p_2 = \frac{a}{2(1-d)} + \frac{c_2 + c_0}{2}. \quad (19)$$

We plug the results of Equation (19) into Equation (17) to imply that

$$p_1 = \frac{a}{2(1-d)} + \frac{c_1 + c_0}{2}. \quad (20)$$

Our findings of Equations (19) and (20) are the same as that derived by an analytic approach as we demonstrate in Equations (9) and (13).

Next, we will begin to find the maximum value.

$$\begin{aligned}
\pi(p_1^*, p_2^*) = &\left(\frac{a}{2(1-d)} - \frac{c_1 + c_0}{2}\right)\left(\frac{a}{2} + \frac{d(c_2 + c_0) - (c_1 + c_0)}{2}\right) \\
&+ \left(\frac{a}{2(1-d)} - \frac{c_2 + c_0}{2}\right) \\
&\left(\frac{a}{2} + \frac{d(c_1 + c_0) - (c_2 + c_0)}{2}\right) \\
= &\frac{a^2}{2(1-d)} - \frac{a}{2}(c_1 + c_2 + 2c_0) \\
&- \frac{d}{2}(c_1 + c_0)(c_2 + c_0) \\
&+ \frac{(c_1 + c_0)^2}{4} + \frac{(c_2 + c_0)^2}{4} \\
= &\frac{1}{4}(a - c_1 - c_0)^2 + \frac{1}{4}(a - c_2 - c_0)^2 \\
&+ \frac{d}{2(1-d)}a^2 - \frac{d}{2}(c_1 + c_0)(c_2 + c_0). \quad (21)
\end{aligned}$$

The first three terms of Equation (21) are squares such that they are positives numbers. However, the fourth term of Equation (21) contains a negative sign. To ensure the positivity of Equation (21), we must add extra conditions.

If we neglect $1-d$ from the denominator of the third term of Equation (21), and then we can rewrite the third and fourth terms of Equation (21) as follows,

$$\frac{d}{2}\left[a^2 - (c_1 + c_0)(c_2 + c_0)\right]. \quad (22)$$

Hence, we will need

$$d > 0, \qquad (23)$$

$$1 - d > 0, \qquad (24)$$

and

$$a^2 > (c_1 + c_0)(c_2 + c_0). \qquad (25)$$

Now we observe Equations (23) and (24) to imply that they should be merge into one compact expression. On the other hand, the condition of Equation (25) had better break into two parts such that $a > c_1 + c_0$ and $a > c_2 + c_0$ are both valid.

From our above derivation, we need three extra conditions:

$$1 > d > 0, \qquad (26)$$

$$a > c_1 + c_0, \qquad (27)$$

and

$$a > c_2 + c_0. \qquad (28)$$

Based on our findings of Equations (27) and (28), we rewrite Equation (21) as

$$\pi(p_1^*, p_2^*) = \frac{1}{4}(a - c_1 - c_0)^2 + \frac{1}{4}(a - c_2 - c_0)^2 + \frac{d}{2}\left[a^2 - (c_1 + c_0)(c_2 + c_0)\right] + \frac{d^2}{2(1-d)}a^2, \qquad (29)$$

to obtain the maximum profit.

IV. NUMERICAL EXAMPLES

From Xiao and Qi (2008), we find the following data: $a = 20$, $c_0 = 5$, $c_1 = 3$, $c_2 = 2$ and $d = 0.5$. Hence, our extra three conditions of Equations (26), (27) and (28) are supported by the numerical examples of Xiao and Qi (2008). Therefore, we receive a strong support from Xiao and Qi (2008) for our added three new restrictions that was overlooked by Xiao and Qi (2008).

V. CONCLUSION

We examined an important supply chain to find three extra conditions that was satisfied by numerical examples of Xiao and Qi (2008). Hence, our findings not only provided an algebraic approach but also to point out three necessary criteria to validate the positivity of the derived maximum point and maximum value.

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