

Purchase Decision for ATUR Broadband Network

Chung Yeh¹, Wen-Chin Liu², Rang-Shing Ting³

1. Associate professor, the Department of Industrial Engineering and Systems Management, Feng Chia University, Taichung, 40724, Taiwan, R.O.C.

cveh@fcu.edu.tw

2. Ph.D. Candidate, the Department of Industrial Engineering and Systems Management, Feng Chia University, Taichung, 40724, Taiwan, R.O.C.

bennyliu@ms.aide.com.tw

3. Associate Engineer, Chunghwa Telecom, No.664, Jhongshan Rd., Shalu Township, Taichung County 433, Taiwan, R.O.C.

d2929@cht.com.tw

Abstract - Along with the booming of telecommunication, Internet web has become a vital media in the current global communication and the communication quality of Internet has been emphasized. To the client user, the Internet ATUR (ADSL Transceiver Unit Remote) is the equipment they usually have to the extent of reliable quality. This study targets on the quality and reliability analysis of ATUR under test environment of a students' apartment complex, failure data is collected from after-service department of various ATUR brands and analysis result addresses the MTBF through Weibull distribution which is used to determine ADSL's quality and reliability among brands individually in order to make purchase decision of an ATUR.

Index Term - Keywords: Internet, ADSL (Asymmetric Digital Subscriber Line, Weibull distribution, MTBF(Mean Time Between Failure), Reliability

1. Introduction

In the 21st century, the consumer pays more attention to product's quality and reliability in evidence of its functionality and durability. Besides high quality and reliability, ISP (Internet Service Provider) has to focus on the consumer satisfaction in the long term; Internet technology has created a critical need for an effective way to troubleshoot a system failure remotely. This has stimulated an ISP services ranging from interactive entertainments and social media to pervasive urban sensing, which are pushing towards a continuous increase in the number of Internet users, and their demand for ubiquitous, reliable, secure and high-speed access to the Internet [1]. To cater the increasing bandwidth demand, network technologies with higher capacity are introduced both by wired and wireless network, especially, the wireless-network technology is treated as a joint effort of industry and academia for increasing the capacity of bandwidth.

Reliability is the major key performance index- KPI to quality of communication equipment, the broadband of ADSL is backbone of Internet with ATUR set up for client end which means ATUR reliability is much more orientated to consumer's satisfaction [2,3]. Given that ATUR used existing broadband connections for client access, such access is as reliable as the primary connection is based. The demand for ISP service is influenced by volume of marketing and technical factors that constitute the overall 'offering' which includes price level, price structure and reliability [4]. The finding of studies examining the use of ATUR to various forms related to reliability and purchased decision has to be made.

In this study we used different brands of ATUR to define their KPI and compare their differences. The principle of the Weibull distribution was used to depict reliability, MTBF, and give more evaluation information. The result of the study was to evaluate the different brands of ATUR to guide the consumer on a purchase decision as well as foster advancements in the ATUR technology.

2. Literature review

The development of a broadband and ubiquitous Internet is mainly based on optical network technologies for building high capacity transport and access networks. ISPs(Internet Service Providers) subjected to reach "socially acceptable" equilibrium points in a large ISP population [5]. It has been argued, by that such approaches can lead eventually to a truly global Internet community [6] matter where subscriber logged in [7]. The characteristics of ATUR reliability, however, as a research field has not yet been much explored. The characteristics of product failure mode can be categorized in different periods, infant, steady, aging, the function

of failure is usually depicted as a bathtub curve [8], and the profile of bath tub can be differentiated with failure mode individually. Many scholars start with function of failure rate and consider the behavior of reliability to determine the failure mechanism and its impact on the failure rate and figure out the characteristics of the bath tub curve. In the model of reliability, function of distribution is usually expressed as probability density function, eg. normal distribution, logarithm normal distribution, Weibull distribution, exponential distribution, etc. Because the failure rate can be treated as a function of time and reliability is also mapped with time singularly, we conclude that failure rate can be a function of reliability in these reliability models, eg. The life-time of adapter for power transfer and its distribution can be verified by Weibull distribution. The reliability for adapter is the important factor not only for economical impact, but also for design consideration. The assessment of product's life for ADSL can usually apply Weibull distribution [9]. The field of application includes electronics, electro-medium polyamide, optics communication, etc.

In electronics product, the electronics devices always cause failure in the system; the failure was resulted from impact of heat consumption and mechanical friction is used to proof that the feasibility of a reasonable distribution can be applied on lifetime distribution. [10].

In the past, studies always applied several different distributions on life time analysis, like normal distribution, logarithm normal distribution, Weibull distribution, exponential distribution, etc, to show the failure mode in order to understand the variation of reliability during product life cycle. A product has the mode of constant failure rate will be easy to acquire better parameter to assess the influence of reliability analysis under a practical condition of failure.

3. Theorem of ADSL network and its reliability

3.1 ADSL introduction

ADSL utilize technology of modulation to precede data transfer or web surfing up to 512 Kbps through ATUR setup and current telephone line in which made from dual twist copper line while telephone is in use [11]. The system infrastructure is shown as Fig: 1.

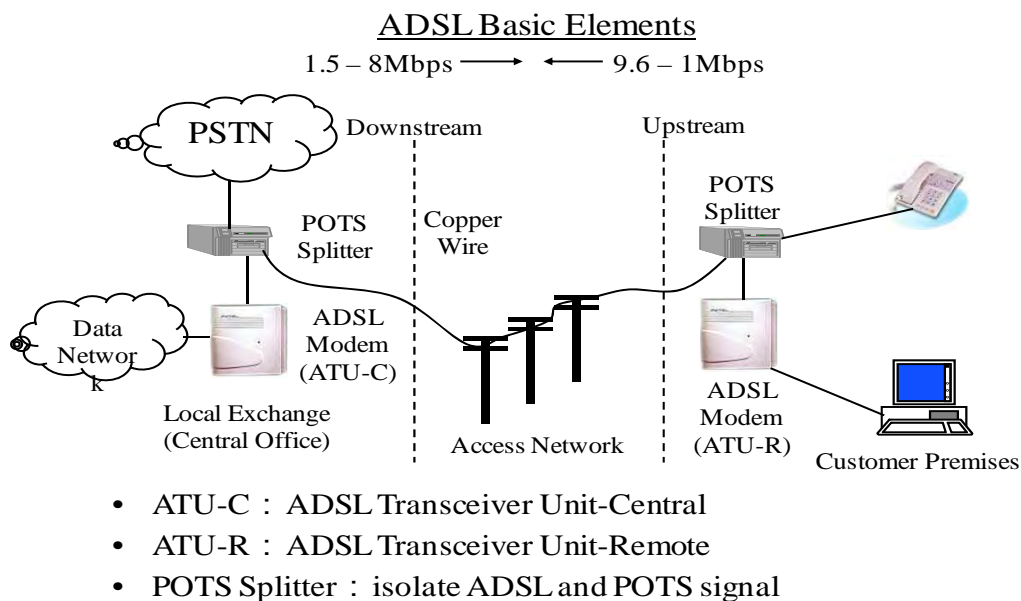


Fig: 1. ADSL network

3.2 Basic concept of ATUR

ATUR stands for ADSL Transceiver Unit Remote, setup on the client's end which equipped with the functions of ADSL Modem and router [12]. The main job of the ATUR is collaboration with local ATUC, ADSL Transceiver Unit Central, which means the type or specification of requirement of ATUR will be determined by the local ATUC.

Generally, ADSL network can be divided into three segments, the first, "base band" is around 4.3KHz on telephone voice, secondly, "upload", its frequency ranges from 30kHz to 138kHz, charge of file transfer of uploading, thirdly, "download", its frequency ranges from 200kHz to 1100kHz, charge of data transfer of downloading, three segments of ADSL are shown as Fig: 2.

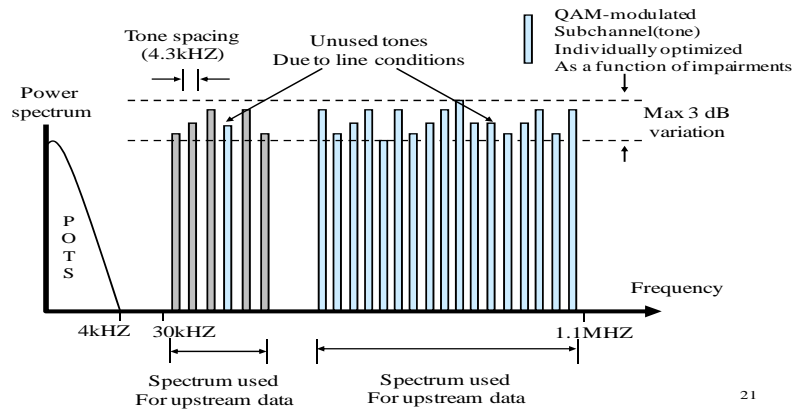


Fig: 2. Band distribution of three segments of ADSL

3.3 Introduction of reliability function

Life-time of product, T , is a random variable and variable P is the probability of life-time T greater than designated time, t ; expressed as probability $P(T > t)$ for which product reliability function can be defined as $R(t) = P(T > t)$, If the probability density function of life-time(T) expressed as $f(t)$, then its accumulative density probability function can be functioned as:

$$F(t) = P(T < t) = 1 - R(t), t \geq 0$$

If t , the timing of failure, features random variable, then the probability density function of failure time $f(t)$ has related to reliability $R(t)$ as,

$R(t) = 1 - F(t) = 1 - \int_0^t f(t)dt$, Mean lifetime is used to evaluate the feature of reliability for component or system. Mean lifetime is also called MTTF(Mean Time To Failure) or MTBF(Mean Time Between Failures).

MTBF- mean time between failure- means the average time between two failures in which the fail can be repaired and reused. If T stands for total operation time and r stands for total times of failure, then MTBF can be expressed as, $MTBF = \frac{T}{r}$, MTTF- mean time to failure- means the product operates till it fails for which the product cannot be repaired but replaced. MTTF is the expected value to fail depicted as the principle theorem of probability. It can also be showed as,

$$MTTF(MTBF) = \int_0^{\infty} R(t)dt = \int_0^{\infty} t \cdot f(t)dt,$$

3.4 Lifetime distribution of Product

There are many genetic types of reliability distribution for certain typical product which are usually adopted as Table 1 and life time distribution for ATUR suits for Weibull distribution. Let x be the lifetime of a product, and it is Weibull distributed with probability density functioned as,

$$f(x; \beta, \alpha) = \frac{\beta}{\alpha} \left(\frac{x}{\alpha}\right)^{\beta-1} \exp\left\{-\left(\frac{x}{\alpha}\right)^{\beta}\right\}, x > 0,$$

where β is the shape parameter and α is the scale parameter. This distribution was first suggested by Weibull [13] and its applicability to various failure situations are discussed again by Weibull [14].

The method of moment is applied on Weibull distribution, shape parameter β and scale parameter α used to evaluate the differences of ATUR's MTTF and their advantage, the analysis result helps us to make purchase decision.

More introductions about principle of moment method would be depicted as following and let T be the total testing time we need, then the expected duration of the test is denoted by

$$E(T) = \alpha^{-1/\beta} \Gamma(1 + \frac{1}{\beta}) = \eta \Gamma(1 + \frac{1}{\beta})$$

$$D(T) = \alpha^{-1/\beta} \sqrt{\Gamma(1 + 2/\beta) - \Gamma^2(1 + 1/\beta)}$$

$$= \eta \sqrt{\Gamma(1 + 2/\beta) - \Gamma^2(1 + 1/\beta)}$$

Where η equals to $\alpha^{-1/\beta}$ and population mean, E (T), and Standard Deviation, D (T), mapped with sample mean and sample SD, used to evaluate population and be called as moment method. The coefficient of variance, CV, can be defined as,

Table 1 Life distribution and typical product

Distribution	Failure rate	Product
Exponential	Constant	Constant failure rate device tested by durability test and maintain periodically.
Normal	Incremental	Aircraft tire, some mechanical bearing, gear, etc...
Exponential and normal	Incremental normally	Semiconductor, Si-crystal tube, aircraft construction
Weibull distribution	Incremental, decrease, or constant	Some capacitor, roller bearing, regulator, volt meter, etc...

$$CV = D(T)/E(T) = \sqrt{\Gamma(1 + 2/\beta) / \Gamma^2(1 + 1/\beta) - 1}$$

As the CV of the sampling can be quoted as the distribution of population above, so for sample size: n, (t₁, t₂,t_n), sample mean and sample S.D., the sample data is collected then CV of sample can be equated as the follows,

$$CV = \frac{S}{\bar{T}} = \frac{\sqrt{\Gamma(1 + \frac{2}{\beta}) / \Gamma^2(1 + \frac{1}{\beta}) - 1}}{\bar{T}}, \text{ where Sample mean } \bar{T} = \frac{\sum_{i=1}^n T_i}{n} \text{ and}$$

$$\text{Sample S.D. (S)} = \sqrt{\frac{\sum_{i=1}^n (T_i - \bar{T})^2}{n - 1}}$$

The function of β help to get the value of Gama distribution for $\Gamma(1 + \frac{1}{\beta})$, shown in **Attachment 1**. By this way, population mean, E(T), and Standard Deviation, D(T), mapped with sample mean and sample S.D., used to evaluate population which is called the moment method, scale parameter, α , is derived by similarity,

$$\alpha = \frac{1}{n} \sum_{i=1}^n t_i^\beta, \eta = \alpha^{-1/\beta}, \text{ the Mean lifetime (MTBF) is derived to } \eta \Gamma(1 + \frac{1}{\beta})$$

4. Data and analysis

4.1 Data and conditions

Data collected from student suites and the specification of ATUR is adopted to evaluate via comprehensive analysis after integrated ten KPIs which are briefly listed as following items,

- MTBF

- Purchase cost
- Power saving
- Heat sink
- Space occupation
- Convenient to use
- Convenient to clean
- Operation interface
- Average failure frequency annually
- Vendor technical support

Test conditions for this study are included as the follows,

- Test location: student suite apartment (15 - 20 suites per ADSL channel for average)
- Brands of ATUR: four brands of A, B, C, and D
- Test Quantities: 50 buildings of suite apartment (Setup and test ATUR, brand A, B, C and D, per each building simultaneously).
- Test temp.: Room temperature at 26±2 degree C.
- Test time: Running for 24hours continuously till failure happening (Unit: Hours), shown as Attachment 2.

4.2 Data analysis and evaluation

4.2.1 MTBF:

Dynamic deviation use to evaluate CV and shape parameter β , as shown on Table 2 which is used to calculate MTBF for each brands as follows,

(1) Brand A

Average = 18767

Sigma value = 8069

CV = 0.43

The characteristics parameter $\beta = 2.499$ for which $CV = \frac{\sqrt{\Gamma\left(1+\frac{2}{\beta}\right)}}{\sqrt{\Gamma^2\left(1+\frac{1}{\beta}\right)}-1}$,

Scale parameter, $\alpha = \frac{1}{n} \sum_{i=1}^n t_i^\beta = 53787497215$ and $\eta = \alpha^{\frac{1}{\beta}} = 19601$

Mean lifetime MTTF(MTBF) = $\alpha^{\frac{1}{\beta}} \Gamma\left(1+\frac{1}{\beta}\right) = 19601 * 0.887 = 17386$

(2) Brand B

Average = 21233

Standard Deviation = 9555

CV = 0.45, where shape parameter β equals to 2.349 satisfied CV value with $CV = \frac{\sqrt{\Gamma\left(1+\frac{2}{\beta}\right)}}{\sqrt{\Gamma^2\left(1+\frac{1}{\beta}\right)}-1} \cdot \text{Scale}$

parameter $\alpha = \frac{1}{n} \sum_{i=1}^n t_i^\beta = 16357585720$ with $\eta = \alpha^{\frac{1}{\beta}} = 24655$ and $MTTF(MTBF) = \alpha^{\frac{1}{\beta}} \Gamma(1$

$$+ \frac{1}{\beta}) = 24655 * 0.886 = 21844$$

(3) Brand C

Average = 21603

Standard Deviation = 10801

CV = 0.50, where shape parameter β equals to 2.111 satisfied CV value with

$$CV = \frac{\sqrt{\Gamma\left(1 + \frac{2}{\beta}\right) - 1}}{\Gamma\left(1 + \frac{1}{\beta}\right)}. \text{ Scale parameter } \alpha = \frac{1}{n} \sum_{i=1}^n t_i^\beta = 1380554259 \text{ with } \eta = \alpha^{\frac{1}{\beta}} = 22422 \text{ and}$$

$$MTTF(MTBF) = \alpha^{\frac{1}{\beta}} \Gamma\left(1 + \frac{1}{\beta}\right) = 22422 * 0.8857 = 19859$$

(4) Brand D

Average = 23594

Standard Deviation = 10801

CV = 0.426 where shape parameter β equals to 2.501 satisfied CV value with

$$CV = \frac{\sqrt{\Gamma\left(1 + \frac{2}{\beta}\right) - 1}}{\Gamma\left(1 + \frac{1}{\beta}\right)}. \text{ Scale parameter } \alpha = \frac{1}{n} \sum_{i=1}^n t_i^\beta = 97689222146 \text{ with } \eta = \alpha^{\frac{1}{\beta}} = 24885, \text{ mean}$$

$$\text{lifetime } MTTF(MTBF) = \alpha^{\frac{1}{\beta}} \Gamma\left(1 + \frac{1}{\beta}\right) = 24885 * 0.887 = 22073$$

Priority is determined by MTTF(MTBF), so the order for advantage of purchasing is $D > B > C > A$.

Table 2 Relations between CV and characteristics parameter β

CV	Characteristics parameter β
2.00	0.55
1.50	0.71
1.00	1.00
0.75	1.35
0.50	2.10
0.45	2.35
0.35	3.11
0.25	4.55

4.2.2 Purchase cost

The data of purchase cost is collected through the purchase department for many lot sizes. The purchase cost per unit is shown as Table 3.

Table 3 Purchase cost

Item	Brand A	Brand B	Brand C	Brand D
Purchase cost per unit	1687	453	529	783

Result : Priority B > C > D > A

4.2.3 Power saving

Power saving counted for the power consumption rate of ATU-R's adapter, then compared their power saving, shown as Table 4. (Assumed NT\$ 2.5 per elec. degree for average) .

Table 4 Power saving

Item	Brand A	Brand B	Brand C	Brand D
Power consumption	22W	16W	8W	6W
Elec. Fee per (NT\$)	1.32	0.96	0.48	0.36

Result : Priority D > C > B > A

4.2.4 Heat sink

Better heat transfer means the better performance for ATUR's data packet transfer and avoidance of thermal shock. The quality of network related to this evaluation item. The test procedure runs on normal working condition and records the surface temperature of ATUR daily as Table 5.

Table 5 Temperature on the surface of ATUR

Days	Brand A	Brand B	Brand C	Brand D	Days	Brand A	Brand B	Brand C	Brand D
1	20	19	20	20	17	20	21	22	23
2	21	20	21	22	18	19	20	23	19
3	19	17	20	20	19	20	17	20	21
4	20	17	21	21	20	21	18	19	21
5	21	20	22	21	21	20	20	20	19
6	21	21	20	20	22	20	19	20	20
7	22	18	19	22	23	21	18	21	20
8	19	19	22	19	24	21	19	19	20
9	20	19	22	20	25	19	20	22	23
10	20	20	23	21	26	19	20	20	21
11	22	20	19	21	27	20	19	21	21
12	19	18	20	20	28	21	18	21	20
13	21	18	22	19	29	22	17	22	22
14	20	22	23	20	30	19	20	21	20
15	22	20	21	21	Sum	608	574	628	618
16	19	20	22	21	Average	20.27	19.13	20.93	20.6

Result: priority B > A > D > C

4.2.5 Space occupation

Size measurement of length, wide and height, is to calculate the volume of space occupation for each ATUR, as shown on Table 6.

Table 6 Size evaluation

Size	Brand A	Brand B	Brand C	Brand D
L(cm)	22.7	13	19.5	13
W(cm)	15.8	18.5	13.5	14.5
H(cm)	3.1	3	4.5	2.5
Volume(cm ³)	1111.85	721.5	1184.63	471.25

Result : Priority D > B > A > C

4.2.6 Convenient to use

Evaluation of convenience to use is kind of function evaluation which point on the functional offering of PPPoE (Point-to-Point Protocol over Ethernet) and Hub to evaluate and analysis the use convenience. Observation is tabulated on Table 7.

Table 7 Functions of PPPoE and Hub

Item	Brand A	Brand B	Brand C	Brand D
PPPoE	YES(Conditional)	No	Yes	Yes
Hub	No	No	No	Yes

Result : Priority D > C > A > B

4.2.7 Convenient to clean

Evaluate the convenient to clean from practical experience of the after service department.

Data is collected from after service clerks by questionnaire and adopted Likert scale to measure degree of satisfactions ranged from 1, very difficult, to 5, very easy. Eventually, 45 questionnaires received after 50 questionnaire released, return rate is high up to 90%, as shown on Table 8.

Table 8 Result of convenient to clean

Item	Brand A	Brand B	Brand C	Brand D
Score	163	79	159	205

Result : Priority D > A > C > B

4.2.8 Operation interface

Operation interface surveys the difficulty of operation interface to setup and maintenance for ATUR.

Data is collected from ADSL maintenance worker by questionnaire and adopted Likert scale to measure degree of satisfactions ranged from 1: very difficult, to 5: very easy. Eventually, 45 questionnaires received after 50 questionnaire released, return rate is high up to 90%, as shown on Table 9.

Table 9 Operation interface from questionnaire

Item	Brand A	Brand B	Brand C	Brand D
Scores	75	197	210	173

Result: priority C > B > D > A

4.2.9 Average frequency of failure annually

The value of average frequency of failure annually lasted for two-year test period in which indicate the failure times of ATU-R in two years and yearly failure frequency will be valued by average, shown on Table 10.

Table 10 Average frequency of failure annually

Item	Brand A	Brand B	Brand C	Brand D
Failure times biannually	4	2	5	3
Average of failure times annually	2	1	2.5	1.5

Result : Priority C > A > D > B

4.2.10 Vendor technical support

Vendor technical support include the items of software upgrading, after service of maintenance. Data is collected from ADSL technician and maintenance worker by questionnaire and adopted Likert scale to measure degree of satisfactions ranged from 1,"few", to 5,"often". Eventually, 80 questionnaires received after 100 questionnaire released, return rate is high up to 80%, as shown on Table 11.

Table 11 Vendor technical support

Item	Brand A	Brand B	Brand C	Brand D
Upgrade S/W offering	356	298	350	367
Maintenance service offering	329	306	323	335
Scores	685	604	673	702

Result : priority D > A > C > B

4.3 Comprehensive analysis

After integrated KPIs which are depicted as para. 4.2.1 through 4.2.10, it results the comparison for their advantages as Table 12.

Table 12 Comparison for advantage

Item	Best	better	worse	Worst
MTBF	D	B	C	A
Purchase cost	B	C	D	A
Power saving	D	C	B	A
Heat sink	B	A	D	C
Space	D	B	A	C
Use convenience	D	C	A	B
Clean convenience	D	A	C	B
Operation interface	C	B	D	A
Average failure frequency annually	B	D	A	C
Vendor's Technical support	D	A	C	B

Each KPI has its weight to influence the priority of ATUR. AHP (Analytic Hierarchy Process) can be used to decide the weighting for each KPI by hierarchy method. Survey is collected from network technician and maintenance worker, eventually, 30 questionnaires received after 40 questionnaire released, return rate is high up to 75%. By implement and calculation of software, Expert Choice 2000, each weight of KPI is shown on **Table 13**.

Table 13 Weight for each item of KPI

Item	Weight
MTBF	0.357
Purchase cost	0.125
Power saving	0.042
Heat sink	0.050
Space	0.019
Convenient to use	0.055
Clean convenience	0.029
Operation interface	0.039
Average failure frequency annually	0.274
Vendor's Technical support	0.010

After integration of Table 12, Comparison of advantage, and Table 13, weight for each item of KPI, the ATUR, we conclude the comprehensive analysis table as Table 14. The advantage result is Brand D > Brand B > Brand C > Brand A, the conclusion helps the purchase decision of ATUR to make.

Table 14 Comprehensive result

Item	Weight	Brand A	Brand B	Brand C	Brand D
MTBF	0.357	0.357	1.071	0.714	1.428
Purchase cost	0.125	0.125	0.500	0.375	0.250
Power saving	0.042	0.042	0.084	0.126	0.168
Heat sink	0.050	0.150	0.200	0.050	0.100
Space	0.019	0.038	0.057	0.019	0.076
Use convenience	0.055	0.110	0.055	0.165	0.220
Clean convenience	0.029	0.087	0.029	0.058	0.116
Operation interface	0.039	0.039	0.117	0.156	0.078
Average failure times annually	0.274	0.548	1.096	0.274	0.822
Vendor's Technical support	0.010	0.030	0.010	0.020	0.040
Score	1	1.526	3.219	1.957	3.298
Notes : The advantage priority is classified as "best" scores 4, "better" scores 3, "worse" scores 2, "worst" scores 1.					

5. Conclusion and future research

After all of the empirical data and analysis, ATUR’s MTBF and other related KPIs conclude the result of Brand ranking individually by using motion method to derive the characteristics parameter(β) and scale parameter(α) of Weibull distribution. The evaluation of the ATU-R testing data has resulted in the following ranking, Brand D>Brand B>Brand C>Brand A. This research not only applies to the purchase of an ATUR but also escalates ADSL network quality. The related issues could be extended to the equipment for the tier of physical and network in the future research.F

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Attachment 1: Matrix of shape parameter β and Gama distribution $\Gamma(1 + \frac{1}{\beta})$

β	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
2.0	0.8862	0.8861	0.8861	0.8860	0.8859	0.8859	0.8858	0.8858	0.8858	0.8857
2.1	0.8857	0.8857	0.8856	0.8856	0.8856	0.8856	0.8856	0.8856	0.8856	0.8856
2.2	0.8856	0.8856	0.8857	0.8857	0.8857	0.8857	0.8857	0.8858	0.8858	0.8859
2.3	0.8859	0.8860	0.8860	0.8861	0.8861	0.8862	0.8862	0.8863	0.8863	0.8864
2.4	0.8865	0.8866	0.8866	0.8867	0.8868	0.8868	0.8869	0.8870	0.8871	0.8872
2.5	0.8873	0.8874	0.8874	0.8875	0.8876	0.8877	0.8878	0.8879	0.8880	0.8881
2.6	0.8882	0.8883	0.8884	0.8885	0.8886	0.8887	0.8888	0.8889	0.8891	0.8892
2.7	0.8893	0.8894	0.8895	0.8896	0.8897	0.8899	0.8900	0.8901	0.8902	0.8903
2.8	0.8905	0.8906	0.8907	0.8908	0.8909	0.8911	0.8912	0.8913	0.8914	0.8916
2.9	0.8917	0.8918	0.8919	0.8921	0.8922	0.8923	0.8925	0.8926	0.8927	0.8928
3.0	0.8930	0.8931	0.8932	0.8934	0.8935	0.8936	0.8938	0.8939	0.8940	0.8942
3.1	0.8943	0.8944	0.8946	0.8947	0.8948	0.8950	0.8951	0.8952	0.8954	0.8955
3.2	0.8957	0.8958	0.8959	0.8961	0.8962	0.8963	0.8965	0.8966	0.8967	0.8969
3.3	0.8970	0.8972	0.8973	0.8974	0.8976	0.8977	0.8978	0.8980	0.8981	0.8982
3.4	0.8984	0.8985	0.8987	0.8988	0.8989	0.8991	0.8992	0.8993	0.8995	0.8996
3.5	0.8997	0.8999	0.9000	0.9002	0.9003	0.9004	0.9006	0.9007	0.9008	0.9010

Attachment 2: Test data on brands (Unit : Hours)

Bldg	Zyxel	Alcatel	CT-511	TECOM	Bldg	Zyxel	Alcatel	CT-511	TECOM
1	11250	13245	17653	17653	26	23763	27652	27650	27856
2	15630	27654	27324	28976	27	14326	19872	28712	11231
3	19890	19345	13241	29875	28	20087	27698	15431	26030
4	10870	27650	26549	15436	29	23219	17535	19890	10063
5	16752	15346	26120	27867	30	26435	10097	28539	27650
6	17222	20012	10087	26510	31	11265	10987	16786	25870
7	17635	27860	25674	11679	32	23568	27965	12376	15632
8	20090	19875	24530	29087	33	16212	16547	25390	30034
9	17777	14325	15436	20087	34	20075	15643	20031	26215
10	16879	13657	23560	26875	35	14563	17698	27650	11256
11	12387	26453	27065	15436	36	19874	27693	15424	30065
12	14562	19876	20030	24990	37	21398	17659	26509	29901
13	17654	27650	27654	26530	38	25340	29876	15219	10023
14	25645	17865	17520	29807	39	16542	21098	10043	30089
15	18760	26530	27865	26250	40	23563	18790	13254	26115
16	16543	16543	28765	26330	41	13256	24310	26590	11562
17	26756	29006	19876	15321	42	25632	19870	16524	25690
18	15432	19876	28765	26090	43	11320	12342	26200	13456
19	26060	27896	19875	29876	44	17760	28090	19987	31230
20	12353	25376	13457	26120	45	20090	21135	27906	30092
21	23126	16547	27652	25360	46	25360	19650	26030	29032
22	17640	25675	15436	17653	47	15530	11786	13428	26540
23	12342	20986	26590	26010	48	24563	26578	18765	28743
24	20897	27650	10089	29543	49	25690	27654	26794	13456
25	16755	19965	26542	25990	50	17990	16543	27659	26500