Multivariable H∞ Robust Controller for a Permanent Magnet Synchronous Machine (PMSM)

J. Khedri, M. Chaabane, M. Souissi

Automatic Control Unit, National School of Engineers of Sfax, ENIS
Road of soukra, km 3.5 - BP.W, 3038, Sfax, Tunisia
E-mail: khedrijamel@.yahoo.fr

Abstract

Analytical modeling of the PMSM often involves many simplifying assumptions. The model thus obtained is a model where the physical phenomena and dynamics are neglected during the identification of the process such that the saturation of the magnet circuit and the skin effect. However, the variation of stator resistances, cyclic inductances and moment of inertia due to the temperature and extensive use induce a difference between the process and the nominal mathematical model. Besides since the load torque is external disturbance that can affect the PMSM model, the traditional methods (Bode, Nyquist design…) fail to satisfy stability and robust performance. To solve this problem, a robust multivariable H infinity controller is used seeing that the desired performance can be merged in the design phase as performance weights. The effectiveness of the proposed controller is applied for the speed tracking during load changing of a PMSM fed by voltage inverter through experimental study.

Keywords- Robust control, uncertainty, PMSM

I. INTRODUCTION

With the recent advances of computer technologies, control theories and material technologies, more and more linear motors are employed in application requiring highly static and dynamic performance, especially in embedded systems. Among these applications, PMSM is frequently used [1]. This is due to its high power density, its higher efficiency, lower inertia, weight reduction and volume [2], [3]. It has the advantage over other DC and AC machines for its constant excitation and its control is simplified because it does not involve any auxiliary device in the inductor formed by the permanent magnet. In these applications, vector control, which allows decoupling between control variables remains the most widely used. Obviously, the presence of uncertainties like external disturbances load and parametric variations [4] especially the stator resistances and cyclic inductances induce a variation of the model process and limit the performance of the control law

Thus, our main contribution in this paper, consist in considering an efficient robust method called multivariable H∞ control that is insensitive to these variations.

First, we consider an augmented model composed primarily of the system which one wishes to control and filter weightings. Second, a robust multivariable H∞ controller is developed. Finally the proposed control law is applied to the regulation of the speed and d-axis current of a PMSM under load changing and parametric variations.

The paper is organized as follows: in section 2, the problem formulation is given and the model of the PMSM is briefly described. Section 3 deals with the design of the proposed robust multivariable H∞ controller. Experimental results applied to a permanent magnet synchronous machine (PMSM) are presented in section 4.

II. PROBLEM STATEMENT

To design the robust multivariable H_∞ controller, we use the state space approach to solve this standard problem [5], [6], [7], [8], [9],[10]. This consist to find an output feedback controller *K(s)* such that the H_∞ norm of the closed-loop transfer function matrix $F_l(P(s), K(s))$ is less than a prescribed positive constant γ

$$\begin{aligned} &F_l(P(s), K(s)) \text{ stable} \\ &\left\| F_l(P(s), K(s)) \right\|_\infty < \gamma \end{aligned} \tag{1}$$

Where P is the interconnection matrix for control design.

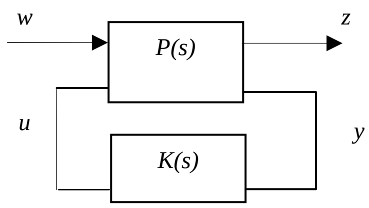

Fig 1 Feedback system

In this approach, the system is represented by a multivariable model with two control inputs and two outputs which we want to enslave

State Model of the Permanent Magnet Synchronous Machine (PMSM)

The used Permanent Magnet Synchronous Machine (PMSM) is fed by a two levels voltage inverter. To obtain non alternatives quantities and in view of conventional assumption, the mathematical model in the rotor reference [11] of the PMSM can be represented by the following set of equations:

$$
\begin{aligned}
&L_d \frac{dI_d}{dt} = -R_s I_d + L_q w_r I_q + vd \\
&L_q \frac{dI_q}{dt} = -R_s I_q - L_d w_r I_d - \phi w_r + vq \\
&J \frac{dw_r}{dt} = p(c_e - c_r) - f_c w_r \\
&J \frac{dw_r}{dt} = p^2 [(L_d - L_q) I_d + \phi] I_q - f_c w_r - p c_r
\end{aligned}
\qquad (2)
$$

where

$$ce = p[(L_d - L_q)I_d + \phi]I_q$$

I_d and I_q are the d-q axis stator currents , vd and vq are the d-q axis stator voltages, R_s is the stator resistance, L_d and L_q are the d-q axis stator inductances, w_r is the motor speed, f_c is the friction coefficient; J is the moment of inertia , ϕ is the flux linkage, c_e is the electromagnetic torque, c_r is the load torque and p is the number of pole .

The synthesis of the robust controller is made commencing with PMSM nominal model. This nominal model is deducted from a linearization of the model (2) around an operating point. Thus the nominal control model is formed by the following transfer matrix of variables to control namely $G_{idnom}(s)$ and $G_{wrnom}(s)$.

$$G_{nom}(s) = \begin{bmatrix} G_{idnom}(s) & 0 \\ 0 & G_{wrnom}(s) \end{bmatrix} \qquad (3)$$

where

$$G_{idnom}(s) = \frac{\frac{1}{R_S}}{1 + \frac{L_d}{R_S} s},$$

$$G_{wrnom}(s) = \frac{\frac{p^2 \phi}{J L_q}}{s^2 + \left(\frac{f_C}{J} + \frac{R_S}{L_q}\right)s + \frac{R_S f_C + p^2 \phi^2}{J L_q}} \qquad (4)$$

As we mentioned above, the machine parameters most likely to vary during operation, in different conditions, are the electrical and mechanical time constants (τ_{elc} and τ_{mec}).

These variations are mainly related to:

- Variations of the stator resistances R_s (depending on the temperature and the skin effect due to the frequency variations of the stator currents especially during the transitional phases).

- Changes in moment of inertia J which appears after prolonged use, recording generally increased.
- Changes of flow due to permanent magnets. Any increase may occur due to manufacturing a new one. In contrast, the permanent flow may decrease after a long term use of the PMSM.

Taking into account these different variations leads us to gather all the model uncertainties in an unstructured multiplicative form. This form takes in consideration a wide range of uncertainties and increases them by weighting function $W_T(s)$ that is to select and specify. Then we define an uncertain model $\tilde{G}(s)$ around the nominal model G(s) and we assume that the uncertainty parameters represent a variation of ±10% of all machine parameters from its nominal values.

One considers disturbance plans below:

Case 1:

$(R_s)_{nom} + \Delta(R_s)_{nom}$,
$(L_q)_{nom} + \Delta(L_q)_{nom}, (\phi)_{nom} + \Delta(\phi)_{nom}$,
$(J)_{nom} + \Delta(J)_{nom}, (f_c)_{nom} + \Delta(f_c)_{nom}$

Case 2:

$(R_s)_{nom} + \Delta(R_s)_{nom}$,
$(L_q)_{nom} - \Delta(L_q)_{nom}, (\phi)_{nom} - \Delta(\phi)_{nom}$,
$(J)_{nom} + \Delta(J)_{nom}, (f_c)_{nom} - \Delta(f_c)_{nom}$

Case 3:

$(R_s)_{nom}, (L_q)_{nom} - \Delta(L_q)_{nom}$,
$(\phi)_{nom} + \Delta(\phi)_{nom}, (J)_{nom}$,
$(f_c)_{nom} - \Delta(f_c)_{nom}$

Case 4:

$(R_s)_{nom} + \Delta(R_s)_{nom}$,
$(L_q)_{nom} - \Delta(L_q)_{nom}, (\phi)_{nom} + \Delta(\phi)_{nom}$,
$(J)_{nom} + \Delta(J)_{nom}, (f_c)_{nom}$

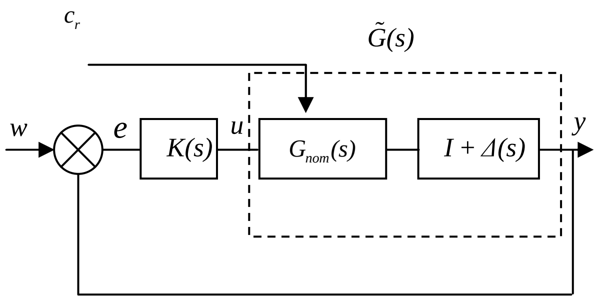

Fig 2 Feedback uncertain configuration

Notice:

- w is the reference signal which is in our study, the current I_{dref} or the speed w_{ref} .
- any disruption due to variations in PMSM parameters can be represented by a function $\Delta(s)$.

c_r is the load torque which is considered as an external disturbance

III. ROBUST MULTIVARIABLE H∞ DESIGN CONTROLLER

To start with the robust multivariable H∞ design controller, let's consider the configuration diagram of Fig 2, where $\tilde{G}(s)$ represents the transfer function of the uncertain plant. $K(s)$ is the controller to be designed

To guarantee H_∞ performance and ensure robustness against all the uncertainties on the closed loop system, one incorporate weighting filters to output signals e, u and y in closed loop configuration of Fig 2. The error e is weighted by the filter $W_s(s)$, the command u by $W_u(s)$ and the output y by $W_T(s)$.

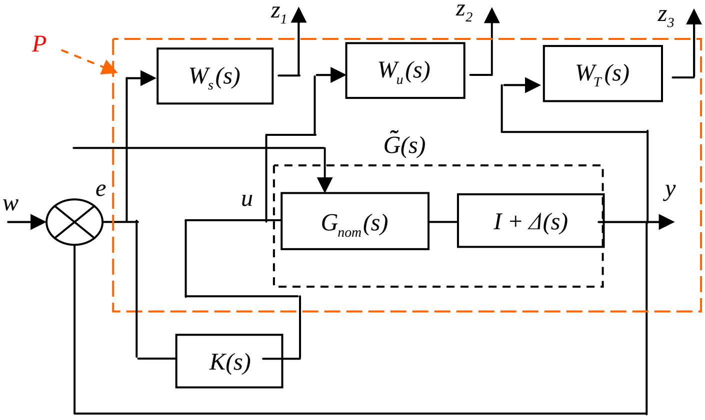

Fig 3 Feedback uncertain configuration with weighting functions

$$\begin{aligned} z_1(s) &= W_s(s)S(s)w(s) \\ z_2(s) &= W_u(s)K(s)S(s)w(s) \\ z_3(s) &= W_T(s)K(s)\tilde{G}(s)S(s)w(s) \end{aligned} \quad (5)$$

Where

$S(s) = \dfrac{1}{1 + G_{nom}(s)K(s)}$ is the sensitivity function

$T(s) = \dfrac{K(s)G_{nom}(s)}{1 + G_{nom}(s)K(s)}$ is the complementary sensitivity function

The interconnexion matrix P shown in Fig 1 has outputs z_1, z_2 and z_3 where have incorporated the weighting functions. The external disturbance c_r, the d-axis current I_{dref} and the speed reference w_{ref} are the inputs. The purpose of the regulation is to achieve a speed response similar to the reference one when the load torque is applied. Thus we can write:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ e \end{bmatrix} = \begin{bmatrix} W_s & -W_s P_{11} & -W_s P_{12} \\ 0 & -W_T P_{11} & W_T P_{12} \\ 0 & W_u & 0 \\ 1 & -P_{11} & -P_{12} \end{bmatrix} \begin{bmatrix} w_{ref} \\ w_r \\ c_r \end{bmatrix}$$

Where $P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix}$

The filter $W_s(s)$ is used to set closed-loop system performance in a way to minimize the effect of disturbances on the sizes to regulate and minimize the tracking error. It was selected so that:

- The modulus margin is at least 6db.
- The tracking of the reference input to the output is of a static error less than 0.01%
- The time response of the system is equal to 100ms and the overshoot of the closed loop is almost null.

$$W_s(s) = \text{diag}(\frac{ks+\alpha}{s}, \frac{ks+\alpha}{s})$$
$$W_s(s) = \begin{bmatrix} \frac{0.003659s+2.1522}{s} & 0 \\ 0 & \frac{0.003659s+2.1522}{s} \end{bmatrix} \quad (6)$$

Notice: α adjusts the rise time. Over α is greater, over the rise time is shorter, over the closed loop is fast

The filter $W_u(s)$ acts directly on the transfer KS connecting the reference input to the command u. In order to limit the complexity of the controller K and reduce its order, $W_u(s)$ is chosen simply as the scalar function:

$$W_u(s) = \text{diag}(\lambda, \lambda)$$
$$W_u(s) = \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix} \quad (7)$$

The filter $W_T(s)$ is selected to set uncertainties norm that the closed-loop system must accept [8, 12].

Based on above fixed rates uncertainties, the weighting function $W_T(s)$ is given as follows:

$$W_T(s) = \text{diag}(\frac{as+b}{cs+d}, \frac{as+b}{cs+d}) \quad (8)$$

where

$a = 0.0033, b = 1, c = 0.00538$ and $d = 0.75$

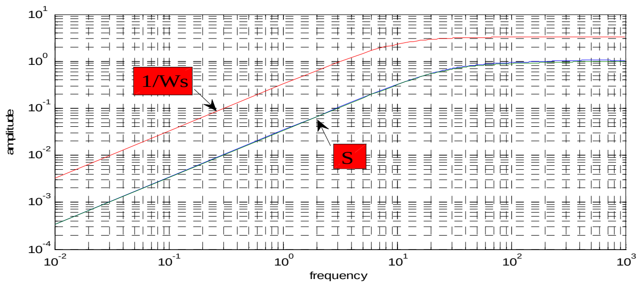

Fig. 4 Frequency response of S and $1/W_s$ singular values

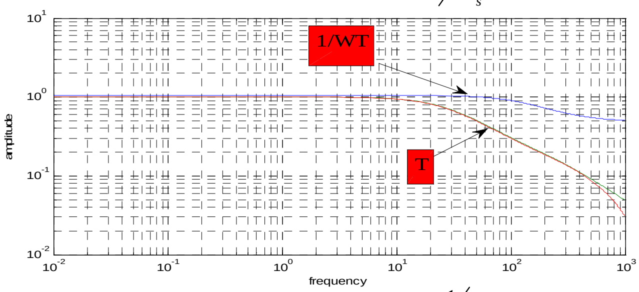

Fig. 5 Frequency response of T and $1/W_T$ singular values

We can notice that the robust stability is ensured because $\bar{\sigma}(T(s)) < \frac{1}{|W_T|}$

where $\bar{\sigma}$ is the maximal singular value

Notice:

- Synchronous machines, does not require a flow model. Thus, the rotor position is the angle of reference. Furthermore, as we have a smooth poles machine, the best selection for its motion is obtained for a value where the internal angle of the machine is equal to $\frac{\pi}{2}$ that means $I_d = 0$

The obtained robust controller calculated by the method H_∞, satisfying the inequality (1) is:

$$K(s)=\begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix} \qquad (9)$$

Where

$$K_{11}=\frac{N_{11}(s)}{D(s)};K_{12}=\frac{N_{12}(s)}{D(s)};K_{21}=\frac{N_{21}(s)}{D(s)};K_{22}=\frac{N_{22}(s)}{D(s)}$$

$$\begin{aligned}
N_{11}(s) &= 1495s^6+3.039e6s^5+3.032e9s^4 \\
&\quad +1.096e12s^3+1.57e14s^2+7.726e015s+87.56 \\
N_{12}(s) &= -9.991s^6-9.542e4s^5-1.513e9s^4 \\
&\quad -5.901096e11s^3-5.392e13s^2-9.869e13s+6031 \\
N_{21}(s) &= 9.9881s^6+9.339e4s^5+1.717e8s^4 \\
&\quad +1.6046e11s^3+4.548e13s^2+3.66e15s+163.6 \\
N_{22}(s) &= 1516s^6+2.542e7s^5+1.343e10s^4 \\
&\quad +2.35e12s^3+1.49e14s^2+1.969e15s-2533 \\
D(s) &= s^7+1.825e4s^6+3.479e7s^5+3.299e10s^4 \\
&\quad +1.119e13s^3+1.208e15s^2+363.7s+5.165e-12
\end{aligned}$$

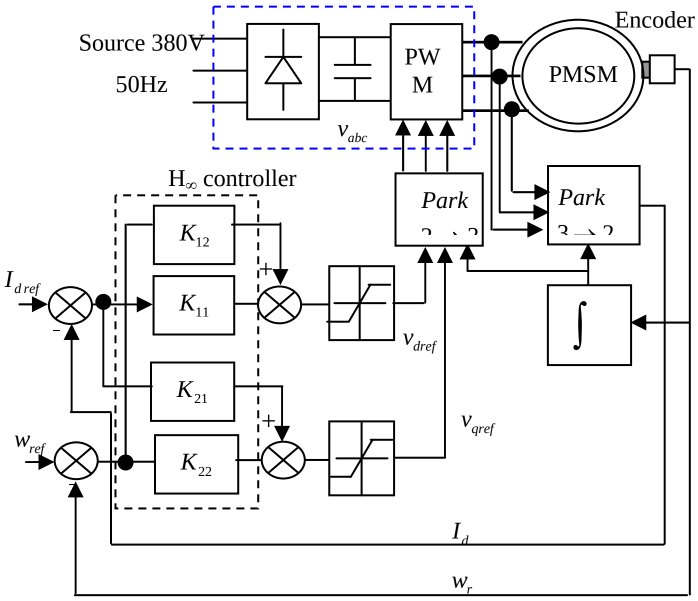

Fig 6 Overall scheme of the multivariable H_∞ robust controller of the PMSM

IV. EXPERIMENTAL RESULTS AND INTERPRETATION

Experimental results are carried out for the PMSM with the specifications listed as in Table 1

TABLE I. PARAMETER OF THE PMSM.

Motor rated power	1KW
Rated current	6.5A
Pole pair number (p)	2
d-axis inductance Ld	4.5mH
q-axis inductance Lq	4mH
Stator resistance	0.56 Ω
Motor inertia J	$2.08.10^{-3}$Kg.m^2
Friction coefficient fc	$3.9.10^{-3}$Nm.s.rad^{-1}
Magnet flux constant ϕ	0.064wb

A Dspace 1104 based Matlab/ Simulink software control algorithm is used for compilation and implementation with a frequency of 10 KHz.

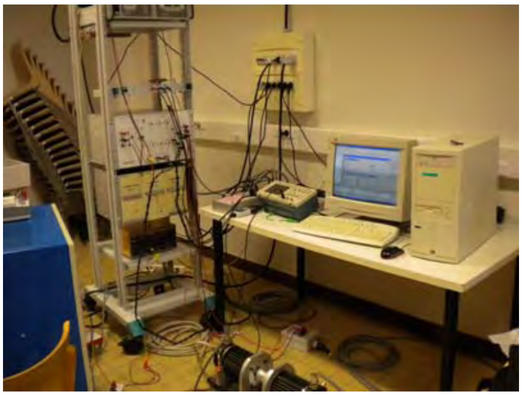

Fig 7 Photo of the test benchmark

Figures (8) and (9) elucidate the experimental results of the d-axis current and the speed

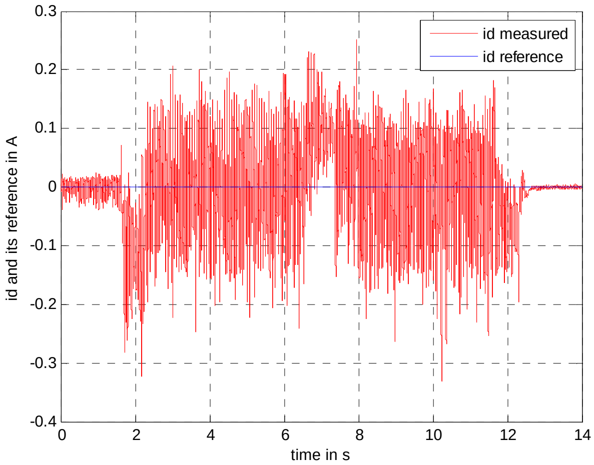

Fig 8 d-axis current and its reference

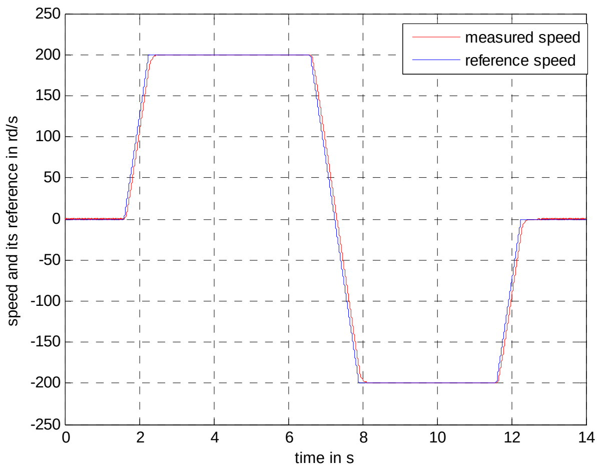

Fig 9 Speed and its reference

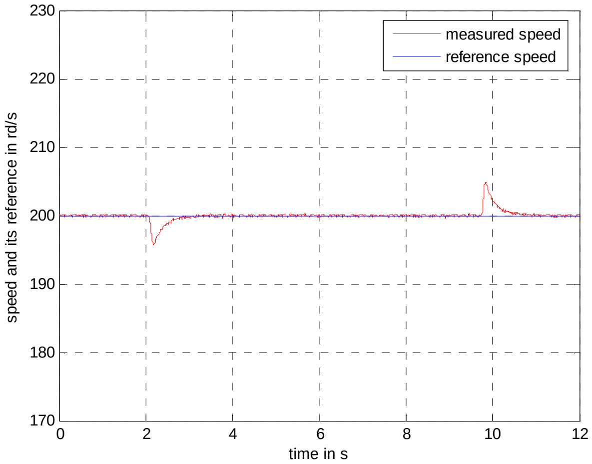

Fig 10 Influence of a disturbance on the speed response

In Fig 10, the experimental result shows the response of the speed after the application of a load torque. It is clear that the controller allows efficient discharge of the applied perturbation. Indeed the speed is maintained at the reference value.

V. Conclusions

This paper, settled a multivariable robust H_∞ controller for current and speed tracking of a PMSM. The controller's dynamics are picked in respect with the integration of weighting functions. The designed controller has been fulfilled in the aim to guarantee closed-loop stability and to provide robust performances through weighting functions. Experimental results are satisfactory

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AUTHORS PROFILE

Jamel Khedri received the Ph. D degree from National School of Engineers of Sfax (ENIS), University of Sfax, Tunisia, in 2010. His currents research interests robust control, fuzzy logic control, D-stability analysis and applications of these techniques to induction motors.

Mohamed Chaabane received the Ph. D degree in Electrical Engineering from the University of Nancy, French in 1991. He was associate professor at the University of Nancy and is are searcher at Center of Automatic Control of Nancy (CRAN) from 1988-1992. Actually is a professor in ENI-Sfax. Since 1997, he is holding are search position at Automatic Control Unit, ENIS. The main research interests are in the filed of robust control, delay systems, descriptor systems and applications of theses techniques to fed-batch processes and agriculture systems. Currently, he is an associate editor of International Journal on Sciences and Techniques of Automatic control computer engineering, IJ-STA (www.sta-tn.com).

Mansour SOUISSI received his Ph. D in Physical Sciences from the University of Tunis in 2002. He is Professor in Automatic Control at Preparatory Institute of Engineers of Sfax, Tunisia. Since 2003, he is holding a research position at Automatic Control Unit, National School of Engineers of Sfax, Tunisia. His current research interests robust control, optimal control, fuzzy logic, linear matrix inequalities and applications of these techniques to agriculture systems. Dr Souissi is a member of the organization committees of several national and international conferences (STA, CASA)