On high order methods for solution of non-linear equation

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***Abstract*— The objective of this paper is two folds, first to derive a new series of third order methods for solving non-linear equation *f(x)=0* involving third, fourth and fifth derivatives of f and secondly to compare the existing methods (for the solution of *f(x)=0*) having various derivatives of *f* by calculating their time estimates on different examples. It is seen that the new methods take the least time among all other methods in its own family of particular derivatives, using same value of tolerance as stopping criteria.**

Keywords - Nonlinear equation; Root-finding Methods; Convergence; Computational efficiency

I. INTRODUCTION

There has been considerable literature on derivation of high order methods involving first, second and third order derivatives of *f(x)* for solution of non-linear equation *f(x)=0*[1]. Most of the methods discussed in literature generally focus on two aspects, one order of the method and second its efficiency.

In real time system many engineering and industry applications exists which needs solution of *f(x) = 0* in minimal possible time, e.g. in rocket propulsion, one needs to solve a non-linear equation in least amount of time so that proper correction to the trajectory can be continuously maintained [2]. Thus solving such type of non-linear equations *f(x) = 0* in minimum amount of time is essential. This aspect does not appear to have been considered anywhere in literature. One of the aspects of the paper is to compare several methods appearing in literature of order *2*, *3* and *4* and having f' , f'' and f''' in their formula by establishing the time taken by a method to solve the equation using the same value of tolerance as stopping criterion. We find that in most cases the Newton's method [1] perform better than all other methods having a single derivatives of *f(x)*, whereas Halley's method [3] perform best when f' and f'' are known. We next derive a series of methods of order *3* having derivatives f''' , f^{iv} and f^{v} , which perform better than all methods of its own family of derivatives. The rates of convergence of methods are proved. In the following we list some of the important methods existing in literature.

Methods based on first derivatives:

Method 1: The second order method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad (1)$$

Proposed by Newton Rapshon method in 1711 [1]. Its efficiency is 1.414.

Method 2: The forth order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = y_n - \frac{f(y_n)}{f'(y_n)} - \frac{1}{2}\left[\frac{f(y_n)}{f'(y_n)}\right]^2 \left[\frac{f'(x_n)}{f(x_n)}\right]\left[\frac{f'(x_n)+f'(y_n)}{f'(y_n)}\right] \qquad (2)$$

is a two-step predictor-corrector type method, given by Noor and Gupta in 2007[4]. Its efficiency is 1.414.

Method 3: The forth order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n + \frac{1}{f'(x_n)}\left[\frac{f^2(x_n)}{f(y_n)-f(x_n)} - \frac{f^2(y_n)}{f(x_n)}\right] \qquad (3)$$

proposed by Maheshweri in 2009 [5]. Its efficiency is 1.5873.

Method 4: The forth order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{3f(x_n)}{2f'(x_n)} + \frac{f(x_n)f'(y_n)}{2[f'(x_n)]^2} \qquad (4)$$

is a two-step iterative method, proposed by Noor-Hassan-Noor in 2007 [6]. Its efficiency is 1.5873.

Method 5: The third order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = -\frac{f(y_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{f(y_n+z_n)}{f'(x_n)} \qquad (5)$$

is a three-step iterative method, proposed by Noor & Noor in 2006 [7]. Its efficiency is 1.31607.

Method 6: The eighth order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = x_n - \left[\frac{f(y_n)}{f'(x_n)}\right]\left[\frac{f(x_n)-f(y_n)}{f(x_n)-2f(y_n)}\right]$$

$$x_{n+1} = z_n - \frac{f(z_n)}{f'(x_n)}\left[\frac{1}{2} + \frac{5f^2(x_n)+8f(x_n)f(y_n)+2f^2(y_n)}{5f^2(x_n)-12f(x_n)f(y_n)}\left(\frac{1}{2} + \frac{f(z_n)}{f(y_n)}\right)\right] \qquad (6)$$

proposed by Wang & Lin in 2010 [8]. Its efficiency is 1.68179.

Method 7: The third order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{f(x_n)\left[2+3f'^2(x_n)-f'(x_n)f'(y_n)\right]}{f'(x_n)+2f'^3(x_n)+f'(y_n)} \qquad (7)$$

was referred by Chun & Kim [9] in 2010. Its efficiency is 1.44219.

Method 8: The third order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{1}{2}\left[3 - \frac{f'(y_n)}{f'(x_n)}\right]\frac{f(x_n)}{f'(x_n)} \tag{8}$$

was referred by Chun & Kim [9] in 2010. Its efficiency is 1.44219.

The other methods based on second derivatives are:

Method 1: The third order method

$$x_{n+1} = x_n - \frac{2f(x_n)f'(x_n)}{2f'^2(x_n) - f(x_n)f''(x_n)}$$

(9)

derived by Halley [3] in 1998. Its efficiency is 1.44219.

Method 2: The third order Chebyshev Method [1] is

$$x_{n+1} = x_n - \frac{f^2(x_n)f''(x_n) + 2f(x_n)f'^2(x_n)}{f'^3(x_n)}$$

(10)

Its efficiency is 1.44219.

Method 3: The third order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n + \frac{-f'(x_n) \pm \sqrt{f'^2(x_n) - 2f(x_n)f''(y_n)}}{f''(y_n)}$$

(11)

Is a two-step predictor-corrector type method, given by Noor & Noor in 2007 [10]. Its efficiency is 1.44219.

Method 4: Noor & Noor gives a three step method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = -\frac{(y_n - x_n)^2}{2f'(x_n)} f''(x_n)$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{(y_n - x_n)^2}{2f'(x_n)} f''(x_n) - \frac{(y_n + z_n - x_n)^2}{2f'(x_n)} f''(x_n)$$

(12)

in 2007 [11].

Method 5: Noor gives another two step method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = -\frac{(y_n - x_n)^2}{2f'(x_n)} f''(x_n)$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{(y_n - x_n)^2}{f'(x_n)} f''(x_n) - \frac{2f(y_n)}{f'(x_n)} + \frac{(y_n - x_n)^2 f'(y_n)}{2f'^2(x_n)} f''(x_n) + \frac{f(y_n)f'(y_n)}{f''^2(x_n)}$$

$$+\frac{(y_n-x_n)f(y_n)}{f'^2(x_n)}f''(x_n)+\frac{(y_n-x_n)^3 f''^2(x_n)}{2f'^2(x_n)} \qquad (13)$$

in 2007 [12].

Method 6: The third order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} - \frac{1}{2}\left[\frac{f^2(x_n)f''(y_n)}{f'^3(x_n) - f(x_n)f'(x_n)f''(x_n)}\right] \qquad (14)$$

is a two-step method, given by Rafis & rafiullah in 2009 [13]. Its efficiency is 1.31607.

Method 7: The sixth order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = y_n - \frac{f(y_n)}{f'(y_n)} - \frac{f^2(y_n)f''(y_n)}{2f'^3(y_n)} \qquad (15)$$

derived by Noor, Noor & momani in 2007 [14]. Its efficiency is 1.43096.

Method 8: The third order method

$$x_{n+1} = x_n - \frac{f(x_n)f'(x_n)\left[f(x_n)f''(x_n)+2+2f'^2(x_n)\right]}{2f'^2(x_n)\left[1+f'^2(x_n)\right]-f(x_n)f''(x_n)}$$

(16)

was referred by Chun & Kim [9] in 2010. Its efficiency is 1.44219.

Method 9: The third order method

$$x_{n+1} = x_n - \frac{f^2(x_n)f''(x_n)+2f(x_n)f'^2(x_n)}{2f'^3(x_n)}$$

(17)

was referred by Chun & Kim [9] in 2010. Its efficiency is 1.44219.

The methods based on fourth derivatives are

Method 1: The fourth order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = y_n - \left[\frac{(y_n-x_n)^2}{2f'(x_n)}f''(x_n) + \frac{(y_n-x_n)^3}{6f'(x_n)}f'''(x_n) + \frac{(y_n-x_n)^4}{24f'(x_n)}f^{iv}(x_n)\right]$$

(18)

proposed by Saeed & Aziz in 2008 [15]. Its efficiency is 1.319507.

Method 2: The fourth order method

$$y_n = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$z_n = -\left[\frac{(y_n-x_n)^2}{2f'(x_n)}f''(x_n) + \frac{(y_n-x_n)^3}{6f'(x_n)}f'''(x_n) + \frac{(y_n-x_n)^4}{24f'(x_n)}f^{iv}(x_n)\right]$$

$$x_{n+1} = y_n - \left[\frac{(y_n+z_n-x_n)^2}{2f'(x_n)} f''(x_n) + \frac{(y_n+z_n-x_n)^3}{6f'(x_n)} f'''(x_n) + \frac{(y_n+z_n-x_n)^4}{24f'(x_n)} f^{iv}(x_n)\right] \quad (19)$$

proposed by Saeed & Aziz in 2008 [15]. Its efficiency is 1.319507.

II. THE PROPOSED METHODS

By Taylor series, we have

$$f(x_n+h) = f(x_n) + hf'(x_n) + \frac{h^2}{2!}f''(x_n) + \frac{h^3}{3!}f'''(x_n) + O(h^4)$$

Retaining on the right hand sides the terms up to h^3 and equating to 0, we have

$$f(x_n+h) = f(x_n) + hf'(x_n) + \frac{h^2}{2!}f''(x_n) + \frac{h^3}{3!}f'''(x_n) = 0 \quad (20)$$

We replace one 'h' in the coefficient of $f''(x_n)$ by Hally's method's 'h' given by [3]

Whereas in coefficient of $f'''(x_n)$, one 'h' is replace by h given as in Hally's method [3] and one 'h' as in Newton-Rapshon method [1], and solving the resulting equation we obtain

$$h = -\frac{3f(2f'^2 - ff'')}{6f'^3 - 6ff'f'' + f^2f'''} \quad (21)$$

Here $$x_{n+1} = x_n - \frac{3f(2f'^2 - ff'')}{6f'^3 - 6ff'f'' + f^2f'''} \quad (22)$$
we call this method is H3. Its efficiency is 1.31607.

We expand equation (1) by taylor series for forth order as

$$f(x_n+h) = f(x_n) + hf'(x_n) + \frac{h^2}{2!}f''(x_n) + \frac{h^3}{3!}f'''(x_n) + \frac{h^4}{4!}f^{iv}(x_n) + O(h^5) = 0 \quad (23)$$

In the coefficient of $f''(x_n)$ in equation (4), take first 'h' from HN method. In the coefficient of $f'''(x_n)$, take first 'h' from H3 method, second 'h' from Halley's method. In the coefficient of $f^{iv}(x_n)$, take first 'h' from H3 method, second 'h' from Hally's method and third 'h' from Newton Rapshon method. Now solve the equation (4) for h that gives

$$h = -\frac{4f(6f'^3 - 6ff'f'' + f^2f''')}{24f'^4 - 36ff'^2f'' + 8f^2f'f''' + 6f^2f''^2 - f^3f^{iv}} \quad (24)$$

$$x_{n+1} = x_n - \frac{4f(6f'^3 - 6ff'f'' + f^2f''')}{24f'^4 - 36ff'^2f'' + 8f^2f'f''' + 6f^2f''^2 - f^3f^{iv}} \quad (25)$$

We call this method H4. Its efficiency is 1.24573.

In similar case we expand taylor series for fifth order, and put the values of 'h'

$$f(x_n+h) = f(x_n) + hf'(x_n) + \frac{h^2}{2!}f''(x_n) + \frac{h^3}{3!}f'''(x_n) + \frac{h^4}{4!}f^{iv}(x_n) + \frac{h^5}{5!}f^{v}(x_n) + O(h^6) = 0 \quad (26)$$
Solve it and get h as

$$h = -\frac{5f(24f'^4 - 36ff'^2f'' + 8f^2f'f''' + 6f^2f''^2 - f^3f^{iv})}{120f'^5 - 240ff'^3f'' + 60f^2f'^2f''' + 90f^2f'f''^2 - 20f^3f''f''' - 10f^3f'f^{iv} + f^4f^{v}} \quad (27)$$

$$x_{n+1} = x_n - \frac{5f(24f'^4 - 36ff'^2f'' + 8f^2f'f''' + 6f^2f''^2 - f^3f^{iv})}{120f'^5 - 240ff'^3f'' + 60f^2f'^2f''' + 90f^2f'f''^2 - 20f^3f''f''' - 10f^3f'f^{iv} + f^4f^v} \qquad (28)$$

This method is called as H5. Its efficiency is 1.200849.

Theorem: Consider for a nonlinear equation f(x) = 0, if *f(x)* is sufficiently differentiable, then the method H3 converges with order 3.

Proof: As given H3 method in equation (23)

$$x_{n+1} = x_n - \frac{3f(2f'^2 - ff'')}{6f'^3 - 6ff'f'' + f^2f'''}$$

On substituting $x_n = \xi + \varepsilon_n$ in this equation and expanding $f(\xi + \varepsilon_n)$, $f'(\xi + \varepsilon_n)$, $f''(\xi + \varepsilon_n)$ and $f'''(\xi + \varepsilon_n)$ in Taylor's series about the point ξ, we obtain

$$\varepsilon_{n+1} = \varepsilon_n$$

$$- \left(\frac{3\left[f(\xi) + \varepsilon_n f'(\xi) + \frac{\varepsilon_n^2}{2} f'(\xi) + \cdots\right]\left[2\left\{f'^2(\xi) + 2\varepsilon_n f'(\xi)f''(\xi) + \varepsilon_n^2\left(f''^2(\xi) + f'(\xi)f'''(\xi)\right) + \cdots\right\} - \left\{\varepsilon_n f'(\xi)f''(\xi) + \frac{\varepsilon_n^2}{2}\left(f''^2(\xi) + 2f'(\xi)f'''(\xi)\right) + \cdots\right\}\right]}{\begin{array}{c} 6\left\{f'^3(\xi) + 3\varepsilon_n f'^2(\xi)f''(\xi) + \varepsilon_n^2\left(3f'(\xi)f''^2(\xi) + \frac{3f'^2(\xi)f'''(\xi)}{2}\right)\cdots\right\} - \\ 6\left\{\begin{array}{c} f(\xi)f'(\xi)f''(\xi) + \left(f(\xi)f'(\xi)f'''(\xi) + f'^2(\xi)f''(\xi) + f(\xi)f''^2(\xi)\right) + \\ \frac{\varepsilon_n^2}{2}\left(2f'^2(\xi)f'''(\xi) + f(\xi)f'(\xi)f^{iv}(\xi) + 3f'(\xi)f''^2(\xi) + 3f(\xi)f''(\xi)f'''(\xi)\right) + \cdots \end{array}\right\} \\ + \left\{f^2(\xi)f'''(\xi) + \varepsilon_n\left(f^2(\xi)f^{iv}(\xi) + 2f(\xi)f'(\xi)f'''(\xi)\right) + \varepsilon_n^2\left(\frac{f^2(\xi)f^v(\xi)}{2} + 2f(\xi)f'(\xi)f^{iv}(\xi) + f'^2(\xi)f'''(\xi) + f(\xi)f''(\xi)f'''(\xi)\right) + \cdots\right\} \end{array}} \right)$$

Put the value $f(\xi) = 0$, and simplifying it

$$\varepsilon_{n+1} = \varepsilon_n - \frac{\varepsilon_n\left\{6f'^3(\xi)\right\} + \varepsilon_n^2\left\{9f'^2(\xi)f''(\xi) + 3f'^2(\xi)f''(\xi)\right\} + \cdots}{6f'^3(\xi) + \varepsilon_n\left\{12f'^2(\xi)f''(\xi)\right\} + \varepsilon_n^2\left\{9f'^2(\xi)f''^2(\xi) + 4f'^2(\xi)f'''(\xi)\right\} + \cdots}$$

$$\varepsilon_{n+1} = \varepsilon_n - \frac{1}{6f'^3(\xi)}\left[\varepsilon_n\left\{6f'^3(\xi)\right\} + \varepsilon_n^2\left\{9f'^2(\xi)f''(\xi) + 3f'^2(\xi)f''(\xi)\right\} + \cdots\right]$$

$$\left[1 + \varepsilon_n\left\{\frac{2f''(\xi)}{f'(\xi)}\right\} + \varepsilon_n^2\left\{\frac{9f''^2(\xi) + 4f'''(\xi)}{6f'(\xi)}\right\} + \cdots\right]^{-1}$$

$$\varepsilon_{n+1} = \varepsilon_n - \left[\varepsilon_n - \varepsilon_n^3\left\{\frac{4f''^2(\xi)}{f'^2(\xi)} + \frac{9f''^2(\xi) + 4f'''(\xi)}{6f'(\xi)}\right\} - O(\varepsilon_n^4)\right]$$

On neglecting ε_n^4 and higher powers of ε_n, we get

$$\varepsilon_{n+1} = \alpha\varepsilon_n^3,$$

Where $\alpha = \frac{4f''^2(\xi)}{f'^2(\xi)} + \frac{9f''^2(\xi) + 4f'''(\xi)}{6f'(\xi)}$

Thus H3 method has third order convergence. Similarly third order convergence of H4 and H5 methods can be proved.

III. NUMERICAL EXPERIMENT

All computations were done on PC using UNIX 32-bit Operating System, GCC 4.6.0 compiler with Intel(R) Core(TM) i3 M330 processor and 4 GB RAM. We accept an approximate solution rather than the exact root. We use the following stopping criteria for computer programs. (i) $|x_{n+1} - x_n| < 10^{-15}$ (ii) $|f(x_n)| < 10^{-15}$ and so on, when the stopping criterion is satisfied x_{n+1} is taken as the exact root α computed.

We use the following test functions to perform several methods of order 2, 3 and 4 available in literature

(a) $f_1(x) = x^{10} - 1$, $x_0 = 0.5$, $\alpha = 1.0$

(b) $f_2(x) = (x-2)^{23} - 1$, $x_0 = 4.5$, $\alpha = 3.0$

(c) $f_3(x) = (x-3)^2 - e^{2x-3} - 3x + 11$, $x_0 = 5.5$, $\alpha = 2.272$

(d) $f_4(x) = e^{x^2+7x-30} - 1$, $x_0 = 3.5$, $\alpha = 3.0$

The numerical results presented in table show that the proposed methods in this contribution have at least equal performance with respect to time as compared with the other methods. Here x_0 represent the approximate starting value of root and IT represents the total number of iterations. In all the tables, the methods are given according its corresponding equation numbers in this paper.

Table 1: Methods based on 1st derivative

Methods	f_1 (sec)		f_2 (sec)		f_3 (sec)		f_4 (sec)	
	Time(10^{-7})	IT	Time(10^{-7})	IT	Time(10^{-7})	IT	Time(10^{-7})	IT
2	51	14	36	9	18	5	21	5
3	Div		57	15	19	6	26	7
4	Div		69	19	25	8	33	9
5	Div		106	23	41	10	48	10
6	Div		39	8	16	4	19	4
1 (NRM)	69	43	46	26	17	11	22	12
7	Div		85	19	26	8	39	9
8	Div		84	19	26	8	39	9

Table 2: Methods based on 2nd derivative

Methods	f_1 (sec)		f_2 (sec)		f_3		f_4 (sec)	
	Time(10^{-7})	IT	Time(10^{-7})	IT	Time(10^{-7})	IT	Time(10^{-7})	IT
2	51	14	36	9	18	5	21	5
3	Div		57	15	19	6	26	7
4	Div		69	19	25	8	33	9
5	Div		106	23	41	10	48	10
6	Div		39	8	16	4	19	4
1 (NRM)	69	43	46	26	17	11	22	12
7	Div		85	19	26	8	39	9
8	Div		84	19	26	8	39	9

Table 3: Methods based on 3rd derivative

Methods	f_1 (sec)		f_2 (sec)		f_3 (sec)		f_4 (sec)	
	Time(10^{-7})	IT	Time(10^{-7})	IT	Time(10^{-7})	IT	Time(10^{-7})	IT
22(H3)	13	4	34	10	14	5	18	5

Table 4: Methods based on 4th derivative

Methods	f_1 (sec)		f_2 (sec)		f_3 (sec)		f_4 (sec)	
	Time(10^{-7})	IT	Time(10^{-7})	IT	Time(10^{-7})	IT	Time(10^{-7})	IT
18	Div		83	18	31	8	35	8
19	Div		76	15	29	7	35	7
25(H4)	16	4	31	7	14	4	18-4	4

Table 5: Methods based on 5th derivative

Methods	f_1 (sec)		f_2 (sec)		f_3 (sec)		f_4 (sec)	
	Time(10^{-7})	IT	Time(10^{-7})	IT	Time(10^{-7})-	IT	Time(10^{-7})	IT
28(H5)	14	3	32	6	13	3	16	3

IV. CONCLUSION

Several examples show that the methods H1, H2 presented in this paper e.g. see tables, perform better than other methods as far as the timing estimates are concerned. Most of the well-known methods diverge on example (a). Our methods H3, H4 and H5 give good results on this example using the same starting value. Although methods (15) and (16) do converge, however time taken by our method is much less. Our methods give better time estimates than all methods in its own family of derivatives. Many of the methods, which have better efficiency than our methods take more time to compute solution of *f(x)=0*.

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