Edge Detection in Gray Level Images Based on Non-Shannon Entropy

El-Owny, Hassan Badry Mohamed A.
Department of Mathematics, Faculty of Science ,Aswan University , 81528 Aswan, Egypt.
Current: CIT College, Taif University, Taif, KSA.
hassanowny02@yahoo.com

***Abstract*— Digital image processing is a subset of the electronic domain, wherein the image is converted to an array of small integers, called pixels, representing a physical quantity. Edges characterize boundaries and edge detection is one of the most difficult tasks in image processing hence it is a problem of fundamental importance in image processing. The problem of edge detection although it is fundamental and is existing since years but it is still an area where there is still scope of research. It has been found that the previous used algorithms or methods were not able to produce ideal or optimized results. This paper presents an efficient techniques based on Non-Shannon measures of entropy for edge detection. Our objective is to find the best edge representation. A set of experiments in the domain of edge detection are presented. The experimental results show that the new technique often yield more efficient results comparing with classic methods.**

Keywords- Edge Detection; Threshold Value ; Non-Shannon Entropy.

I. INTRODUCTION

Computer vision is currently finding wide application in a large variety of fields such as aerospace, biology, medical science, geology, astronomy, engineering and etc. Since the increasing demands on production automation and quality control, industrial application of digital image processing system are also rapidly growing. Detection of edge in an image is a very important step toward complete image understanding. Since edges are important features of an image, there is a lot of significant information contained in edges of an image. Edges often correspond to object boundaries, shadow transition. It helps to extract useful features for pattern recognition.

Many algorithms for edge detection have been proposed[1-12]. In general, edge detection can be classified in two categories: gradient operators and second derivative operators[2-10]. Due to an edge in an image corresponds to an intensity change abruptly or discontinuity, step edge contain large first derivatives and zero crossing of the second. In the case of first derivative operation, edge can be detected as local maximum of the image convolved with a first derivative operator[24-27]. Prewitt, Robert, Sobel[9] and Canny[10] implement their algorithm using this idea. For the second derivative case, edges are detected as the location where the second derivative of the image crosses zero. The using of Laplacian of Gaussian convolution mask is the most common method of the second derivative operator[2,5,8].

The goal of this paper is to introduce a new approach based on information theory, which is entropy based thresholding. The proposed method is decrease the computation time as possible as can and the results were very good compared with the other methods.

The paper is organized as follows: Section 2 describes in brief the basic concepts of Shannon and non-Shannon entropies. Section 3 is devoted to the proposed method of edge detection. In Section 4, the details of the edge detection algorithm is described. In Section 5, some particular images will be analyzed using proposed method based algorithm and moreover, a comparison with some existing methods will be provided for these images. Finally, conclusions will be drawn in Section 6.

II. SHANNON AND NON-SHANNON ENTROPY

The entropy is a basic thermodynamic concept that is associated with the order of irreversible processes in the universe. Physically it can be associated with the amount of disorder in a physical system. Shannon [13] redefined the entropy concept of Boltzmann/Gibbs as a measure of uncertainty regarding the information content of a system. He defined an expression for measuring quantitatively the amount of information produced by a process.

In accordance with this definition, a random event A that occurs with probability $P(A)$ is said to contain $I(A) = \ln[1/P(A)] = -\ln[P(A)]$ units of information. The amount $I(A)$ is called the self-information of event A . The amount of self information of the event is inversely proportional to its probability. The basic concept of entropy in information theory has to do with how much randomness is in a signal or in a random event. An alternative way to look at this is to talk about how much information is carried by the signal [14]. Entropy is a measure of randomness.

Let $p_1, p_2, \cdots\cdots, p_k$ be the probability distribution of a discrete source. Therefore, $0 \le p_i \le 1, i = 1,2,\cdots,k$ and $\sum_{i=1}^{k} p_i = 1$, where k is the total number of states. The entropy of a discrete source is often obtained from the probability distribution.

The Shannon Entropy can be defined as

$$H(p) = -\sum_{i=1}^{k} p_i \ln(p_i) \quad (1)$$

This formalism has been shown to be restricted to the domain of validity of the Boltzmann–Gibbs–Shannon (BGS) statistics. These statistics seem to describe nature when the effective microscopic interactions and the microscopic memory are short ranged. Generally, systems that obey BGS statistics are called extensive systems. If we consider that a physical system can be decomposed into two statistical independent subsystems A and B, the probability of the composite system is $p^{A+B} = p^A \cdot p^B$, it has been verified that the Shannon entropy has the extensive property (additive):

$$H(A+B) = H(A) + H(B). \quad (2)$$

From [16]

$$H_\alpha(A+B) = H_\alpha(A) + H_\alpha(B) + \psi(\alpha) \cdot H_\alpha(A) \cdot H_\alpha(B), \quad (3)$$

where $\psi(\alpha)$ is a function of the entropic index. In Shannon entropy $\psi(\alpha) = 1$.

Rènyi entropy[17] for the generalized distribution can be written as follows:

$$H_\alpha^R(p) = \frac{1}{1-\alpha} \ln \sum_{i=1}^{k} (p_i)^\alpha, \ \ \alpha > 0,$$

this expression meets the BGS entropy in the limit $\alpha \longrightarrow 1$. Rènyi entropy has a nonextensive property for statistical independent systems, defined by the following pseudo additivity entropic formula

$$H_\alpha(A+B) = H_\alpha(A) + H_\alpha(B) + (\alpha - 1) \cdot H_\alpha(A) \cdot H_\alpha(B).$$

Tsallis[18,19] has proposed a generalization of the BGS statistics, and it is based on a generalized entropic form,

$$H_\alpha^T(p) = \frac{1 - \sum_{i=1}^{k} (p_i)^\alpha}{\alpha - 1},$$

where k is the total number of possibilities of the system and the real number α is an entropic index that characterizes the degree of nonextensivity. This expression meets the BGS entropy in the limit $\alpha \longrightarrow 1$. The Tsallis entropy is nonextensive in such a way that for a statistical independent system, the entropy of the system is defined by the following pseudo additive entropic rule

$$H_\alpha(A+B) = H_\alpha(A) + H_\alpha(B) + (1 - \alpha) \cdot H_\alpha(A) \cdot H_\alpha(B)$$

The generalized entropies of Kapur of order α and type β [20,21] is

$$H_{\alpha,\beta}(p) = \frac{1}{\alpha - \beta} \ln\left(\frac{\sum_{i=1}^{k} p_i^\alpha}{\sum_{i=1}^{k} p_i^\beta}\right), \ \alpha \ne \beta, \alpha, \beta > 0 \quad (4)$$

In the limiting case, when $\alpha \longrightarrow 1$ and $\beta \longrightarrow 1$, $H_{\alpha,\beta}(p)$ reduces to $H(p)$ and when $\beta = 1$, $H_{\alpha,\beta}(p)$ reduces to $H_\alpha^R(p)$. Also, $H_{\alpha,\beta}(p)$ is a composite function which satisfies pseudo-additivity as:

$$H_{\alpha,\beta}(A+B) = H_{\alpha,\beta}(A) + H_{\alpha,\beta}(B) + (1-\alpha)(1-\beta) \cdot H_{\alpha,\beta}(A) \cdot H_{\alpha,\beta}(B). \quad (5)$$

III. IMAGE THRESHOLDING BASED ON KAPUR ENTROPY

A gray level image can be represented by an intensity function, which determines the gray level value for each pixel in the image. Specifically, in a digital image of size $M \times N$ an intensity function $f(x,y)$ $\{ f(x,y) | \ x \in \{1,2,\cdots,M\}, \ y \in \{1,2,\cdots,N\}\}$, takes as input a particular pixel from the image, and outputs its gray level value, which is usually in the range of 0 to 255 (*if* 256 *levels are used*).

Thresholding produces a new image based on the original one represented by *f*. It is basically another function $g(x,y)$, which produces a new image (i.e. *the thresholded image).* A threshold is calculated for each pixel value. This threshold is compared with the original image (i.e. f) to determine the new value of the current pixel. g can be represented by the following equation [22,23].

$$g(x,y) = \begin{cases} 0, & \text{if} \ \ f(x,y) \le t \\ 1, & \text{if} \ \ f(x,y) > t \end{cases}, \ t \text{ is the thresholding value.}$$

When Entropy applied to image processing techniques, entropy measures the normality (i.e. normal or abnormal) of a particular gray level distribution of an image. When a whole image is considered, the Kapur entropy as defined in (4) will indicate to what extent the intensity distribution is normal. When we extend this concept to image segmentation, i.e. dealing with foreground(Object) and background regions in an image, the entropy is calculated for both regions, and the subsequent entropy value provides an indication of the normality of the segmentation. In this case, two equations are need for each region, each of them called priori.

In image thresholding, when applying maximum entropy, every gray level value is a candidate to be the threshold value. Each value will be used to classify the pixels into two groups based on their gray levels and their affinity, as less or greater than the threshold value (t).

Let $p_1, p_2, \ldots, p_t, p_{t+1}, \ldots, p_k$ be the probability distribution for an image with k gray-levels, where p_t is the normalized histogram i.e. $p_t = h_t/(M \times N)$ and h_t is the gray level histogram. From this distribution, we can derive two probability distributions, one for the object (class A) and the other for the background (class B), are shown as follows:

$$p_A : \frac{p_1}{P_A}, \frac{p_2}{P_A}, \ldots, \frac{p_t}{P_A},$$
$$p_B : \frac{p_{t+1}}{P_B}, \frac{p_{t+2}}{P_B}, \ldots, \frac{p_k}{P_B}, \tag{6}$$

where

$$P_A = \sum_{i=1}^{t} p_i \ , \ P_B = \sum_{i=t+1}^{k} p_i, \ t \text{ is the threshold value.} \tag{7}$$

In terms of the definition of Kapur entropy of order α and type β, the entropy of Object pixels and the entropy of background pixels can be defined as follows:

$$H_{\alpha,\beta}^{A}(t) = \frac{1}{\alpha-\beta} \ln\left(\frac{\sum_{i=1}^{t}\left(\frac{p_i}{P_A}\right)^{\alpha}}{\sum_{i=1}^{t}\left(\frac{p_i}{P_A}\right)^{\beta}}\right), \ \alpha \neq \beta, \alpha, \beta > 0$$
$$H_{\alpha,\beta}^{B}(t) = \frac{1}{\alpha-\beta} \ln\left(\frac{\sum_{i=t+1}^{k}\left(\frac{p_i}{P_B}\right)^{\alpha}}{\sum_{i=t+1}^{k}\left(\frac{p_i}{P_B}\right)^{\beta}}\right), \ \alpha \neq \beta, \alpha, \beta > 0 \ . \tag{8}$$

The Kapur entropy $H_{\alpha,\beta}(t)$ is parametrically dependent upon the threshold value t for the object and background. It is formulated as the sum each entropy, allowing the pseudo-additive property for statistically independent systems, as defined in (5). We try to maximize the information measure between the two classes (object and background). When $H_{\alpha,\beta}(t)$ is maximized, the luminance level t that maximizes the function is considered to be the optimum threshold value. This can be achieved with a cheap computational effort.

$$t^{opt} = \operatorname{Arg\,max}\left[H_{\alpha,\beta}^{A}(t) + H_{\alpha,\beta}^{B}(t) + (1-\alpha)\cdot(1-\beta)\cdot H_{\alpha,\beta}^{A}(t)\cdot H_{\alpha,\beta}^{B}(t)\right]. \tag{9}$$

When $\alpha \longrightarrow 1$ and $\beta \longrightarrow 1$, the threshold value in (4), equals to the same value found by Shannon Entropy. Thus this proposed method includes Shannon's method as a special case. The following expression can be used as a criterion function to obtain the optimal threshold at $\alpha \longrightarrow 1$ and $\beta \longrightarrow 1$.

$$t_{Sh}^{opt} = \operatorname{Arg\,max}\left[H_{\alpha,\beta}^{A}(t) + H_{\alpha,\beta}^{B}(t)\right]. \tag{10}$$

Now, we can describe the Kapur Threshold algorithm to determine a suitable threshold value t^{opt} and α and β as follows:

Algorithm 1: Threshold Value Selection (Kapur Threshold)

1. Input: A digital grayscale image I of size $M \times N$.
2. Let $f(x,y)$ be the original gray value of the pixel at the point $(x,y), (x = 1,2,\cdots,M,\ y = 1,2,\cdots,N)$
3. Calculate the probability distribution $p_i,\quad 0 \le i \le 255$.
4. For all $t \in \{0,1,\ldots,255\}$,
  I. Apply Equations (6) and (7) to calculate $P_A,\ P_B, p_A$ and p_B.
  II. ***if*** $0 < \alpha < 1$ and $0 < \beta < 1$ ***then***
    Apply Equation (9) to calculate optimum threshold value t^{opt}.
  else
    Apply Equation (10) to calculate optimum threshold value t_{Sh}^{opt}.
  end-if
5. **Output**: The suitable threshold value t^{opt} of I, for $\alpha,\beta > 0,\ \alpha \neq \beta$.

IV. EDGE DETECTION ALGORITHM

The process of spatial filtering consists simply of moving a filter mask w of order $m \times n$ from point to point in an image. At each point (x, y), the response of the filter at that point is calculated a predefined relationship. We will use the usual masks for detection the edges. Assume that $m = 2a + 1$ and $n = 2b + 1$, where a, b are nonnegative integers. For this purpose, smallest meaningful size of the mask is 3 × 3, as shown in Fig. 1.

$w(-1,-1)$	$w(-1,0)$	$w(-1,1)$
$w(-0,-1)$	$w(0,0)$	$w(0,1)$
$w(1,-1)$	$w(1,0)$	$w(1,1)$

Fig. 1: Mask coefficients showing coordinate arrangement

$f(x-1, y-1)$	$f(x-1, y)$	$f(x-1, y+1)$
$f(x, y-1)$	$f(x, y)$	$f(x, y+1)$
$f(x+1, y-1)$	$f(x+1, y)$	$f(x+1, y+1)$

Fig. 2

Image region under the above mask is shown in Fig. 2. In order to edge detection, firstly classification of all pixels that satisfy the criterion of homogeneousness, and detection of all pixels on the borders between different homogeneous areas. In the proposed scheme, first create a binary image by choosing a suitable threshold value using Kapur entropy. Window is applied on the binary image. Set all window coefficients equal to 1 except centre, centre equal to × as shown in Fig. 3.

1	1	1
1	×	1
1	1	1

Fig. 3

Move the window on the whole binary image and find the probability of each central pixel of image under the window. Then, the entropy of each Central Pixel of image under the window is calculated as $H(CP) = -p_c \ln(p_c)$.

Table 1 . p and H of central under window.

$\boldsymbol{p}$	1/9	2/9	3/9	4/9	5/9	6/9	7/9	8/9
$\boldsymbol{H}$	0.2441	0.3342	0.3662	0.3604	0.3265	0.2703	0.1955	0.1047

where, p_c is the probability of central pixel CP of binary image under the window. When the probability of central pixel $p_c = 1$ then the entropy of this pixel is zero. Thus, if the gray level of all pixels under the window homogeneous, then $p_c = 1$ and $\boldsymbol{H} = 0$. In this case, the central pixel is not an edge pixel. Other possibilities of entropy of central pixel under window are shown in Table 1.

In cases $p_c = 8/9$, and $p_c = 7/9$, the diversity for gray level of pixels under the window is low. So, in these cases, central pixel is not an edge pixel. In remaining cases, $p_c \leq 6/9$, the diversity for gray level of pixels under the window is high. So, for these cases, central pixel is an edge pixel.

Thus, the central pixel with entropy greater than and equal to 0.2441 is an edge pixel, otherwise not.

The following Algorithm summarize the proposed technique for calculating the optimal threshold values and the edge detector.

Algorithm 2: Edge Detection

1. Input: A grayscale image I of size $M \times N$ and t^{opt} , that has been calculated from algorithm 1.

2. Create a binary image: For all x, y,

$$\textbf{\textit{if}} \;\; I(x,y) \leq t^{opt} \; \textbf{\textit{then}} \; f(x,y) = 0 \; \textbf{\textit{else}} \; f(x,y) = 1.$$

3. Create a mask w of order $m \times n$, in our case ($m = 3, n = 3$)

4. Create an $M \times N$ output image g: For all x and y, Set $g(x,y) = f(x,y)$.

5. Checking for edge pixels:

Calculate: $a = (m-1)/2$ and $b = (n-1)/2$.

For all $y \in \{b+1, \ldots, N-b\}$, and $x \in \{a+1, \ldots, M-a\}$,

$sum = 0$;

For all $l \in \{-b, \ldots, b\}$, and $j \in \{-a, \ldots, a\}$,

if $(f(x,y) = f(x+j, y+l))$ ***then*** $sum = sum + 1$.

if $(sum > 6)$ ***then*** $g(x,y) = 0$ ***else*** $g(x,y) = 1$.

6. Output: The edge detection image g of I.

The steps of our proposed technique are as follows:

Step 1: Find global threshold value (t_1) using Kapur entropy . The image is segmented by t_1 into two parts, the object (Part1) and the background (Part2).

Step 2: By using Kapur entropy, we can select the locals threshold values (t_2) and (t_3) for Part1 and Part2, respectively.

Step 3: Applying Edge Detection Procedure with threshold values t_1, t_2 and t_3.

Step 4: Merge the resultant images of step 3 in final output edge image.

In order to reduce the run time of the proposed algorithm, we make the following steps: Firstly, the run time of arithmetic operations is very much on the $M \times N$ big digital image, I , and its two separated regions, Part1 and Part2. We are use the linear array p (probability distribution) rather than I , for segmentation operation, and threshold values computation t_1, t_2 and t_3. Secondly, rather than we are create many binary matrices f and apply the edge detector procedure for each region individually, then merge the resultant images into one. We are create one binary matrix f according to threshold values t_1, t_2 and t_3 together, then apply the edge detector procedure one time. This modifications will reduce the run time of computations.

V. EXPERIMENTAL RESULTS

To demonstrate the efficiency of the proposed approach, the algorithm is tested over a number of different grayscale images and compared with traditional operators. We selected a real-world images and synthetic images(Fig. 4). The images detected by Canny, LOG, Sobel, Roberts, Prewitt and the proposed method, respectively. All the concerned experiments were implemented on Intel® Core™ i3 2.10GHz with 4 GB RAM using MATLAB R2007b. As the algorithm has two main phases – global and local enhancement phase of the threshold values and detection phase, we present the results of implementation on these images separately.

The proposed scheme used the good characters of Kapur entropy, to calculate the global and local threshold values. Hence, we ensure that the proposed scheme done better than the traditional methods.

In order to validate the results, we run the Canny, LOG, Sobel, Roberts and Prewitt methods and the proposed algorithm 10 times for each image with different sizes. As shown in Fig. 5. It has been observed that the proposed edge detector works effectively for different gray scale digital images as compare to the run time of Canny method.

Some selected results of edge detections for these test images using the classical methods and proposed scheme are shown in Fig.(6-11). From the results; it has again been observed that the performance of the proposed method works well as compare to the performance of the previous methods (with default parameters in MATLAB).

(a) (b) (c) (d)

Fig. 4 Original images

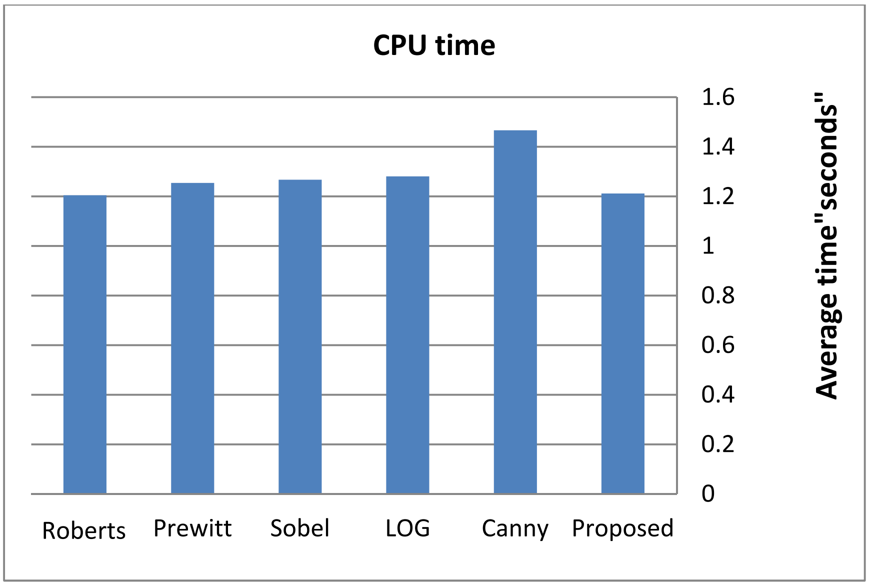
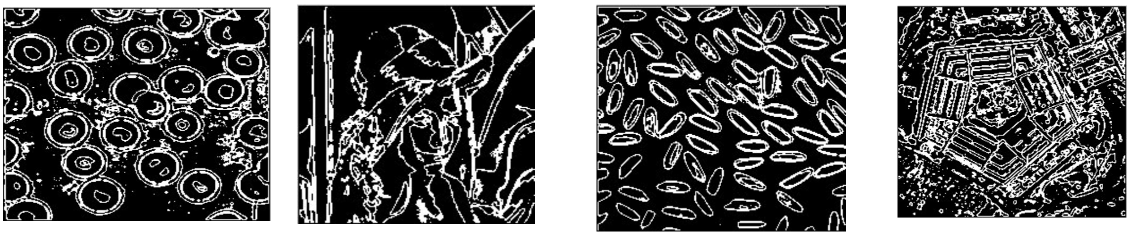


Fig.5: Chart time for proposed method and classical methods
with 512×512 pixel test images



(a) (b) (c) (d)

Fig. 6: Proposed method applied to the test images.

(a) (b) (c) (d)

Fig. 7: Sobel edge detector applied to the test images

(a) (b) (c) (d)

Fig. 8: LOG edge detector applied to the test images

(a) (b) (c) (d)

Fig. 9: Canny edge detector applied to the test images

(a) (b) (c) (d)

Fig. 10: Roberts edge detector applied to the test images

(a) (b) (c) (d)

Fig. 11: Prewitt edge detector applied to the test images

VI. CONCLUSION

An efficient approach using Kapur entropy for detection of edges in grayscale images is presented in this paper. The proposed method is compared with traditional edge detectors. On the basis of visual perception and edgel counts of edge maps of various grayscale images it is proved that our algorithm is able to detect highest edge pixels in images. The proposed method is decrease the computation time as possible as can with generate high quality of edge detection. Also it gives smooth and thin edges without distorting the shape of images. Another benefit comes from easy implementation of this method.

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