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A multi-period capacity reservation contract model using sampling average approximaion

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Abstract - We consider a multi-period capacity reservation contract practiced between a buyer and multiple suppliers, where the buyer buys multiple types of product and sells it to end-customers, and the suppliers produces and replenishes the products as agreed upon contractually.

In this paper, we developed mathematical model to detemine the optimal capacity reservation limits on a rolling-horizon basis. We use sampling average approximation method Numerical experiments based on a test problem motivated from apparel industry are provided.

Keywords - Supply chain management; Stochastic programming

I. INTRODUCTION

It is common practice in many industries to use a replenishment contract with a mechanism of capacity reservation. Although there are several forms of capacitiy reservation contracts, the problem discussed in this paper is the one used in apparell industry. Most of apparel firms are fabless and make outsourcing deals for productions with factories of their subcontractors. Every month, firms make a reservation for production lines and manpower in the factories of suppliers. The reservation is made a few months in advance based on a demand prediction and a monthly production planning at that time. After observing the actual demand, firms make a purchase replenishment order within a upper and lower limit of reserved production capacity. Typically apparel firms a variety of products with different selling seasons, such as (AB) all-season basic product, (SS) spring/summer-season product, (FW) fall/winter-season product, and (SP) spot-demand product, and thus, detemining future capacity range considering demand uncertainty with different selling seasons is very difficult.

Early research of the problem is based on two-stage models. These models include backup agreement [1], quick response [2] buy back [3], minimum commitment [4], quantity flexibility [5]-[8] and revenue-sharing [12] contracts.

There are several multi-stage models. Themse models include Long-term contract [9], Lower limit [10], Cancellation [11], Nonstationary demand [12] – [14], Spot market demand [15], rolling-horizon implementation strategy [16]. Even with these progress, however, they only deal with a single type product. To the best of our knowledge, our paper is the first to consider the capacity reservation problem under multi-period multi-prodution multi-supllier settings. By doing so, solving the problem is much more difficult.

In this paper, we developed mathematical model to detemine the optimal capacity reservation limits on a rolling-horizon basis. Numerical experiments based on a test problem motivated from apparel industry are provided.

The reminder of paper is as follows. In section 2, we give a proposed model and the solution algorithm based on sampling average approximation. In section 3, we describe the numerical exiperiments, and in section 4, we give a conclusion.

II. PROPOSED MODEL

A. Assumption

We model that a buyer buys multiple types of products $i = 1, \cdots, n$ from suppliers $j = 1, \cdots, m$ and sells it to end-customers. For each period, the buyer observes demand from customers. Let d_{it} denote demand of product i at period t, and x_{it} denote inventory amount of product i at period t. If the buyer has sufficient amount of inventory $x_{it} \geq d_{it}$, the buyer sells the amount d_{it} and if $x_{it} < d_{it}$, the buyer sells the amount x_{it}. Let s_{it} be the lost-demand of product i at period t and p_{it} be the per-unit revenue of product i. The lost-sales of produt i at period t is $p_{it} \times s_{it}$ with $s_{it} = \max(d_{it} - x_{it}, 0)$.

We assume that a known deterministic production lead-time, call it L, and for a production request made at period t, supplier j completes the production at time $t+L$. Therefore, inventory dynamics is modeld as

$$x_{it} = x_{i(t-1)} - d_{it} + \sum_{j=1}^{m} y_{ij(t-L)} + s_{it}, \quad \forall i, j, t$$

where y_{ijt} is the order quantity of product i to supplier j at period t.

We assume that a known reservation term B, and for each period, say t, the buyer makes a capacity-reseravation to supplier j with respect to the upper limit $u_{(t+B)j}$ and lower limit $l_{(t+B)j}$. After the capacity-reservation at period $t+B$ is determined, order-quantity at period $t+B$ can be changed only in the booked range. This reserved capacity limit impose the constraint at period $t+B$,

$$l_{j(t+B)} \le \sum_{i=1}^{n} a_i y_{ij(t+B)} \le u_{j(t+B)},$$

where a_i is per-unit man-hour of producing product i. We assume there is a cost incurred with respect to the flexibility of reservation, expressed as

$$f_j(u_{jt} - l_{jt}), \quad \forall j, \forall t,$$

where f_j is per-unit cost for flexibility of supplier j. We call this as flexibility cost. The trade-off is as follows. If $(u_{jt} - l_{jt})$ is large, the buyer has a greater flexibility but has greater cost, and vice versa. Each supplier has different ability of flexibility, and has limitation with respect to discrepancy as

$$\frac{|u_{jt} - l_{jt}|}{l_{jt}} \le b_j,$$

where b_j is mamximum range of flexibility. We call b_j as flexibility capability. Note that above inequality can be reduced to linear constraints as

$$-b_j l_{jt} \le u_{jt} - l_{jt} \le b_j l_{jt}$$

We assume that each product has differnt selling-season, volume and demand predictability. We model that demand follows the normal distribution $d_{it} \sim N(\mu_{it}, \sigma_{it}^2)$. We assume that the longer the horizon, the larger the error, and thus assume the following relations

$$\sigma_{it}^2 = k(\tau - t)\sigma_{i\tau}^2, \quad t \le \tau$$

with a coefficient $k > 0$.

The overall decision process is modeled as rolling-horizon model, which is motivated by model predictive control in the control engineering. The planning horizon is $t = 1, \cdots, H$. At each period t, the buyer update demand predction $d_{i(t+1)}, \cdots, d_{i(t+T)}$, and solve the planning problem to get order quantity $y_{ijt}, \cdots, y_{ij(t+T)}$, inventory x_{it}, and capacity range $l_{j(t+B)}, u_{j(t+B)}, \cdots, l_{j(t+T)}, u_{j(t+T)}$. We interpret these as plan of action for next T periods. We take l_{t+B} and u_{t+B} as actual implementation for the period t.

The overall process is summarizes as follows:

Repeat for $t = 1, \cdots, H$:

1) Observe demand d_t and update demand forecasting $[d_{i(t+1)}, \cdots, d_{i(t+T)}]$
2) Solve (robust) optimization problem to get
   - order quantity $y_t, \cdots, y_{t+T}$
   - inventory $x_t, \cdots, x_{t+T}$
   - capacity range $l_{t+B}, u_{t+B}, \cdots, l_T, u_T$
3) Take order quantity u_t and booking l_{t+B} and u_{t+B}

B. An optimization model at each stage

An optimization model at each stage is formulated as follows:

Minimize

$$\sum_{t=1}^{T} (p_t^T s_t + c^T u_t + h^T x_t + f(u_t - l_t)) \tag{1}$$

subject to

$$x_t = x_{(t-1)} + \sum_{j=1}^{m} y_{j(t-L)} - d_t + s_t, \tag{2}$$

$$t = 1, \cdots, T$$

$$l_{tj} \le m^T y_{jt} \le u_{tj}, \tag{3}$$

$$t = 1, \cdots, T,\ j = 1, \cdots, m$$

$$\frac{|u_{jt} - l_{jt}|}{l_{jt}} \le b_j \tag{4}$$

$$t = 1, \cdots, T,\ j = 1, \cdots, m$$

$$x_t, s_t, y_{jt}, l_t, u_t \ge 0, \quad t = 1, \cdots, T \tag{5}$$

- Parameters:
  - $t = 1, \cdots, T$: period index
  - $i = 1, \cdots, n$: product index
  - $j = 1, \cdots, m$: suppliers index
  - $p_t = [p_{t1}, \cdots, p_{tn}]^T$: Price of product at time t
  - $c = [c_1, \cdots, c_n]^T$: Unit production cost
  - $h = [h_1, \cdots, h_n]^T$: Unit inventory holding cost
  - $d_t = [d_{1t}, \cdots, d_{nt}]^T$: Demand at time t
  - f_j : Flexibility cost of supplier j
  - $a = [a_1, \cdots, a_m]^T$: Man-hour for products
  - $y_{-L+1}, \cdots, y_0$:order quantities that are fixed in the past period
  - $x_0 = [x_{01}, \cdots, x_{0n}]$: Initial inventory
  - L : Lead time

- B : Reservation term
- b_j : Maximum range of flexibility

- Decision variable:
  - $s_t = [s_{1t}, \cdots, s_{nt}]^T$: Shortage (amound of unmet demand) at time t
  - $x_t = [x_{1t}, \cdots, x_{nt}]^T$: Inventory at time t
  - $y_{jt} = [u_{t1j}, \cdots, u_{tnj}]^T$: Order quantity at time t to supplier j
  - $l_t = [l_{t1}, \cdots, l_{tm}]$:Researved capacity upper limit at time t
  - $u_t = [u_{t1}, \cdots, u_{tm}]$:Researved capacity lower limit at time t

The objective function (1) is composed of lost-sales, production cost, inventory cost, and flexibility cost. The constraint (2) express the inventory update equation. The constraint (3) is capacity-reservation constraint. The constraint (4) is maximum flexibility constraints. The constraint (5) is nonnegativity constraint.

C. Sampling average approximation

The above problem (1)-(5) is multi-stage stochastic programming problem. Stochastic programming problem is the problem where objective and constraint functions depend on decision variable and uncertain parameters. Since analytical solution in special cases, $e.g.$, when expectations can be found analytically. In general case, approximate solution via (Monte Carlo) sampling is used. This approach is called sampling average approximation (SAA)

SAA is a general method for (approximately) solving stochastic programming problem. In our problem setting, we generate N samples of demand scenarios $d_t^k, \cdots, d_t^k$, with associated probabilities $\pi_1, \cdots, \pi_N$ with $\pi_k = 1/N$.

Let z_k is the optimal value of the problem (1)-(5) under demand scenario k, and we can get sample average approximations solution as

$$z_{SAA} = \frac{1}{N} \sum_{k=1}^{N} z_k$$

It is known that $z_{SAA} \to z^\star$ as $N \to \infty$.

III. NUMERICAL EXPERIMENTS

In this section, we show numerical experiments motivated from an industrial example of monthly production planning of a SPA firm of fast fashions.

A. Input data

The reservation term B is 2 months, the planning horizon H is 12 months, the optimizatoin term T is 6 months, and the lead-time L is 2.

There are 5 suppliers and each supplier has flexibility cost $f_j = 1, j = 1, \cdots, 5$ and maximum range of flexibility $b_1 = 0.35, b_2 = 0.20, b_3 = 0.20, b_4 = 0.10, b = 5 = 0.10$.

There are thousands of SKUs for the company, but to simplify the problem, we cluster them into 10 product groups, say $n = 10$. Each product group has different characteristic of demand predictability, seasonality, and volume. Seasonality pattern we consider in this experiment are grouped into four types: (AB) all-season basic product, (SS) spring/summer-season product, (FW) fall/winter-season product, and (SP) spot-demand product. Basic trends of these demands are shown in Table I. These values are used for average demand μ_{it}.

To model demand predictability of next period, we use coefficient of variation denoted as $v_i = \sigma_{i(t+1)} / \mu_{i(t+1)t}$. We grouped products into four types with respect to the demand predictability as very high ($v_i = 0.45$), high ($v_i = 0.15$), middle ($v_i = 0.30$), low ($v_i = 0.05$). For each period t, we get $\sigma_{i(t+1)}$ from μ_{it} and v_i. For $\sigma_{i(t+2)}, \cdots$ we use the relation $\sigma_{i\tau} = k(\tau - t)\sigma_{it}$ with $k = 1.2$. Other parameters are summarized in table II.

B. Results and discussion

The capacity reservation and monthly production planning for each supplier for each period is determined as TABLE III. The total profit which is an objective function of this model is calculated in TABLE IV.

As TABLE III shows, at suppliers wigh higher flexibility capability (e.g. Supplier A), the range between upper-limit and lower-limit is wider. Spot products and seasonality products, demand predictability of which is high tend to be produced at these suppliers. On the other hand, at suppliers with lower flexibility capability (*e.g.* Supplier D, E), the range is narrower. all-season basic product, demand predictability of which is low tend to be produced at these suppliers. As TABLE IV shows, while there are opportunity losses because of limited capacity of suppliers in each month, the variations between months were moderated effectively.

IV. CONCLUSION

In this paper, we developed a simulation tool to make a decision of monthly production planning and capacity-reseravation which is useful for apparel firms to determine monthly production planning every month. We suggest a multi-period, multi-item, multi-supplier capacity reservation model to minimize total consisting of production cost, inveonty cost, lost-sales and flexibility cost. In conclusion, we could develop a simulation tool which can determine effective monthly production planning and strategic capacity-booking.

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