A Numerical Stability Analysis for Application of Particle Swarm Optimization to Facility Layout Problem

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Abstract - This paper performs the numerical stability analysis for the application of Particle Swarm Optimization (PSO) to the Facility Layout Problem (FLP). FLP has many practical applications and a number of algorithms have been proposed to solve this problem. Most of those algorithms try to solve this problem by encoding layout candidates and using combinational optimization techniques to obtain the best one among those encoded candidates. However, since there exist layouts which cannot be represented by those encoding techniques, there is possibility of missing the searching opportunity for the optimal solution. As one approach to overcome this problem, this paper is concerned with the development of algorithm to solve FLP by PSO, so that it can search continuously optimal coordinate of each department. In particular, this paper attempts to discover the effects of the parameter settings on the convergence performance of particles in PSO applied to FLP.

Keywords - Supply chain management; Stochastic programming

I. INTRODUCTION

The Facility Layout Problem (FLP) is one of the most concerned areas in design of production system. The goal of FLP is to discover the best locations of departments in order to minimize some objective function, subject to some constraints. In the context of production system, the objective is to minimize the total material handling cost of moving materials between the departments. The importance of material handling is stated in [1] as it corresponds to 20-50% of the total operating cost within manufacturing. The common constraints needed to account for is non-overlap constraints between departments, while the other constraints depends widely on manufacturing environments. FLP has also many practical applications in such areas as office layout or VLSI (Very Large Scale Integrated) circuit floor planning, further to the design of production system.

In the optimization point of view, FLP is known to be NP-hard, which implies that, in general, it is difficult to solve. Hence, exact algorithms can be applied only for small size problems and it is reported in [2] that the optimal solution can only be found for problems with less than 7 departments. Therefore, one of the biggest challenges most of researches on FLP are facing is the development of approximation algorithms to find as good solution as possible.

This paper attempts the application of Particle Swarm Optimization (PSO) to FLP, as it is known to be effective for many types of continuous optimization problem. Further, since the parameter settings have great influences on the performance of PSO, this paper performs the numerical stability analysis to discover the effects of the parameter settings on the convergence performance of particles when it is applied to FLP.

II. LITERATURE REVIEW

This section provides the literature review on FLP. The review does not focus on the exact algorithm approaches, since most of the algorithms that has been researched are the approximation algorithms as above-mentioned. Also, this review does not focus on researches on QAP (Quadratic Assignment Problem) based FLP, as they are not similar to the model considered in this paper.

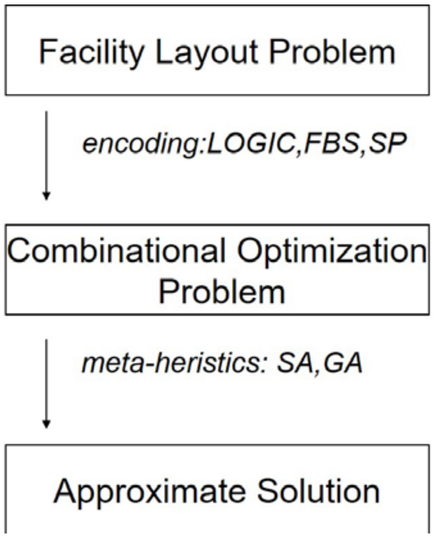

Fig 1. Scheme in Combinational Optimization Approaches.

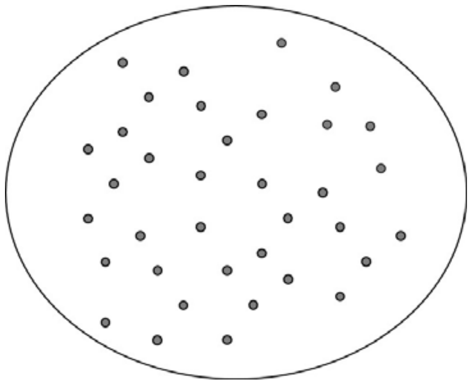

Fig 2. Solution space of FLP (outer-circle) and layout represented by encoding techniques (dots)

A. Combinational optimization approaches

Most of the algorithms which have been proposed are based on the combinational optimization approaches. They try to solve FLP by encoding layout candidates and using combinational optimization techniques to obtain the best one among those encoded candidates (see Figure 1).

Tam [3] proposed the encoding technique called LOGIC (Layout Optimization using Guillotine-Induced Cut), where the layout is represented as a collection of rectangular partitions organized as a slicing tree. A slicing tree consists of branches (departments) and branching operators that specify whether the departments on opposite sides of a branch are to the left, right, above, or below. With a given slicing tree, the layout can be determined by partitioning facility according to the operators. It uses SA (Simulated Annealing) to find the best slicing tree to minimize the objective function. Tate and Smith [4] proposed encoding technique called FBS (Flexible Bay Structure) to represent layout based on other partitioning rule. It is used together with GA (Genetic Algorithm). Murata, Fujiyoshi, Nakatake, and Kajitani [5] proposed SP (Sequence-Pair) for the VLSI floorplaninng problem, where the layout can be represented by the pair of sequences which determines the relative positions between departments. In addition, a lot of encoding techniques such as BL (Bottom-Left) algorithm [6], Boundary method [7], O-tree [8], has been proposed.

One of the major weaknesses of those approaches is the presence of the layout which cannot be represented by those encoding techniques. Thus, there is possibility of missing the searching opportunity for the optimal solution (see Fig.2.).

B. Continuous Optimization Approaches

Some of the researches have attempted the development of algorithm to solve FLP by searching continuously optimal coordinate of each department. [9] proposed the algorithm utilizing non-linear programming. [10] introduced the concept of convex approximation to FLP.

However, those researches have been facing the difficulty because FLP is non-convex problem, as non-overlapping constraint makes the feasible space non-convex set. Therefore, the algorithm which makes it possible to search effectively the solution space of FLP is required.

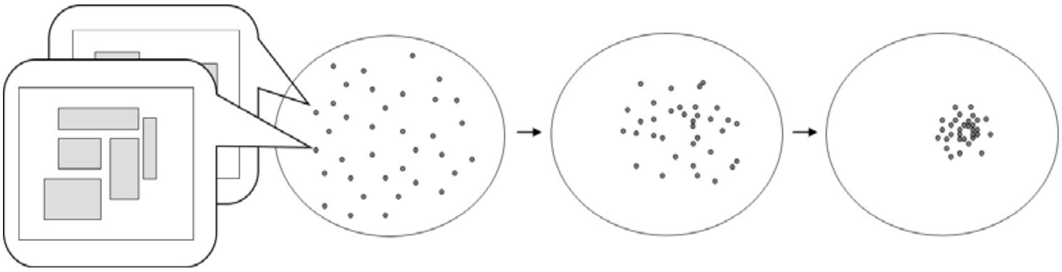

Fig 3. Iteration of Particle Swarm in PSO

III. NOTATION AND FORMULATION

This section provides the mathematical formulation of FLP that this paper focus on.

A. Constants

- N : number of departments
- w_i : width of department i
- h_i : height of department i
- W : witdh of facility
- H : height of facility
- f_{ij} : flow of material between departments i and j

B. Decision Variables

- x_i : x -coordinate of center of department i
- y_i : y -coordinate of center of department i

C. Objective Function and Constraints

minimize

$$\sum_{i=1}^{N}\sum_{j=1}^{N} f_{ij}d_{ij} \tag{1}$$

subject to

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2} \tag{2}$$

$$X_{ij} = \max\{(w_i + w_j)/2 - |x_i - x_j|, 0\} \tag{3}$$

$$Y_{ij} = \max\{(h_i + h_j)/2 - |y_i - y_j|, 0\} \tag{4}$$

$$w_i / 2 - x_i \le 0 \tag{5}$$

$$h_i / w - y_i \le 0 \tag{6}$$

$$x_i - (W - w_i / 2) \le 0 \tag{7}$$

$$y_i - (H - h_i / 2) \le 0 \tag{8}$$

The objective is the minimization of the total material handling cost associated with material flows and distances between departments. The non-overlapping constraints are given in (2)-(4). These constraints require that the distances between two departments in x (or y) directions must be greater than half of sum of their widths (or heights). The constraints (5)-(8) ensure that all departments must remain inside the facility.

IV. APPLICATION OF PSO TO FLP

To overcome the above-mentioned difficulty, this paper applies PSO to FLP. The explanation of the application is provided in this chapter.

A. General Description of PSO

PSO is one of the meta-heuristics proposed by [11] in 1995. It is based on the evolutionary computation and designed to solve continuous non-convex optimization problems. Like the other evolutionary computation methods, PSO is a population-based search algorithm and is initialized with a population of random solutions, called particles. Unlike in the other evolutionary methods, each particle in PSO is also associated with a velocity. Particles fly through the search space with velocities which are dynamically adjusted according to their historical behaviors. The usage of historical behaviors are composed of three factors, which are previous velocities, the best position of each particle (called Pbest), and the best position among all particles (called Gbest). Therefore, the particles have a tendency to fly towards the better search area over the course of search process.

B. Layout Representation by Particle

Let D denote the number of particles, K denote the number of iterations (generations), $p = 1, \cdots, P$ denote the particle number, $k = 1, \cdots, K$ denote the iteration number, $r_{pk} \in \mathbb{R}^{2N}$ denote the position of p -th particle after the k -th iteration. In the proposed method, r_{pk} , whose components are x and y coordinates of departments, represents the layout candidate as

$$r_{pk} = [x_1, \cdots, x_N, y_1, \cdots, y_N]^N$$

The positions of particles are updated in the process of PSO. (see Figure 3).

C. Penalty Function for Satisfaction of Constraints

Since PSO is unconstrained optimization technique, this paper proposes the penalty function in order to satisfy the constraints. Let $h : \mathbb{R}^2 \to \mathbb{R}$ denote the left-hand side in (4), $g_m : \mathbb{R}^2 \to \mathbb{R}$ denote the left-hand side in (5)-(8) and t denote the weight of penalty. The penalty function defined is as follows in (9).

$$P(r) = t(\sum_{i,j} |h(r)| + \sum_{i} \sum_{m=1}^{4} \max(g_m(r), 0))$$

The penalty function becomes zero only if the constraints (4)-(8) are satisfied, otherwise positive (see Fig.4).

D. Detailed Description of Proposed Algorithm

The detailed description of the proposed algorithm is given as follows (see Fig.3)

given $P, K, k = 0, r_{p0}, v_{p0}, w, c_1, c_2$

repeat

1) update the position of Pbest and Gbest:

$$r_p^{pb} = \arg\inf_k f(r_{pk}), \quad \forall p$$

$$r^{gb} = \arg\inf_k f(r_p^{pb})$$

2) update the velocity of particles:

$$v_{p(k+1)} = wv_{pk} + c_1(r_{pk} - r_p^{pb}) + c_2(r_{pk} - rgb), \quad \forall p$$

3) update the position of particles:

$$r_{p(k+1)} = r_{pk} + v_{pk}$$

4) $k \to k + 1$

were $f : \mathbb{R}^{2N} \to \mathbb{R}$ denotes sum of the objective function and penalty function, and w, c_1, c_2 denotes the parameters to control the influence of previous velocity, Pbest, and Gbest.

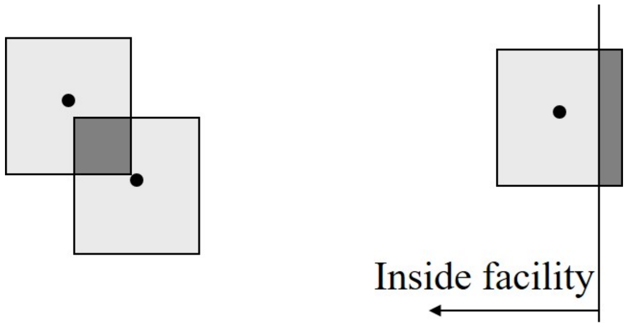

Fig 4. Description of when penalty function becomes positive.

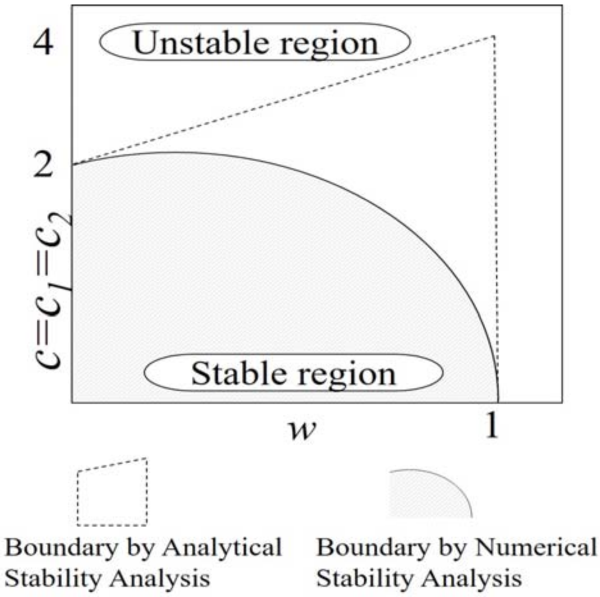

Fig 5. Boundary by Stability Analysis

V. NUMERICAL STABILITY ANALYSIS ON APPLICATION OF PSO TO FLP

This section provides the result of the numerical stability analysis of PSO to FLP.

Table I. Product Groups and seasonal trends

$w \setminus c$	0.0	0.4	0.8	1.2	1.6	2.0	2.4	2.8	3.2	3.6
0.0	1000000000.0	293304.3	255234.5	∞	∞	∞	∞	∞	∞	∞
0.2	222506439.4	1340954.0	262917.3	∞	∞	∞	∞	∞	∞	∞
0.4	29655758.6	272866.1	267755.3	∞	∞	∞	∞	∞	∞	∞
0.6	757118221.7	286150.4	267359.0	∞	∞	∞	∞	∞	∞	∞
0.8	899203651.3	289359.2	407230.8	∞	∞	∞	∞	∞	∞	∞

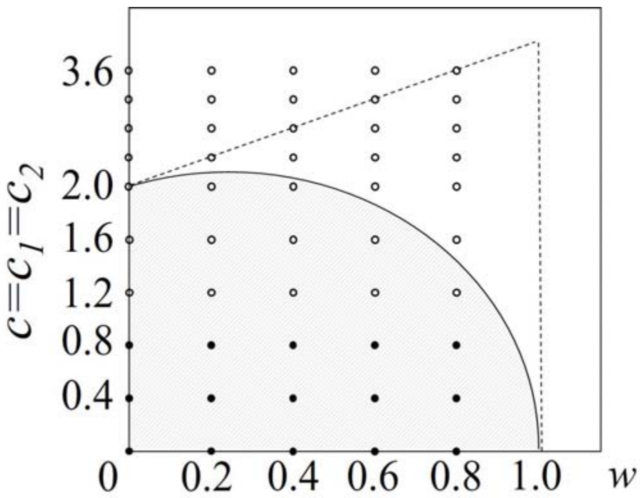

Fig 6. Solution space of FLP (outer-circle) and layout represented by encoding techniques (dots)

A. Stability Analysis of PSO

As is the case with the other meta-heuristics algorithm, it is known that the parameter settings have great effects on the performance of PSO. In particular, without proper selection of parameters, the particles will diverge and no improvement of objective function occurs over the course of computation. As one approach to avoid such consequences, the application of the stability analysis to PSO has been proposed in several researches. The goal of the stability analysis is to discover the boundary between the stable region where the particles will converge and the unstable region where not (see Fig.5). Clerc and Kennedy[12] performs the analytical stability analysis based on the linear eigenvalue analysis. It provides the stability region as in (10)-(12)

$$0 \le w \le 1 \tag{9}$$

$$0 \le c \le 2w + 2 \tag{10}$$

$$c = c_1 = c_2 \tag{11}$$

Ueno and Yasuda[13] performs the numerical stability analysis based on their computational experiments carried out for the benchmark problems. It provides the stability region as in Fig.5.

Based on the concepts of these previous works, this paper performs the numerical stability analysis for the application of PSO to FLP. The two main objectives of the analysis are to specify the boundary between the stable region and the unstable region of PSO when it is applied to FLP, and to discover which parameter settings will be the most suitable for FLP.

For the width and height of each department, the data for the MCNC (Microelectronic Center of North Carolina) benchmark problem (apte) are used. The distances between departments are measured by the distance between centroids and the flows between departments are set to one.

The program is written in C language, and the experimental conditions are Intel (R) Core™ 2 Duo CPU E6850, 3.00GHz, 3.00GHz, with 4.00GB memory.

B. Result of Stability Analysis

The numerical stability analysis is performed to the following parameter settings. Note that the parameter settings out of (10)-(12) need not to be examined as they are proven to result in the particles diverging as above-mentioned.

- $w = \{0.0, 0.4, 0.6, 0.8\}$
- $c = \{0.0, 0.4, 0.8, 1.2, 1.6, 2.0, 2.4, 2.8, 3.2, 3.6\}$

The other parameters are determined as $K = 1000, P = 10000$, according to the pre-experiment, so that PSO can search the solution space sufficiently enough. The initial locations and initial velocities of each particle are obtained randomly.

The results of 10 times trial of the numerical stability analysis are shown in Table.1. The values in the table represent the average values of the objective function of Pbest at K -th iteration, i.e. the last iteration. Fig.6 represents the plots of the results of numerical stability analysis, as well as stable and unstable region obtained by [13] and [14].

Comparing the results obtained by the experiments shown in Fig.6, the stable region of PSO applied to FLP is smaller than those obtained by the previous works. Further, it can be seen from Table 1 that the parameter settings of provide good results compared to others. On the other hand, the parameter settings of provide worse results. In particular, the parameters with provide the bests results among obtained results. This implies that the particles should account for the Pbest and Gbest more than the previous vectors.

The reason for this tendency can be considered as follows. At the beginning of the searching process, each department moves closer one another to minimize the distance between other departments. After getting closer one another, each particle needs t o change the direction of velocity drastically so that it can avoid the overlap. Therefore, taking over the direction of the previous velocity vector does not contribute the improvement.

VI. CONCLUSION

This paper applies Particle Swarm Optimization (PSO) to the Facility Layout Problem (FLP) and performs the numerical stability analysis for the PSO to FLP. The method does not use encoding techniques, and instead, it searches coordinates of each department continuously, so that it can represent any possible layout. In the numerical stability analysis, it is found that the stable region is smaller than those obtained by the previous works. Also, according to the results of experiments, this paper can propose the parameter settings suitable to the application of PSO to FLP.

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