Optimal Tasks Allocation Process Based on Fusion of the Unallocated Tasks for Distributed Systems

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Abstract- Tasks partitioning and systematic tasks allocation to processors over the distributed real time system for proper utilization of processor’s capacity are very much important factors in distributed systems. Efficient use of available processors enhances the performances of the distributed systems and provide an optimal solution. This paper presents a new mathematical model consisting of a set of m tasks to n processors (where, $m >> n$) which performs static allocation of such software program modules that minimizes the application program’s parallel EC, ITCC and maximizes the overall throughput of the system in such a way that the allocated load on all the processors are balanced. The heuristic model deals with the optimal tasks allocation by the fusion of the unallocated tasks with the other allocated tasks.

Keywords: DCS, Task Allocation, Parallel Computation, EC, ITCC, Processors Utilization, Throughput, Load Balancing.

I. INTRODUCTION

In the past years, Distributed Computing Systems (DCS) have become a key platform in order to achieve optimal cost by allocating number of tasks to different processors for execution in task allocation problems. The configuration of a DCS is the set of autonomous processors inter-connected by a high bandwidth communication link to coordinate their actions to achieve a common goal. DCS has become a dominant paradigm in computer architecture, mainly in the form of multi-core processors. If two, or three, or more processors are available, than many of the programs can be executed more quickly. While one processor is doing one aspect of computation, others can work on another and share the data. In this way DCS employs different processing components simultaneously to solve a problem. In this form of computing, the work is accomplished by breaking the large problems into the smaller ones, which are than solved concurrently. DCS has attracted many researchers by posing several challenging problems. A major problem encountered in DCS is the task allocation to the processors so that the system cost i.e. sum of execution cost and communication cost is minimized without violating any of the system constraints [1]. A best allocation of tasks leads to a best balancing of the system [2]. The performance of the DCS may suffer if the distribution of resource is not carefully implemented. In order to make the best use of resources in a DCS, it becomes essential to maximize the overall throughput by allocating the tasks to processors in such a way that the allocated load on all the processors should be balanced [3]. The tasks partitioning and task allocation activities influence the distributed software properties such as IPC [4, 5]. Many approaches have been reported for solving the task assignment problems in DCS. The problem of finding an optimal dynamic assignment of a modular program for a two processor system is analyzed by [6]. The main incentives for choosing DCS are higher throughput, improved availability and better access to a widely communicated web of information [7]. One measure of usefulness of a general purpose DCS is the system’s ability to provide a level of performance commensurate to the degree of multiplicity of resources present in the system. Taxonomy of approaches to the resource management problem is
reported [8]. The taxonomy, while presented and discussed in terms of distributed scheduling, is also applicable to most types of resource management. A model for allocating information files has been reported by [9] the model considers storage cost, transmission cost, file lengths, and request rates, as well as updating rates of files.

The coordination of dynamic task allocation based on available resources is very challenging in DCS. Task allocation and load balancing has been extensively researched in past decades and a large number of related studies and results have been presented concerning this topic [10, 11, 12, 13]. A task allocation model that allocates application tasks among processors in DCS satisfying: minimum inter-processor communication cost, balanced utilization of processors and all engineering requirements has been reported by Perng –Yi Richard Ma et. al. [14]. This problem of task allocation in heterogeneous distributed systems with goal of maximizing the system reliability has been addressed [11]. The model is based on well-known simulated annealing (SA) technique. Yadav et. al. [15] have formulated an algorithm for Reliability Evaluation of Distributed System based on Failure Data Analysis. J. B. Sinclair [16] asserted for finding an optimal assignment of the modules of a program to processors in a distributed system by multiprocessor scheduling with the aid of the network flow algorithms. A module incurs an execution cost that may be different for each processor assignment, and modules that are not assigned to the same processor but that communicate with one another incur a communication cost. Sagar et. al. [17] used the matrix reduction technique and reported that a task is selected randomly to start with and then assigned to a processor. Kumar et. al. [18, 19] proposed algorithms for distributed processing system and they clustered heavily communicated tasks and allocate them to same processors. DCS offers the potential for improved performance and resource sharing. To make the best use of the computational power available in multi processing system, it is essential to assign the tasks dynamically to that processor whose characteristics are most appropriate for the execution of the tasks in distributed processing system [13, 20]. Elsadek et al. [21] proposed a new methodology which expands the Maximally Linked Module algorithm by incorporating simulated annealing and produces complete allocations for assigning static modules to processors of a distributed system.

Communication technology has completely changed the modern life standard and web services play a crucial role in this technology. It has opened new avenues for processing, sharing and transferring of data. Apart from sharing data and I/O devices, DCS also share the computational power among the nodes which improves the system performance [22]. Yadav et. al. [23, 24] and Singh et. al. [24,25] reported task scheduling and represented ANN based task scheduling model for load distribution in fully connected client server network using feedback neural network architecture. Load distribution is important for performance enhancement of the distributed systems. It means the allocation of workload over the distributed system optimally [26]. Analysis of load distribution in DCS through systematic Allocation Task and an exhaustive approach of performance analysis to the distributed systems based on cost assignments have been reported by Kumar et. al. [27, 28].

Our objective of this paper is to minimize the total program Execution Cost(EC). A heuristic model utilized the mathematical programming technique for execution of the tasks assuming, when a task is assigned to a processor, it remains there while the characteristic of computation change, and a new assignment must be computed. During formation of an algorithm it is assumed that the number of tasks is greater than the number of processors. Let it be  \( m > n \), where  \( m \) tasks is to be executed on a set of  \( n \) processors, which have different processing capabilities. It is considered that the EC of a task on each processor is known, if a task is not executable on any of the processor due to the reason of some missing resources than the EC of that task is taken to be infinite (\( \infty \)). When a task is executed on a processor, it stores the output data in its local memory and when needed, the processor read it from there. Whenever a group of task is assigned to the same processor, the Inter Task Communication Cost(ITCC) between them is zero. Completion of a program from computational point of view means that all related tasks or modules have got executed. The developed model is based on matrix reduction technique and the performance, effectiveness and efficiency of it has been implemented on several sets of input data. It is found that the algorithm is suitable for arbitrary number of tasks and processors with random program structure.

II. NOTATIONS

\( T \) : the set of tasks of a parallel program to be executed
\( P \) : the set of processors in DCS
\( n \) : the number of processors
\( m \) : the number of tasks forming a program
\( k \) : the number of clusters
\( t_i \) : \( i^{th} \) task of the given program
\( P_j \) : \( j^{th} \) processor in set of processor  \( P \)
\( X_{ij} \) : the decision variable s.t.  \( X_{ij} = 1 \), if \( i^{th} \) task is allocated to \( j^{th} \) processor,  \( X_{ij} = 0 \), otherwise.
\( ec_{ik} \) : incurred execution cost (EC), if \( i^{th} \) task is executed on \( k^{th} \) processor
\( cc_{ij} \) : incurred inter task communication cost between task \( t_i \) and \( t_j \), if they are executed on separate processor
III. FORMULATION OF TASK ASSIGNMENT PROBLEM

The problem being addressed here is concerned with an optimal task allocation in DCS, which consists of an application program of a set of communicating tasks \( T = \{ t_1, t_2, t_3, \ldots, t_m \} \) and a DCS with \( n \) processors \( P = \{ p_1, p_2, p_3, \ldots, p_n \} \) interconnected by communication network and let \( m > n \). An allocation of tasks to processors is defined by a function \( A_{alloc} \) from the set \( T \) of tasks to the set \( P \) of processors s.t.:

\[
A_{alloc} : T \rightarrow P, \text{ where } A_{alloc}(i) = j \text{ if task } t_i \text{ is assigned to processor } p_j, 1 \leq i \leq m, 1 \leq j \leq n.
\]

Our objective is to minimize the ITCC and balance the processing load. The processing cost of these tasks on all the processors is given in the form of Execution Cost Matrix \([ECM(,)]\) of order \( m \times n \), the ITCC is taken in the form of a symmetric matrix named as Inter Task Communication Cost Matrix \([ITCCM(,)]\). It is very essential to optimize the overall throughput of the processors by assigning the tasks in a manner that the allocated load on all the processors shall have to be balanced. The proposed methodology includes:

- Identification of allocation by Minimally-linked modules MLM
- Calculate total processing cost of each processor
- Calculate the throughput of the processors
- Identification of optimal response time of the system
- Calculation of average load on each processors

A. Definitions

1. Execution Cost (EC): The EC \( ec_{ij} \) where \( 1 \leq i \leq m, 1 \leq j \leq n \) of each task \( t_i \) depends on the processor \( p_j \) to which it is assigned and the work to be performed by each of tasks of that processor \( p_j \). The processing Execution cost \( EC \) of the tasks on all the processors is given in the form of \([ECM(,)]\) of order \( m \times n \). The EC of a given assignment on each processors are calculated by Equation (1)

\[
PEC(j) = \sum_{j=1}^{m} ec_{ij}x_{ij}, \quad i = 1, 2, 3, \ldots, m, \quad \ldots(1)
\]

where,

\[
x_{ij} = \begin{cases} 1, & \text{if task } t_i \text{ is assigned to processor } p_j \\ 0, & \text{otherwise} \end{cases}
\]

2. Inter Task Communication Cost (ITCC): The ITC cost \( cc_{ik} \) of the interacting tasks \( t_i \) and \( t_k \) is the minimum cost required for the exchange of data units between the processors during the process of execution.

\[
ITCC(j) = \sum_{i<k} cc_{ik}x_{ij}, \quad j = 1, 2, 3, \ldots, n, \quad \ldots(2)
\]

where,

\[
x_{ij} = \begin{cases} 1, & \text{if task } t_j \text{ is assigned to processor } p_j \\ 0, & \text{otherwise} \end{cases}
\]

3. Response Time Cost of the System (RTC): The RTC of the system is a function of amount of computation to be performed by each processor and the computation time. This function is defined by considering the processor with the heaviest aggregate computation and communication load. Response time of the system for a given assignment is defined by

\[
RTC(A_{alloc}) = \max_{A_{alloc}} \left\{ PEC(A_{alloc})_j + IPC(A_{alloc})_j \right\} \quad \ldots(3)
\]

4. Average Load: For each node there are \( N \) values representing the execution cost required for the corresponding task to be processed on each of the \( N \) processors. These values are in the matrix \([ECM(,)]\). Each edge is labeled with a value that represents the communication cost needed to exchange the data when the tasks reside on different processors. Average load on a processor is the sum of EC of the tasks assigned to it then the average load that must be assigned to each processor \( P_j \) is calculated as

\[
L_{avg}(P_j) = \frac{W_j}{m}, \quad j = 1, 2, 3, \ldots, n \quad \ldots(4)
\]

where,

\[
W_j = \sum_{i=1}^{m} ec_{ij}, \quad j = 1, 2, 3, \ldots, n.
\]

The total workload \( W \) is the summation of the maximum module execution cost on the different processors as shown
The average load on a processor depends upon the different tasks on each processor. The system is considered to be balanced if the load on each processor is equal to the processor average load within a given (small percentage) tolerance.

### B. Assumptions

Several assumptions have been made to keep the algorithm reasonable in size while designing. The program is assumed to be collection of $m$ tasks to be executed on a set of $n$ processors, which have different processing capabilities. A task may be portion of executable code or a data file. The number of tasks to be allocated is more than the number of processors ($m > n$), as normally is the case in real life distributed processing environment. It is assumed that the EC of a task on each processor is known, if a task is not executable on any of the processor due to absence of some resources. The EC of that task is on that processor is taken to be infinite ($\infty$). We assume that once a task has completed its execution on a processor, the processor stores the output data of the task in its local memory, if the data is needed by some other task being computed on the same processor, it reads the data from the local memory. Using this fact, the algorithm tries to assign heavily communicating tasks to same processor. Whenever groups of tasks or cluster are assigned to the same processor, the data transfer between them is zero. Completion of a program from computational point of view means that all related tasks have got executed.

### IV. THE TASKS ALLOCATION PROCESS

Initially the average load on the processors $P_j$ is obtained by using equation (4) and the total load may be calculated by using the equation (5). The load on each processor is equal to the average load within a reasonable tolerance. In the present study a tolerance factor of 20% of average load has been considered. Determine the $n$ Minimally Linked Task (MLT) by using the equation (6) and store the result in two-dimensional array $[MLT(i,j)]$ the first column represents the task number and second column represents the sum of ITCC of task $t_i$ with all task $t_k$, $i$. Rearrange the $[MLT(i,j)]$ in ascending order assigning the second column as sorted key.

$$Tlod = \sum_{j=1}^{n} Lavg(P_j) \quad \ldots(5)$$

Augmented ECM($\cdot$) by introducing the MLM($\cdot$) and sort ECM($\cdot$) in increasing order considering MLT as sorting key. To determine initial allocation apply the Yadav et. al. algorithm [28] and store in an array $Tass(j)$ (where $j = 1, 2, \ldots, n$). The processor position are also store in another linear array $Aalloc(j)$. The value of $Tass(j)$ is also computed by adding the values of $Aalloc(j)$ if a task $t_i$ is assigned to processor $p_j$ otherwise continue. The remaining $(m-n)$ task are then store in a linear array $Tnon-ass(\cdot)$. Tasks assigned to processors $p_j$ and stored in $Tnon-ass(\cdot)$ which are obviously:

$$T = Tass(\cdot) \cup Tnon-ass(\cdot)$$

All the tasks stored in $Tnon-ass(\cdot)$ fused with those assigned tasks stored in $Tass(\cdot)$ on the bases of minimum average of EC and ITCC.

The FEC of a task $t_a \in Tnon-ass(\cdot)$ with some other task $t_i \in Tass(\cdot)$ on processor $p_j$ is obtained as:

$$FEC(j)_{ai} = [ec_{aj} + ec_{ij}], \quad (1 \leq a \leq m, 1 \leq i \leq m, 1 \leq j \leq n, i \neq a)$$

Let $cc_{ai}$ be the ITCC between $t_a \in Tnon-ass(\cdot)$ and $t_i \in Tass(\cdot)$. The FITCC for $t_a$ with $t_i$ is computed as:

$$FITCC(j)_{ai} = \sum_{l \neq t_{ass(i)}} \lfloor cc_{ai} \rfloor$$

Here, $cc_{ai} = 0$ if fused with $t_i$ or $a = i$ and remaining $cc_{ai}$ value are added and MAFC is calculated as:

$$MAFC(j)_{ai} = \text{min}\{(FEC_{a1} + FITCC_{a1}), (FEC_{a2} + FITCC_{a2}), \ldots, (FEC_{an} + FITCC_{an})\}$$

This process will be continued until all the tasks stored in $Tnon-ass(\cdot)$ are fused. After complete allocation is achieved PEC(j) and ITCC(j) by using equation (1) and (2) respectively. Finally, calculate the Total Optimal Cost(TOC) by summing up the values of PEC(j) and ITCC(j), and store the result in a linear array over TOC(j), where $j = 1, 2, \ldots, n$. The maximum value of TOC(j) will be the optimal cost of the system:

$$TOC(j) = \max\{PEC(j) + ITCC(j)\}$$

The Mean Service Rate(MSR) of the processors in terms of $Tass(j)$ to be computed as and store the results in $MSR(j)$, where $j = 1, 2, \ldots, n$.

$$MSR(j) = \frac{1}{PEC(j)}, \quad \text{where, } j = 1, 2, \ldots, n.$$
\[ T_{RP}(j) = \frac{T_{\text{task}}(j)}{PEC(j)} \]

where, \( j = 1, 2, \ldots, n \).

V. IMPLEMENTATION OF THE ASSIGNMENT MODEL, RESULTS AND DISCUSSIONS

Algorithm has been formulated to model the execution process of tasks in the distributed computing environment. The algorithm can be used for efficient execution of the tasks allocation in distributed computing system without overloading the processors. To justify the application and usefulness of the present method an example of a distributed computing system is considered with the following Inputs:

Number of processors of the system (\( n \)) = 3
Number of tasks to be executed (\( m \)) = 4

ECM (\( , ) \), ITCCM (\( , ) \), MLM (\( , ) \))

<table>
<thead>
<tr>
<th>( ECM(,) )</th>
<th>ITCCM(,)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 p_2 p_3 )</td>
<td>( t_1 ) ( t_2 ) ( t_3 ) ( t_4 )</td>
</tr>
<tr>
<td>( 9 ) ( 2 ) ( 6 )</td>
<td>( 1 ) ( 0 ) ( 4 ) ( 6 )</td>
</tr>
</tbody>
</table>

Actual average load to be assign to the processor after introducing the 20% Tolerance Factor (TF) the average load may be assign to the processors is calculated and same may be store in linear array \( L_{avg}() \).

\[
L_{avg}() = \begin{bmatrix}
p_1 \\
p_2 \\
p_3 \\
\end{bmatrix} = \begin{bmatrix}
9 \\
9 \\
10 \\
\end{bmatrix}
\]

Determine the Minimally Link Task (MLT) and store the result in MLM(\( , ) \) as follows:

\[
MLM(,) = \begin{bmatrix}
t_1 & t_2 & t_3 & t_4 \\
11 & 3 & 14 & 14 \\
\end{bmatrix}
\]

Augmented ECM(\( , ) \) by introducing the MLM(\( , ) \) and sort ECM(\( , ) \) in increasing order considering MLT as sorting key.

<table>
<thead>
<tr>
<th>( ECM(,) )</th>
<th>( E_{\text{MLT}}(,) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 p_2 p_3 )</td>
<td>( t_1 ) ( t_2 ) ( t_3 ) ( t_4 )</td>
</tr>
<tr>
<td>( 9 ) ( 2 ) ( 6 )</td>
<td>( 11 )</td>
</tr>
<tr>
<td>( 3 ) ( 8 ) ( 7 )</td>
<td>( 3 )</td>
</tr>
<tr>
<td>( 7 ) ( 10 ) ( 3 )</td>
<td>( 14 )</td>
</tr>
<tr>
<td>( 3 ) ( 4 ) ( 9 )</td>
<td>( 14 )</td>
</tr>
</tbody>
</table>

Apply Yadav et.al. [28] algorithm to determine the initial allocation. The initial allocation than store in a linear array \( T_{\text{ass}}() \) and the processor position are store in \( A_{\text{alloc}}() \). The remaining m-n tasks are stored in \( T_{\text{non-ass}}() \). The results are as follows:

\[
T_{\text{ass}}(j) = \{ t_1, t_2, t_3 \} \\
A_{\text{alloc}}() = \{ p_2, p_1, p_3 \} \\
T_{\text{non-ass}}() = \{ t_4 \}
\]

After getting the initial allocation the task stored in \( T_{\text{non-ass}}() \) has been selected for assignment i.e., \( t_4 \) FEC(\( j \)), for \( t_4 \) with all the stored in \( T_{\text{ass}}() \) are:

<table>
<thead>
<tr>
<th>Tasks</th>
<th>Processors</th>
<th>FEC(j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( t_4 + t_1 )</td>
<td>( p_2 )</td>
<td>( 6 )</td>
</tr>
<tr>
<td>( t_4 + t_2 )</td>
<td>( p_1 )</td>
<td>( 6 )</td>
</tr>
<tr>
<td>( t_4 + t_3 )</td>
<td>( p_1 )</td>
<td>( 12 )</td>
</tr>
</tbody>
</table>

FITCC(\( j \)) for \( t_4 \) with the other assigned tasks stored in \( T_{\text{ass}}() \) are:
MAFC(j) by summing up the values of FEC(j) and FITCC(j) are:

<table>
<thead>
<tr>
<th>Tasks</th>
<th>Processors</th>
<th>FEC(j)</th>
<th>FITCC(j)</th>
<th>MAFC(j)</th>
</tr>
</thead>
<tbody>
<tr>
<td>t4 + t1</td>
<td>p2</td>
<td>6</td>
<td>13</td>
<td>19</td>
</tr>
<tr>
<td>t4 + t2</td>
<td>p1</td>
<td>6</td>
<td>17</td>
<td>23</td>
</tr>
<tr>
<td>t4 + t3</td>
<td>p3</td>
<td>12</td>
<td>12</td>
<td>24</td>
</tr>
</tbody>
</table>

The MAFC(2)_{41} is minimum i.e. 19. Therefore task t4 is fused with task t1 executing on processor p2. We compute PEC(j), ITCC(j), TOC(j), MSR(j) and TRP(j) given in table final results.

The maximum of TOC(j) is 19 i.e. the total busy cost of the system is 19 which corresponds to processor p2. The model deals with the problem of optimal task allocation and load balancing in DCS. The load balancing mechanism is introduced in the algorithm by fusing the unallocated tasks on the basis of minimum of the average impact of EC and ITCC. It can also be perceived from the example presented here that wherever the algorithm of better complexity is encountered [21] present technique gets an upper hand by producing better optimal results with slight enhancement in the cost due to minor in complexity factor. The worst case run time complexity of the algorithm suggested by [21] is O(n^2+m^2+ m+n) and the run complexity of the algorithm presented in this paper is O(m^2n+2mn). Figure1 shows the run time complexity of present methods and [21] considering different cases of tasks and processors. The utility of our algorithm will ultimately depend on how it can be incorporated into the designing and managing activities of DCS.

![Figure: Run time complexity compression](image)

VI. REFERENCES

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