

# Automating the computation of optimal solutions to transportation problems

John Imhonikhe Lawal<sup>1</sup>, Adanna Cecilia Eberendu<sup>2</sup>

Department of Computer Science  
Madonna University, Nigeria

[1johnimholawal@gmail.com](mailto:johnimholawal@gmail.com), [2aceberendu@gmail.com](mailto:aceberendu@gmail.com)

## ABSTRACT

Transportation problems arise each time there is a need to distribute a commodity from several sources to several destinations, where the management is focused on identifying the distribution route that will optimize some objectives such as minimization of total cost or maximization of profit. Conventional methods of computing solutions to such problems are time consuming, challenging and error prone. In this study, a desktop-based system is developed to address these drawbacks. The system is designed to facilitate the processes and manage challenges surrounding the manual computation of optimal solutions to transportation problems. The conceptual design including the Flowcharts, the Entity-Relationship Model and the Use Cases for the developed system is also presented. The programming tools used for the development of the system are Java and NetBeans 8.1 integrated development environment. Apache Derby was also used for the development of the database. Performance assessment was carried out by a number of reviewers and the average feedback indicates that the system is reliable, robust and user-friendly. The reviewers of the system comprised of software developers and students. The test proved the validity of using this system to find optimal solutions to transportation problems.

*Keywords: Transportation problem, optimal solution, programming, minimization and maximization, feasible solution*

## INTRODUCTION

In industry, the problems associated with the economic distribution of goods and raw materials from their sources to their destinations receive a lot of attention as a large proportion of the sales revenue is absorbed by the physical distribution costs (Elsie, 2007). Industries involved in the transportation of goods from different sources to different destinations are often faced with the problem of how to minimize cost or maximize profit in the transportation of their goods. The problem results from an industry which produces a huge amount of homogeneous commodity stored at different origins having to transport these products to different destinations with all the constraints attached. In order to solve this problem, an American mathematician and physicist Frank Lauren Hitchcock (1875-1957) formulated transportation problems as linear programs which gave solution similar to the Simplex Method (Alexander & Schrijver, 1998), where the entities involved in the transportation process are converted into a mathematical model. Overtime, several methods for finding the initial basic solution and the optimal solution to transportation problem were formulated. Some of the methods used for finding the initial basic solution are Northwest corner method, Minimum Matrix Method (MMM), Least Cost Method (LCM) and Column Minima Method (CMM). For the optimal solution we have Stepping Stone Method and Vogel's Approximation Method (VAM).

Each time there is a need to distribute a commodity from multiple sources to multiple destinations, the problem of identifying the distribution route that will optimize some objectives such as minimization of total cost or maximization of profit tends to arise. In order to solve this problem, the entire problem is modeled into a linear programming problem where the mathematical models are used in the representation of physical entities which can now be solved using the mathematical techniques. Carrying out this process manually proves to be herculean and time consuming. Also, the complex nature of the manual calculations makes them error-prone. Therefore, an efficient system is required to automate the entire processes involved in order to speed up the computing time, provide ease of computation and provide accurate results.

This study aims at providing a software solution to transportation problems with the objective of automating the process of computing optimal solutions to transportation problems in order to provide more accurate results, speed up computation time and provide ease of computation.

The robust software to be developed will make the computation of optimal solutions to transportation problems easier, faster and more accurate. In addition to these, the software will provide user interfaces to enable users understand the various processes involved in solving their problems. This study will be of great importance to (but not limited to) the following set of people: Operation Researchers / Companies – They will be able to use

the software developed during this study to calculate optimal solutions to their transportation problems in a faster and more reliable way. Lecturers – They will be able to use the software to speed up calculation of optimal solutions to transportation problems. They can use the software to know what the outcome of questions they set should be since the software enables the user use different methods of their choice, view the results of these methods and view details of iterations. Students – They will be able to use this software to know if their manual calculations are accurate probably when given home works. They too can see how different methods for solving transportation problems work and understand the difference between them.

The study will cover the various processes involved in solving transportation problems which include setting up a transportation table, finding an initial feasible solution, handling degeneracy, and providing the optimal solution. This study will implement three methods for getting the Initial Basic Feasible Solution (IBFS) of a transportation problem and two methods for getting the optimal solution. The methods are NorthWest Corner Method (NWCN), Least Cost Method (LCM), Vogel's Approximation Method (VAM), Modified Distribution (MODI) and Stepping Stone Method. Other methods for getting the initial basic feasible solutions (IBFS) and optimal solutions to transportation problems will not be implemented due to time constraint and the fact that research showed that the aforementioned methods are mostly used worldwide.

## LITERATURE REVIEW

Transportation is an important domain of human activity as it supports and makes possible most other social and economic activities and exchanges. It is also a complex domain with several players and levels of decision where investments are capital intensive and usually require long implementation delays (Crainic, 1998). Transportation problems arise each time there is a need to distribute a commodity either from the manufacturing warehouses (source) to the distribution warehouses (destination) or from the distribution warehouses (source) to local distribution outlets or retail stores (destination). Transportation problems are among the most pressing strategic development problems in many cities, and can be addressed with a consistent and comprehensive approach and planning methodology that helps to design strategies for sustainable cities (Fedra, 2004). Hence, management is always focused on identifying the method of distribution that will optimize some objectives such as minimization of total cost or maximization of profit.

Various researches aimed at discovering methods of solving transportation problems have been carried out over time. The Transportation problem is a special kind of the network optimization problems. It has the special data structure in solution, characterized as a transportation group (Shweta, Dubey, & Rajesh, 2012). Some of the initial work on transportation problems was done by Hitchcock (1941) and Koopmans (1947) as quoted by Lee (1973). The common conclusion is that transportation problem can be expressed as a linear programming problem. Transportation problem is one of the original applications of linear programming models (Uba & Abdussalam, 2013). The basic Transportation problem was originally developed by Hitchcock (Hitchcock, 1941). Linear programming is the method of solving practical problems as a means of linear functions where the variables involved are subject to constraints. It is a mathematical technique used in computer modeling to find the best possible solution in allocating limited resources (Ezennorom & Nitah, 2013). Nitah & Ezennorom (2013) listed some characteristics of a problem that makes it qualified to be called a linear programming problem. These characteristics include:

1. The presence of a well-defined objective function which is to be optimized and can be represented as linear function of decision variables.
2. The presence of constraints or restrictions on the extent of reaching the objective. Also, these constraints must be representable as linear equations or inequalities in terms of variables.
3. The presence of alternative courses of action. That is, there should be different routes to take that can lead to a particular destination.
4. All decision variables must be interrelated and non-negative.
5. The resources must be limited in supply.

We shall therefore illustrate a practical transportation problem and show that it is a linear programming problem satisfying each characteristic of linear programming problems.

## Illustration

Triumph Inc. has three manufacturing plants which are located at Elele, Okija and Akpugo, these plants supply products to three regional distribution warehouses located at Anambra, Edo and Benue. Let's denote the cost of transporting one unit of product from plant  $i$  to destination  $j$  as  $C_{ij}$ , the number of units transported as  $X_{ij}$ . The information can be illustrated table 1:

Table 1 Representation of a transportation problem

	1 Anambra	2 Edo	3 Benue	Supply
1 Elele	$X_{11} C_{11}$	$X_{12} C_{12}$	$X_{13} C_{13}$	$S_1$
2 Okija	$X_{21} C_{21}$	$X_{22} C_{22}$	$X_{23} C_{23}$	$S_2$
3 Akpugo	$X_{31} C_{31}$	$X_{32} C_{32}$	$X_{33} C_{33}$	$S_3$
Demand	$D_1$	$D_2$	$D_3$	

This type of table is usually referred to as a transportation table.

### Objective function

Let  $Z$  be the total cost of transportation from the different sources to the different destinations, then we can say:

$$Z = C_{11}X_{11} + C_{12}X_{12} + C_{13}X_{13} + C_{21}X_{21} + C_{22}X_{22} + C_{23}X_{23} + C_{31}X_{31} + C_{32}X_{32} + C_{33}X_{33}$$

Therefore,

$$Z = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} \cdot X_{ij}$$

Hence, the objective function which can then be optimized (i.e. minimize total cost of transportation) has been gotten.

### Constraints

#### Supply Limitation

None of the three plants can supply more than it has in stock. Hence, there is a supply limitation which can be represented as:

$$X_{11} + X_{12} + X_{13} \leq S_1 \text{ (for Elele)}$$

$$X_{21} + X_{22} + X_{23} \leq S_2 \text{ (for Okija)}$$

$$X_{31} + X_{32} + X_{33} \leq S_3 \text{ (for Akpugo)}$$

#### Demand Limitation

The allocation made to each of the three destinations is limited to the amount of products demanded by such destination. Hence, there is a demand limitation which can be represented as:

$$X_{11} + X_{21} + X_{31} \leq D_1 \text{ (for Anambra)}$$

$$X_{12} + X_{22} + X_{32} \leq D_2 \text{ (for Edo)}$$

$$X_{13} + X_{23} + X_{33} \leq D_3 \text{ (for Benue)}$$

It is already obvious at this stage that the problem satisfies the 3<sup>rd</sup>, 4<sup>th</sup> and 5<sup>th</sup> characteristics.

Therefore, the linear programming formulation of the problem above is:

$$\text{Min } Z = \sum_{i=1}^3 \sum_{j=1}^3 C_{ij} \cdot X_{ij}$$

Subject to the constraints

$$X_{11} + X_{12} + X_{13} \leq S_1$$

$$X_{21} + X_{22} + X_{23} \leq S_2$$

$$X_{31} + X_{32} + X_{33} \leq S_3$$

$$X_{11} + X_{21} + X_{31} \leq D_1$$

$$X_{12} + X_{22} + X_{32} \leq D_2$$

$$X_{13} + X_{23} + X_{33} \leq D_3$$

$$X_{11}, X_{12}, \dots, X_{33} \geq 0$$

The manual computation of the derived linear programming problem as one can anticipate at this point will be very herculean and time consuming.

## TYPES OF TRANSPORTATION PROBLEMS

Transportation problems come in two types:

1. The balanced case
2. The unbalance case. (Eze, Obiegbu, & Jude-Eze, 2005)

The balanced case is when the total supply is equal to the total demand while unbalanced case is when total supply is not equal to the total demand. That is,

$$\sum_{i=1}^m S_i = \sum_{j=1}^n D_j \quad \text{For the balanced case while,}$$

$$\sum_{i=1}^m S_i \neq \sum_{j=1}^n D_j \quad \text{For the unbalanced case,}$$

Where  $S_i$  represents the number of products at source  $i$  and  $D_j$  represents the number of demand at destination  $j$  for  $m$  sources and  $n$  destinations.

Before computing solutions to a transportation problem, it must be balanced.

An unbalanced problem can be transformed into a balanced problem. When the total supply exceeds total demand, the problem can be balanced by creating a fictitious destination (also called dummy destination) which has a demand equal to excess supply. Since nothing is being transported to this destination, the corresponding transport cost is zero. An additional column would be required in the transportation table to handle the dummy destination. When the optimal solution has been gotten finally, any product transported to the dummy destination from a particular plant only means there is an unused capacity at the plant. A similar thing is done when the total demand exceeds the total supply. A dummy source is created which can supply an amount equal to the excess demand. An additional row would be added to the transportation table and transport costs from this source to any destination would be zero. It is important to understand that balancing a problem is only for computational purposes only. Though the total demand is not equal to total supply, the objective function would still be optimized at the end with the available resources.

## SETTING UP THE TRANSPORTATION TABLE

Setting up the transportation table is to summarize conveniently and concisely all relevant data and keep track of algorithm computations (Imam, Elsharawy, Gomah, & Samy, 2009). Each cell in the table represents the amount of products transported from one source to one destination. The amount placed in each cell is the value of a decision variable for that cell (Fedra, 2004). Therefore, the transportation table for the Triumph problem would be like this in table 2:

Table 2 Transportation table for the Triumph Inc. problem

	Anambra	Edo	Benue	Supply
Elele	1	8	6	50000
Okija	3	7	8	45000
Akpugo	4	9	10	40000
Demand	35000	55000	45000	135000

When computation begins, the values for each cell would be placed in them accordingly.

## FINDING THE INITIAL FEASIBLE SOLUTION

A set of variables is said to be a feasible solution if it satisfies the constraints as well as the non-negativity restrictions (Ezennorom & Nitah, 2013). Several methods have been introduced over time for finding a starting solution and they include: Northwest Corner Method (NWCM), Least Cost Method (LCM), Russell's Approximation Method (RAM) developed by Russell L. Ackoff, West Churchman, and Leonard Arnoff (Russell, West, & Leonard, 1957) and Vogel's Approximation Method (VAM) developed by Reinfeld and Vogel (Reinfeld & Vogel, 1958) etc. The "quality" of results obtained from these methods differ but they yield an acceptable initial feasible solution. An acceptable initial feasible solution needs to be basic (Uba & Abdussalam, 2013). A feasible solution to a transportation problem with  $m$  origins and  $n$  destinations is said to be basic if the number of positive allocations are equal to  $m+n-1$ , Imam *et al.* (2009). Solutions with less than  $m+n-1$  allocations are called degenerate solutions while those with more than  $m+n-1$  allocations are called non-basic solutions (Eze, Obiegbu, & Jude-Eze, 2005). Vogel's Approximation Method is not quite as simple as the northwest corner approach or Least Cost Method, but it facilitates a very good initial solution - one that is often the optimal solution (Uba & Abdussalam, 2013).

**NORTHWEST CORNER METHOD (NWCM)**

This method starts in the upper left-hand corner of the transportation table and allocates as much units to each route as possible subject to not exceeding the amount available from the source and amount required at the destination (Elsie, 2007). Any satisfied row or column is crossed out. If the supply is exhausted first, an allocation is made to the destination from the next plant. However, if the allocation satisfies the demand of the destination, an allocation is made to the next destination. As this process is continued, there is a movement towards the southeast corner of the transportation table with all the supplies and demands satisfied eventually.

**LEAST COST METHOD (LCM)**

Here, the cost per unit in each cell is taken into consideration. We first select the cell with the lowest cost for allocation. The maximum allowable unit for that column is allocated to this cell to satisfy the demand of that destination. Then allocation is made to next lowest cost cell while taking into consideration the remaining capacities and requirements. If a tie occurs between two or more least-cost cells, the cell where the largest allocation quantity can be made is selected in preference to the others for allocation. This process is then continued till all demands are met and supplies are emptied.

**VOGEL'S APPROXIMATION METHOD (VAM)**

Vogel's Approximation Method is an improved version of the least cost method that generally produces better starting solutions (Imam et al., 2009). It makes use of the minimum cost allocation. It involves a series of steps repeated in the same manner until the final allocation is made (Eze, Obiegbu, & Jude-Eze, 2005). The steps involved are given as:

1. Inspect the costs in each row and take the difference between the two smallest of these costs in the same row. (For a maximization problem, the difference between the two largest is taken). These differences are shown at the end of each row.
2. Perform the same operation along the columns- subtracting the smallest cost from the second smallest in the same column. These differences are shown at the end of each column.
3. The needed units are allocated to the column with the largest difference. The allocation is made in the row with the lowest cost or highest profit. However, if the highest difference had occurred in a row instead of a column, the allocation will be made to the cell in that row with the lowest cost or highest profit.
4. Any satisfied row or column is marked out and the row and column differences are disregarded. Then we continue from step one continuously until all allocations are completed. (adapted from (Eze, Obiegbu, & Jude-Eze, 2005)):

The differences are called opportunity loss or penalty resulting from not choosing the smallest cost route by either source or destination (Eze, Obiegbu, & Jude-Eze, 2005).

**TESTING THE STARTING SOLUTION FOR OPTIMALITY**

Searching for optimal solutions entails us examining each empty cell in the transportation table to see if we can get any improvement in the total cost of transportation by allocating some units to these cells. Any solution gotten from these new allocations still has to satisfy the demand and supply constraints. Allocation of units to an unused cell may yield better and more economical solutions than the one initially gotten from the methods for finding starting solutions (Eze, Obiegbu, & Jude-Eze, 2005). We shall consider two methods of testing for optimality of starting solutions. The methods are:

1. Modified distribution methods (MODI method)
2. Stepping stone algorithm (or method)

**MODIFIED DISTRIBUTION (MODI) METHOD**

The Modified Distribution (MODI) method allows us to compute improvement indices for each unused square quickly without drawing all of the closed paths. A closed path or loop is a sequence of cells in the transportation table such that the first cell is unused and all the other cells are used with the following conditions:

- i. Each pair of consecutive used or occupied cells lie in the same row or column.
- ii. No three consecutive used or occupied cells lie in the same row or column.
- iii. The first and last cells of a sequence lies in the same row or column.
- iv. No cell appears more than once in a sequence

v. Only horizontal and vertical moves are allowed and can only change directions at used or occupied cells. This makes it possible to save time as compared to other methods for solving transportation problems.

MODI provides a new means of finding the unused route with the largest negative improvement index. Once the largest index is identified, we are required to trace one closed path only. This path helps determine the maximum number of units that can be shipped via the best unused route.

### **STEPPING-STONE METHOD**

Steps involved are;

1. Select an unused cell and draw a closed path through cells that are currently used and back to the unused cell.
2. For every traced path or loop, begin with a plus (+) sign at the starting unused cell and alternately place a minus (-) and plus (+) sign at each used cell.
3. Calculate an Improvement Index by first adding the unit-cost figures found in each cell containing a plus sign and subtracting the unit costs in each square containing a minus sign.
4. If all indices calculated are greater than or equal to zero (positive) then, an optimal solution had been reached. If not, select the path or loop that has the most negative value and use this to further improve the solution. To further improve the current solution, select the smallest number found in the path or loop containing a minus (-) sign. This number is added to all cells on the closed path or loop with plus (+) signs and subtracted from all cells on the path with minus (-) signs.
5. A new basic feasible solution will be computed.
6. Steps 1 through 4 are repeated to calculate an Improvement Index for all unused or empty cells in order to test whether an optimal solution has been reached (University of the City of Manila, 2012).

### **DEGENERACY**

As stated earlier, sometimes the number of allocations turns out to be less than  $m+n-1$ . This condition is known as degeneracy. When there is a degenerate solution, further improvements on the problem will not be possible. Hence, degeneracy needs to be resolved before proceeding further with the optimality test (Eze, Obiegbo, & Jude-Eze, 2005). In order to resolve degeneracy, the conventional method is to allocate an infinitesimally small amount  $\epsilon$  to that unoccupied cell, which has the minimum transportation cost. Once this is done, the test of optimality is applied and if necessary, the solution is improved in the normal way until optimality is reached.

### **MAXIMUM PROFIT ALLOCATION PROBLEM**

In most cases, solutions to transportation problems are focused on minimizing the total cost of transportation. The transportation problem solving technique was originally developed to handle problems of minimization type (Eze, Obiegbo, & Jude-Eze, 2005). However, recent developments have made it possible to adapt to problems of maximization type. Two methods of achieving this is considered.

#### **METHOD 1**

This method is based on the concept that the minimization of the negative of a function is equivalent to the maximization of the problem (Eze, Obiegbo, & Jude-Eze, 2005). Hence, when there is a need to maximize a production output or profit return, all we need to do is multiply the profits by -1 and apply the minimization approach accordingly. We can also perform the optimality test on the allocation obtained using any method for doing so. One important thing to note is that optimality is reached whenever all the improvement indexes are positive.

#### **METHOD 2**

This method is based on the concept of opportunity cost. Using this method, we select the largest profit value or the largest contribution per unit in the upper-right hand of the cells in the transportation table. We then obtain the difference between this largest value and other profit elements in each cell. This is called the opportunity cost associated with each cell. The opportunity cost is then used to substitute the original profit per unit allocation at each cell. When this is done, the problem has been transformed to a minimization of the opportunity cost and any procedure for obtaining optimal solutions to minimization problems can be applied to the transportation data (Eze, Obiegbo, & Jude-Eze, 2005).

From the information provided above, it can be seen that the whole process involved in computing optimal solutions for transportation problems is very herculean and can contain arithmetic errors. Most researches

carried out on transportation problems often produce more methods of solving for optimal solutions but very few develop efficient software capable of using these methods. The few who have developed software for this purpose either develop web-based software or desktop based software which is not user friendly. The details of calculations displayed by this software were not clearly displayed to the user. Also, none of the software encountered during the study implemented security. Therefore, this study aims at developing a piece of software that will fill this gap.

## METHODOLOGY

The data collected during this study were collected through the internet and by going through textbooks. Textbooks were studied to understand how the manual computation of solutions to transportation problems is done. From these textbooks and internet sources, various exercises and examples of transportation problems were gotten. The program design is illustrated with the use of flowcharts, diagrams, tables and charts. The user is expected to provide certain information for the system to carry out its functions. This information will be gotten with the aid of a graphical user interface which the user will interact with.

The user will provide his username and password here in order to gain access to the software. Unregistered users can click on the sign up button to go to the sign up form. The main outputs of the system are the optimal solution, the initial basic feasible solution, and the details of iterations. The system also enables the user to know those who developed the software.

### The Triumph Inc. problem

Triumph Inc. has three manufacturing plants which are located at Elele, Okija and Akpugo, these plants supply products to three regional distribution warehouses located at Anambra, Edo and Benue. The transportation table for this problem is shown in table 3:

Table 3 Triumph Inc. Problem

	Anambra	Edo	Benue	Supply
Elele	1	8	6	50000
Okija	3	7	8	45000
Akpugo	4	9	10	40000
Demand	35000	55000	45000	135000

## RESULTS AND DISCUSSION

The system was developed using the Java programming language. Java is a high level programming language with several characteristics which were favorable to the development of the system. The codes were written in the NetBeans 8.1 IDE which at the time of coding was the latest version of NetBeans.

If the user clicks on the sign-in button without entering any value in the username and password fields or the user does not exist in the database or the user leaves any input field blank, the system will bring up an error message requiring the user to do the correct thing. Also if the user enters two different passwords, the system will bring up an error message informing the user of the difference in the inputted passwords.

If the user enters the correct data into the fields, the system outputs a message showing registration successful.

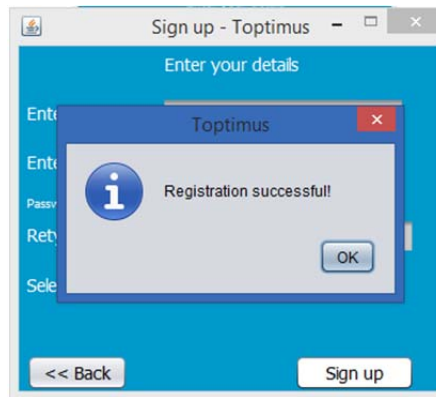


Figure 1 User Registration

The system was tested by different people such as software developers and students using the following process. First the user signed up on the sign up form.

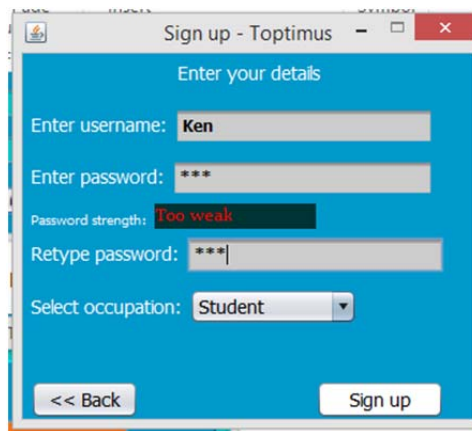


Figure 22 The user signs up

Next, the user created a new study by clicking on the Start new study button.

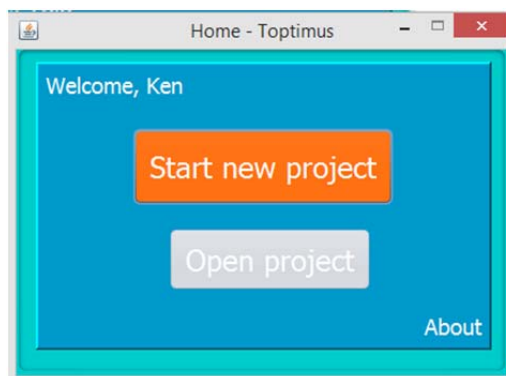


Figure 3 The user starts a new study

The user then provided a study name study name form.



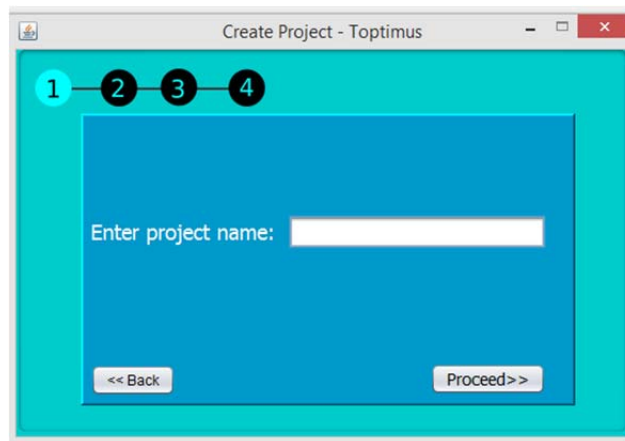


Figure 4 The user enters a study name

The user then entered the number of sources and destinations and then added the name and capacity of source and destination, saves the input then clicked next.

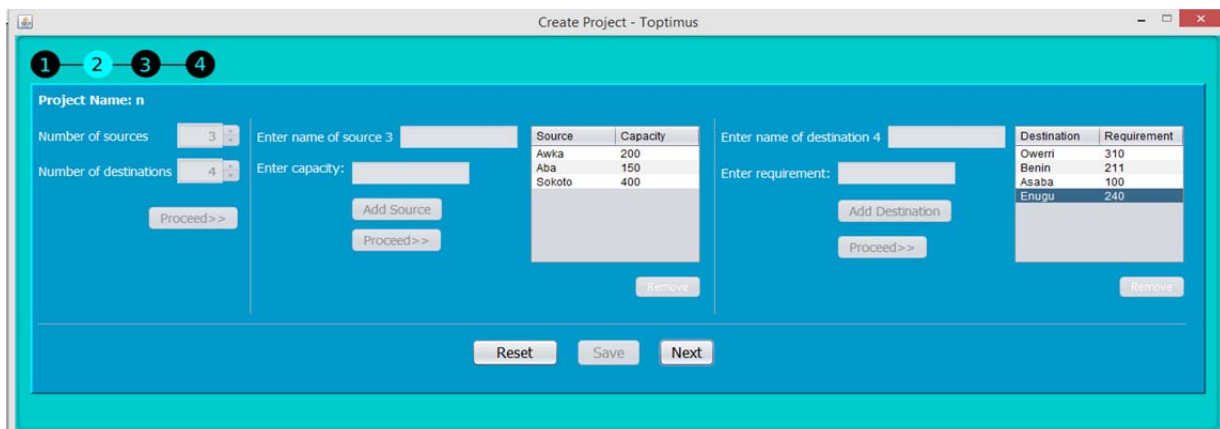


Figure 5 The user enters the details of the transportation problem

Next, the cost of the transportation was added and the currency code was selected also.

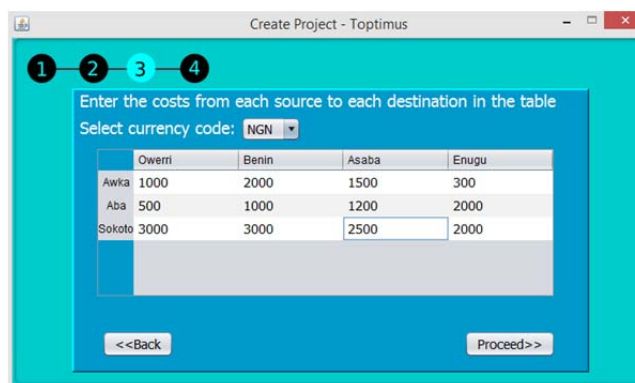


Figure 6 The user enters the transportation costs

Next, the user selected the objective function and methods for solving the problem and clicked on finish.

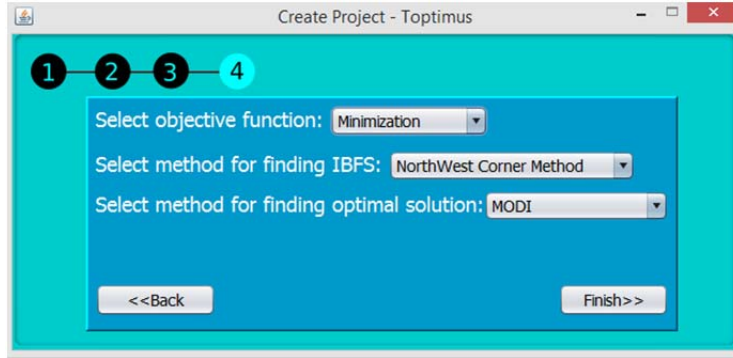


Figure 7 The user selects the objective function and methods to use

This is the first output form which shows the optimal solution gotten by using the selected objective function and IBFS method and also the allocation.

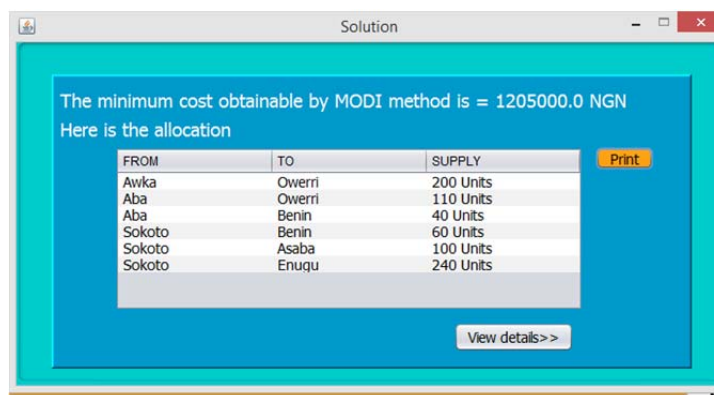


Figure 83 The solution is displayed to the user

And finally, the details of the solution were displayed on the Details form.

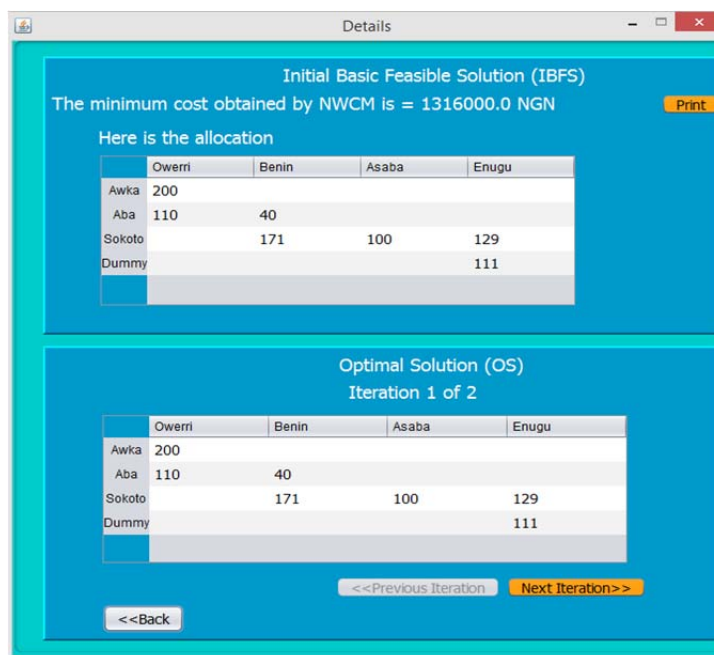


Figure 9 The details of iterations is displayed to the user

## CONCLUSION

This report analyzes the processes and the problems associated with manual computation of optimal solutions to transportation problems. It explained several methods used in obtaining initial basic feasible solutions, testing for optimality and finding the optimal solution. The report describes how the processes involved were automated by providing details of the system analysis including the methods of data collection, the system requirements, the functional requirements, non-functional requirements, the system design including the Flowcharts, the Entity-Relationship Model and the Use Cases for the developed system is also presented. The way the system was implemented and was also presented.

This study has been able to automate the North-West Corner, Least Cost, Vogel's Approximation, Modified Distribution and Stepping Stone methods for finding optimal solutions to transportation problems. However, other methods used in computing optimal solutions still exist but due to time constraints, such methods were not automated. We recommend that such methods can be automated by researchers who wish to continue this work. Also, the system developed in this study was desktop-based which is limited to usage by only users who have the software installed on their computers. It will however be of more advantage if the system is made web-based as this will enable different users access the system from different locations and with different devices.

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