Observer Design for Simultaneous State and Faults Estimation

Wafa JAMEL, Atef KHEDHER, Nasreddine BOUGUILA and Kamel BEN OTHMAN
ATSI/ENIM
14 Av Ibn Eljazzar, 5019 Monastir, TUNISIA
(wafa_jamel, khedher_atef, kamelbenothman)@yahoo.fr
nasreddine.bouguila@gmail.com

***Abstract*—This paper addresses the problem of state and faults estimation for Takagi-Sugeno nonlinear systems. Based on this structure for modeling, a proportional integralmultiple observer with unknown inputs is proposed in order to estimate simultaneously the state variables and actuator and sensor faults. Estimation error dynamics stability is studied with the Lyapunov theory and the observer gains are obtained by solving linear matrix inequalities (LMI). Academic examples are provided in order to illustrate the proposed method.**

***Index Terms*—Takagi-Sugeno structure modeling, proportional integral observer with unknown inputs, actuator and sensor faults, estimation error.**

I. INTRODUCTION

State estimation methods are mostly based on linear models of the studied systems [1], [2]. However, it turns out that linear models only describe the dynamic behavior of the system around an operating point. Hence, the use of nonlinear models becomes unavoidable because it allows an accurate representation of the system on a wide operating range [3].

Multiple model approach constitutes a tool which is largely used in the modeling of nonlinear systems [4], [5]. The principle of this approach is to reduce the system complexity by the decomposition of its operation space into a finished number of operating zone; which is described by a local model. Each local model can be a linear time invariant system valid around an operation point [6]. The local models are then aggregated by means of an interpolation mechanism. The global model is the sum of the local models weighted by activation functions associated to each one, [7]. These weighting functions quantify the relative contribution of each submodel to the global model according to the current operating point of the system. It should be noted that various realisations of multiple models can be employed in order to generate the global output of the multiple model [8], [9]. The Takagi-Sugeno representation is the most structure used in the multiple model approach.

The Takagi-Sugeno allows representing the behavior of nonlinear systems by the interpolation of a set of linear submodels. The relative contribution of each submodel is quantified with the help of a weighting function $\mu_i\left(\xi(t)\right)$.

In the case of Takagi-Sugeno systems, state estimation is based on the design of a nonlinear multiple observer using the same nonlinear functions as the Takagi-Sugeno model.Approaches using Takagi-Sugeno model are the object of many works in different contexts including the taking into account of unknown inputs [10], [11] or parameter uncertainties [12], [13]. Various studies dealing with the presence of unknown inputs acting on the system have been proposed in the litterature [12], [14].

In [17] a proportional integral observer with unknown inputs for descriptor systems described by multiple model is proposed.

We propose in this work the problem of observer design (proportional integral observer with unknown inputs) for nonlinear system described by Takagi-Sugeno model and submitted to perturbations. The main contribution is to estimate simultaneously actuator and sensor faults. Sensor faults estimation is made using the extension of the mathematical transformation proposed in [14] and used in [15], [16] to this kind of models. The stability conditions are derived from the Lyapunov theory and expressed as matrix inequality (LMI) problem.

The paper is organized as follows. The section 2 is devoted to introduce the Takagi-Sugeno structure for modeling. It is followed by the design of the proportional integral observer with nuknown inputs in section 3. This section is interested to estimate system state and actuator faults and / or sensor faults. Numerical examples and some simulation results are given to show the proposed methods. Conclusions are detailed in section 4.

II. TAKAGI SUGENO STRUCTURE FOR MODELING

A nonlinear system given by

$$\begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = g(x(t)) \end{cases} \quad (1)$$

can be written under the following Takagi-Sugeno form

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) \end{cases} \quad (2)$$

where $x(t) \in R^n$ is the state vector, $u(t) \in R^m$ is the input vector, $y(t) \in R^p$ represents the output vector. $A_i \in R^{n*n}$ is the state matrix, $B_i \in R^{n*m}$ is the matrix of input and $C \in R^{p*n}$ is the output matrix of the system. $\mu_i(\xi(t))$ are the weighting functions depending on the decision variable which can be measurable (as the input or the output of the system) or non measurable (as the state of the system). r is the number of local models. It depends on the precision of desired modeling, the complexity of the nonlinear system and the choice of the structure of the weighting functions. The weighting functions satisfy the convex sum properties described by

$$\sum_{i=1}^{r} \mu_i(\xi(t)) = 1 \quad \text{and} \quad 0 \le \mu_i(\xi(t)) \le 1 \quad (3)$$

III. DESIGN OF A PROPORTIONAL INTEGRAL OBSERVER WITH UNKNOWN INPUTS

This section is devoted to the synthesis of proportional multiple integral observer for nonlinear systems submitted to disturbances. Three cases are distinguished: actuator fault, sensor fault and both. A mathematical transformation is used to consider sensor faults as actuator faults of an augmented system.

A. Case of actuator fault

1) *Problem formulation*: Consider the following nonlinear Takagi-Sugeno system affected by unknown inputs described by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + E_i d(t)) \\ y(t) = Cx(t) \end{cases} \quad (4)$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^p$ are respectively the state vector, the input vector and the measured output. $d(t) \in R^q$ is the vector of unknown inputs. A_i, B_i, E_i, and C are known constant matrices with appropriate dimensions.

The structure of the proportional integral observer with unknown inputs is given as follows [17]

$$\begin{cases} \dot{z}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(N_i z(t) + G_i u(t) + L_i y(t) + H_i \hat{d}(t)) \\ \hat{x}(t) = z(t) + My(t) \\ \dot{\hat{d}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\phi_i(y(t) - \hat{y}(t)) \end{cases} \quad (5)$$

where $\hat{x}$ is the estimated state vector, z represents the state vector of the observer, $\hat{y}$ is the estimated output vector and $\hat{d}$ are the unknown inputs estimated. N_i, G_i, L_i, H_i, ϕ_i, and M have to be defined so that the reconstructed state converges asymptotically to the actual state $x(t)$.

For arbitrary initial conditions $x(0)$, $z(0)$ and arbitrary input $u(t)$, the following equations are verified

$$\lim_{t \to \infty} (x(t) - \hat{x}(t)) = 0 \quad (6)$$

$$\lim_{t \to \infty} (d(t) - \hat{d}(t)) = 0, \ \forall \ \hat{d}(0) \quad (7)$$

One notes $e(t) = x(t) - \hat{x}(t)$ and $f(t) = d(t) - \hat{d}(t)$.

Using the expressions of $\hat{x}(t)$ and $y(t)$ given respectively by (5) and (4), one obtains:

$$e(t) = (I - MC)x(t) - z(t) = Px(t) - z(t) \qquad (8)$$

with $P = I - MC$.

The state estimation error dynamics is given by

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))(P(A_i x(t) + B_i u(t) + E_i d(t)) \\ &- \sum_{i=1}^{r} \mu_i(\xi(t))(N_i z(t) + G_i u(t) + L_i y(t) + H_i \hat{d}(t)) \end{aligned} \qquad (9)$$

The equation (9) can be written as

$$\begin{aligned} \dot{e}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))(N_i e(t) + (PA_i - N_i - K_i C)x(t) + H_i f(t)) \\ &+ \sum_{i=1}^{r} \mu_i(\xi(t))((PB_i - G_i)u(t) + (PE_i - H_i)d(t)) \end{aligned} \qquad (10)$$

with $K_i = L_i - N_i M$.

If the following conditions are satisfied

$$N_i = PA_i - K_i C \qquad (11a)$$

$$H_i = PE_i \qquad (11b)$$

$$G_i = PB_i \qquad (11c)$$

$$L_i = K_i + N_i M \qquad (11d)$$

$$P = I - MC \qquad (11e)$$

Equation (10) is reduced to

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(N_i e(t) + H_i f(t)) \qquad (12)$$

We assume that the unknown inputs are bounded and slowly varying, i.e. $\dot{d}(t) \approx 0$.

The fault estimation error can be expressed as

$$\dot{f}(t) = \dot{d}(t) - \dot{\hat{d}}(t)$$

$$\dot{f}(t) = -\sum_{i=1}^{r} \mu_i(\xi(t))\phi_i C(x(t) - \hat{x}(t)) \qquad (13)$$

The dynamics of the unknown inputs estimation error is given by

$$\dot{f}(t) = -\sum_{i=1}^{r} \mu_i(\xi(t))\phi_i C e(t) \qquad (14)$$

The equations (12) and (14) can be written in the following form

$$\begin{bmatrix} \dot{e}(t) \\ \dot{f}(t) \end{bmatrix} = \sum_{i=1}^{r} \mu_i(\xi(t)) \begin{bmatrix} N_i & H_i \\ -\phi_i C & 0 \end{bmatrix} \begin{bmatrix} e(t) \\ f(t) \end{bmatrix} \qquad (15)$$

The state estimation error in equation (15) converges asymptotically towards zero if the stability of the matrices $\begin{bmatrix} N_i & H_i \\ -\phi_i C & 0 \end{bmatrix}$ is guaranteed.

2) *Method of resolution:* Four steps are needed to determine the matrices of the multiple observer.

1. From (11e),we have

$$I = P + MC = \begin{bmatrix} \mathrm{P} & M \end{bmatrix} \begin{bmatrix} I_1 \\ C \end{bmatrix} \qquad (16)$$

The matrix I_1 is an identity matrix of full rank.

Then, we obtain:

$$\begin{bmatrix} \mathrm{P} & M \end{bmatrix} = \begin{bmatrix} I_1 \\ C \end{bmatrix}^+ \qquad (17)$$

where $\begin{bmatrix} I_1 \\ C \end{bmatrix}^+$ is the pseudo-inverse of $\begin{bmatrix} I_1 \\ C \end{bmatrix}$.

2. Knowing P, these matrices are deduced

$$G_i = PB_i$$
$$H_i = PE_i$$

3. To calculate the gains K_i and ϕ_i, estimation errors are rewritten as follows

$$\begin{bmatrix} \dot{e}(t) \\ \dot{f}(t) \end{bmatrix} = \sum_{i=1}^{r} \mu_i(\xi(t))(\bar{A}_i - \bar{K}_i\bar{C}) \begin{bmatrix} e(t) \\ f(t) \end{bmatrix} \qquad (18)$$

Equation (18) is written as

$$\dot{e}_a(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(\bar{A}_i - \bar{K}_i\bar{C})e_a(t) \qquad (19)$$

with

$$\bar{A}_i = \begin{pmatrix} PA_i & H_i \\ 0 & 0 \end{pmatrix}, \ \bar{K}_i = \begin{pmatrix} K_i \\ \phi_i \end{pmatrix},$$
$$e_a(t) = \begin{pmatrix} e(t) \\ f(t) \end{pmatrix}, \quad \bar{\mathrm{C}} = \begin{pmatrix} \mathrm{C} & 0 \end{pmatrix}.$$

Theorem 1: The proportional integral observer with unknown inputs is determined if there exists a symmetric positive definite matrix X and matrices $W_i = X\bar{K}_i$ such that the following LMI hold $\forall\ i \in \{1,...,r\}$

$$\bar{A}_i^T X + X\bar{A}_i - \bar{C}^T W_i^T - W_i\bar{C} < 0 \qquad (20)$$

The gains $\bar{K}_i$ of the observer are computed from $\bar{K}_i = X^{-1}W_i$.

4. The matrices N_i and L_i are deduced respectively from (11a) and (11d).

3) *Simulation example:* Consider the system (4) with $r = 2$ and two components of the actuator fault $d(t)$, defined by the following matrices

$$A_1 = \begin{pmatrix} -0.6 & -2 \\ 0.5 & -0.2 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -0.7 & -0.3 \\ 2 & -0.3 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},$$
$$B_1 = \begin{pmatrix} 0.1 \\ 0.2 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 0.3 \\ 0.4 \end{pmatrix}, \quad E_1 = \begin{pmatrix} 5 & 3 \\ 1 & 6 \end{pmatrix}, \quad E_2 = \begin{pmatrix} 2 & 7 \\ 5 & 3 \end{pmatrix}.$$

The known input $u(t)$ is defined by: $u(t) = 0.25 * \sin(\pi t)$.

The computation of the matrices of the multiple observer (5) yields

$$N_1 = \begin{pmatrix} -0.5697 & 0.4471 \\ -0.6286 & -0.5641 \end{pmatrix}, \quad N_2 = \begin{pmatrix} -0.5970 & -0.4940 \\ 0.3339 & -0.5547 \end{pmatrix},$$

$$L_1 = \begin{pmatrix} -0.0288 & -1.4240 \\ 0.5844 & -0.1644 \end{pmatrix}, \quad L_2 = \begin{pmatrix} -0.5606 & 0.5188 \\ 0.6287 & -0.4001 \end{pmatrix},$$
$$K_1 = \begin{pmatrix} 0.1097 & -1.7168 \\ 0.9486 & -0.0645 \end{pmatrix}, \quad K_2 = \begin{pmatrix} -0.2230 & 0.5970 \\ 0.6061 & -0.1114 \end{pmatrix},$$
$$\phi_1 = \begin{pmatrix} 3.0752 & -3.6290 \\ 0.7173 & 0.9386 \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} 0.3354 & 1.3233 \\ 3.5283 & -3.6301 \end{pmatrix},$$
$$H_1 = \begin{pmatrix} 2.8 & 0.6 \\ -0.6 & 1.8 \end{pmatrix}, \quad H_2 = \begin{pmatrix} 0.2 & 3.6 \\ 1.6 & -0.2 \end{pmatrix}, \quad G_1 = \begin{pmatrix} 0.02 \\ 0.06 \end{pmatrix},$$
$$G_2 = \begin{pmatrix} 0.1 \\ 0.1 \end{pmatrix} \quad \text{and} \quad M = \begin{pmatrix} 0.4 & -0.2 \\ 0.2 & 0.4 \end{pmatrix}.$$

where K_i are proportional gains and ϕ_i are integral gains. Simulation results are given in Figures (1) to (4). This method allows to estimate well the states and the actuator fault affecting the system.

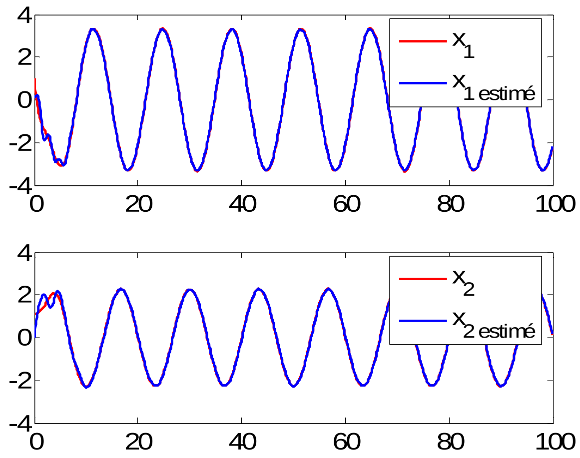

Fig.1 States and their estimation

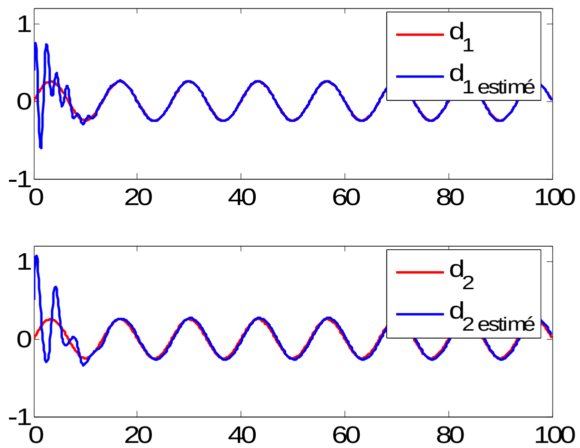

Fig.2 Actuator fault and its estimation

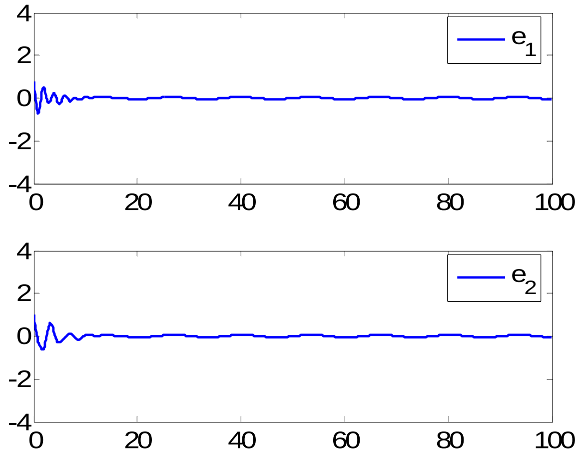

Fig.3 State estimation errors

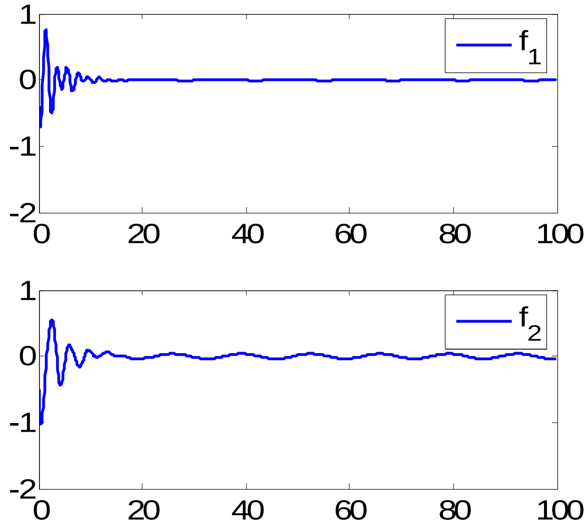

Fig.4 Actuator estimation error

B. Case of sensor fault

This section is devoted to the design of an observer for systems affected by sensor faults. A mathematical transformation is used to consider this perturbation as unknown inputs of an augmented system.

1) Problem formulation: Consider a nonlinear system submitted to a sensor fault

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t)) \\ y(t) = Cx(t) + D\bar{u}(t) \end{cases} \tag{21}$$

where $x(t) \in R^n$, $u(t) \in R^m$ and $y(t) \in R^p$ are respectively the system state, the system input and the measured output. $\bar{u}(t)$ represents sensor fault. A_i, B_i, C and D are known constant matrices with appropriate dimensions.

Consider the new state $z(t)$ [15], [16], [18], [19] given by

$$\dot{z}(t)=\sum_{i=1}^{M} \mu_i(\xi(t)) \, (-\bar{A}_i z(t)+\bar{A}_i Cx(t)+\bar{A}_i D\bar{u}(t)) \tag{22}$$

where $-\bar{A}_i \quad \forall\, i \in \{1,\ldots,r\}$ are stables matrices.

Let's consider the augmented system $X(t) = \begin{pmatrix} x(t) \\ z(t) \end{pmatrix}$ that can be modelled as

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_{ai}X(t) + B_{ai}u(t) + D_{ai}\bar{u}(t)) \\ Y(t) = C_a X(t) \end{cases} \quad (23)$$

with

$$A_{ai} = \begin{bmatrix} A_i & 0 \\ \bar{A}_i C & -\bar{A}_i \end{bmatrix}, \quad B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad D_{ai} = \begin{bmatrix} 0 \\ \bar{A}_i D \end{bmatrix}$$

and $C_a = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix}$.

The structure of the chosen observer is as follows

$$\begin{cases} \dot{Z}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(N_i Z(t) + G_i u(t) + L_i Y(t) + H_i \hat{\bar{u}}(t)) \\ \hat{X}(t) = Z(t) + MY(t) \\ \dot{\hat{\bar{u}}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\phi_i(Y(t) - \hat{Y}(t)) \end{cases} \quad (24)$$

where $\hat{X}$, Z, Y and $\hat{\bar{u}}$ represent respectively the augmented

state vector estimated, the state vector of the observer, the augmented measured output estimated and the estimated sensor faults.

Let us define the state estimation error $\tilde{X}$ and the fault estimation error $\tilde{f}$

$$\tilde{X}(t) = X(t) - \hat{X}(t) \quad (25a)$$

$$\tilde{f}(t) = \bar{u}(t) - \hat{\bar{u}}(t) \quad (25b)$$

The dynamics of the state reconstruction error is

$$\begin{aligned} \dot{\tilde{X}}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))(N_i \tilde{X}(t) + (PA_{ai} - N_i - K_i C_a)X(t)) \\ &+ \sum_{i=1}^{r} \mu_i(\xi(t))((PB_{ai} - G_i)u(t) + (PD_{ai} - H_i)\bar{u}(t) + H_i \tilde{f}(t)) \end{aligned} \quad (26)$$

with $K_i = L_i - N_i M$.

If the following conditions are fulfilled:

$$N_i = PA_{ai} - K_i C_a \quad (27a)$$

$$G_i = PB_{ai} \quad (27b)$$

$$H_i = PD_{ai} \quad (27c)$$

$$L_i = K_i + N_i M \quad (27d)$$

$$P = I - MC_a \quad (27e)$$

The equation (26) is reduced to

$$\dot{e}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(N_i \tilde{X}(t) + H_i \tilde{f}(t)) \qquad (28)$$

The dynamics of the fault reconstruction error is

$$\dot{\tilde{f}}(t) = \dot{u}(t) - \dot{\hat{u}}(t)$$
$$= -\sum_{i=1}^{r} \mu_i(\xi(t)) \phi_i C_a \tilde{X}(t) \qquad (29)$$

where $\dot{u}(t)$ is assumed that is nul.

The equations (28) and (29) can be written in the following form

$$\begin{bmatrix} \dot{\tilde{X}}(t) \\ \dot{\tilde{f}}(t) \end{bmatrix} = \sum_{i=1}^{r} \mu_i(\xi(t)) \begin{bmatrix} N_i & H_i \\ -\phi_i C_a & 0 \end{bmatrix} \begin{bmatrix} \tilde{X}(t) \\ \tilde{f}(t) \end{bmatrix} \qquad (30)$$

The estimation error (30) converges asymptotically towards zero if the matrices $\begin{bmatrix} N_i & H_i \\ -\phi_i C_a & 0 \end{bmatrix}$ are stable.

2) *Method of resolution:* In order to solve the system (27), four steps are needed.

1. Using the expression (27e), one has

$$I = \begin{bmatrix} P & M \end{bmatrix} \begin{bmatrix} I_1 \\ C_a \end{bmatrix} \qquad (31)$$

The matrix I_1 is an identity matrix of full rank.

Then, one deduces

$$\begin{bmatrix} P & M \end{bmatrix} = \begin{bmatrix} I_1 \\ C_a \end{bmatrix}^+ \qquad (32)$$

where $\begin{bmatrix} I_1 \\ C_a \end{bmatrix}^+$ is the pseudo-inverse of $\begin{bmatrix} I_1 \\ C_a \end{bmatrix}$.

2. G_i and H_i are deduced knowing P

$$G_i = PB_{ai}$$
$$H_i = PD_{ai}$$

3. The estimation errors (28) and (29) are rewritten in the following form

$$\begin{bmatrix} \dot{\tilde{X}}(t) \\ \dot{\tilde{f}}(t) \end{bmatrix} = \sum_{i=1}^{r} \mu_i(\xi(t))(\bar{A}_{ai} - \bar{K}_i \bar{C}) \begin{bmatrix} \tilde{X}(t) \\ \tilde{f}(t) \end{bmatrix} \qquad (33)$$

(33) can be written as

$$\dot{e}_a(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(\bar{A}_{ai} - \bar{K}_i \bar{C}) e_a(t) \qquad (34)$$

with

$$\bar{A}_{ai} = \begin{pmatrix} PA_{ai} & H_i \\ 0 & 0 \end{pmatrix}, \; \bar{K}_i = \begin{pmatrix} K_i \\ \phi_i \end{pmatrix}, \; e_a(t) = \begin{pmatrix} \tilde{X}(t) \\ \tilde{f}(t) \end{pmatrix}$$

and $\bar{C} = \begin{pmatrix} C & 0 \end{pmatrix}$.

Theorem 2: The proportional integral observer with unknown inputs (21) for the system (24) is determined if there exists a symmetric positive definite matrix X and matrices $W_i = X\bar{K}_i$ such that the following LMI are checked

$$\bar{A}_{ai}{}^{T} X + X\bar{A}_{ai} - \bar{C}^{T} W_i^{T} - W_i \bar{C} < 0 \qquad (35)$$

The gains of the observer are determined via the resolution of $\bar{K}_i = X^{-1} W_i$..

4. The observer matrices N_i and L_i are then obtained by (27a) and (27d).

3) *Simulation example:* Consider the nonlinear system described by the Takagi-Sugeno model given by the equation (21) with

$$A_1 = \begin{pmatrix} -1.8 & 0.4 & -2 \\ 4 & -1 & 1 \\ 2 & 0.81 & -1 \end{pmatrix}, \quad A_2 = \begin{pmatrix} -1 & 2 & -3 \\ -0.1 & -0.2 & -0.3 \\ 0.25 & -0.35 & 0.1 \end{pmatrix},$$

$$B_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 2 \\ 2 \\ 4 \end{pmatrix}, \quad D = \begin{pmatrix} 0.1 \\ 0.35 \\ 0.25 \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

One chooses $\bar{A}_1 = 30 * I$ and $\bar{A}_2 = 20 * I$, with $I = eye(3,3)$.

The known input u(t) is defined by: u(t) = 0.3sin(0.1_t). The sensor fault is defined by:

$$\bar{u}(t) = \begin{cases} 0, t \le 20s \\ 0.4 * \sin(\pi t), 20s < t \le 55s \\ 0, 55s < t \le 75s \\ 0.5, t > 75s \end{cases}$$

The obtained observer gains are

$$K_1 = \begin{pmatrix} -0.62 & 0 & 0 & 0 & 0 & 0 \\ 1.998 & 0 & 0 & 0 & 0 & 0 \\ 0.66 & 0.906 & 0.3 & 0 & 0 & 0 \\ 15 & 0 & 0 & -14.469 & 0.127 & 0.091 \\ 0 & 15 & 0 & 0.085 & -14.125 & 0.319 \\ 0 & 0 & 15 & 0.061 & 0.214 & -14.309 \end{pmatrix},$$

$$\phi_1 = \begin{pmatrix} 0 & 0 & 0 & 1.519 & 5.325 & 3.809 \end{pmatrix},$$

$$K_2 = \begin{pmatrix} 0.05 & 0 & 0 & 0 & 0 & 0 \\ 0.82 & 0.40 & 0 & 0 & 0 & 0 \\ -2.03 & -0.76 & 1.16 & 0 & 0 & 0 \\ 10 & 0 & 0 & -9.479 & 0.085 & 0.060 \\ 0 & 10 & 0 & 0.057 & -9.250 & 0.213 \\ 0 & 0 & 10 & 0.040 & 0.143 & -9.372 \end{pmatrix},$$

$$\phi_2 = \begin{pmatrix} 0 & 0 & 0 & 1.016 & 3.561 & 2.548 \end{pmatrix}$$

Figure (5) depicts the system states and their estimates. Figure (6) represents the sensor fault and its estimate. One can note that the system states and the sensor fault are correctly estimated.

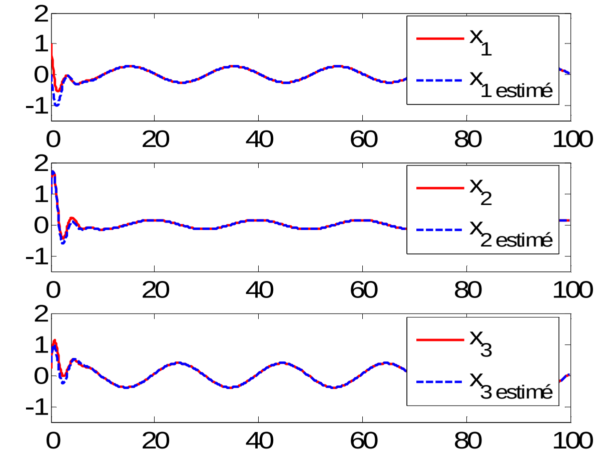

Fig.5 System states and their estimates

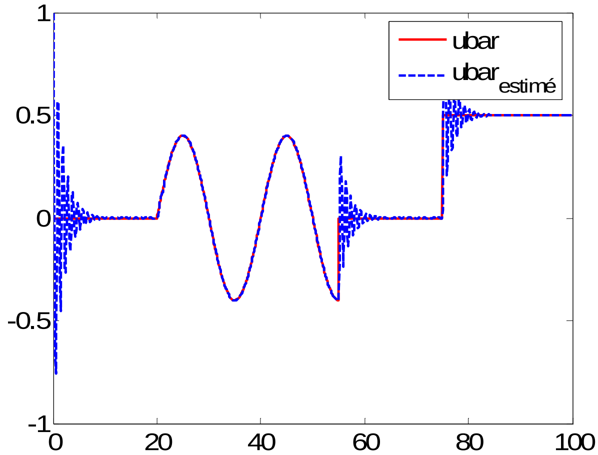

Fig.6 Sensor fault and its estimate

C. Case of actuator and sensor faults

1) *Problem formulation:* Consider the Takagi-Sugeno multiple model affected by actuator faults and sensor faults described by

$$\begin{cases} \dot{x}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_i x(t) + B_i u(t) + E_i d(t)) \\ y(t) = Cx(t) + D\bar{u}(t) \end{cases} \qquad (36)$$

where $x(t)$ is the state vector, $u(t)$ represents the input vector, $y(t)$ represents the measured output, $d(t)$ represents sensor fault. A_i, B_i, C and D are known constant matrices with appropriate dimensions.

Consider the new state $z(t)$ which satisfies the equation (22). The dynamics of the augmented state $X(t) = \begin{pmatrix} x(t) \\ z(t) \end{pmatrix}$ is governed by

$$\begin{cases} \dot{X}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(A_{ai} X(t) + B_{ai} u(t) + D_{ri}\gamma(t)) \\ Y(t) = C_a X(t) \end{cases} \qquad (37)$$

with

$$A_{ai} = \begin{bmatrix} A_i & 0 \\ \bar{A}_i C & -\bar{A}_i \end{bmatrix}, \quad B_{ai} = \begin{bmatrix} B_i \\ 0 \end{bmatrix}, \quad D_{ri} = \begin{bmatrix} E_i & 0 \\ 0 & \bar{A}_i D \end{bmatrix},$$

$$C_a = \begin{bmatrix} C & 0 \\ 0 & I \end{bmatrix} \quad \text{and} \quad \gamma(t) = \begin{bmatrix} d(t) \\ \bar{u}(t) \end{bmatrix}.$$

The proposed proportional integral observer with unknown inputs takes the following form

$$\begin{cases} \dot{Z}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(N_i Z(t) + G_i u(t) + L_i Y(t) + H_i \hat{\gamma}(t)) \\ \hat{X}(t) = Z(t) + MY(t) \\ \dot{\hat{\gamma}}(t) = \sum_{i=1}^{r} \mu_i(\xi(t))\phi_i(Y(t) - \hat{Y}(t)) \end{cases} \quad (38)$$

where $\hat{X}$ and $\hat{\gamma}$ respectively denote the estimates of X and γ. Z is the state vector of the multiple observer. $e_a(t)$ denotes the state estimation error, that is defined by $e_a(t) = X(t) - \hat{X}(t)$. $\tilde{f}(t)$ is the fault estimation error that is defined by $\tilde{f}(t) = \gamma(t) - \hat{\gamma}(t)$.

The dynamic of the state estimation error is expressed as follows

$$\begin{aligned} \dot{e}_a(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))(N_i e_a(t) + (PA_{ai} - N_i - K_i C_a)X(t) + H_i \tilde{f}(t)) \\ &+ \sum_{i=1}^{r} \mu_i(\xi(t))((PB_{ai} - G_i)u(t) + (PD_{ri} - H_i)\gamma(t)) \end{aligned} \quad (39)$$

with $K_i = L_i - N_i M$.

If the following conditions are fulfilled:

$$N_i = PA_{ai} - K_i C_a \quad (40a)$$

$$G_i = PB_{ai} \quad (40b)$$

$$H_i = PD_{ri} \quad (40c)$$

$$L_i = K_i + N_i M \quad (40d)$$

$$P = I - MC_a \quad (40e)$$

The state reconstruction error is reduced to

$$\dot{e}_a(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(N_i e_a(t) + H_i \tilde{f}(t)) \quad (41)$$

It is assumed that the faults are bounded and slowly varying, i.e $\dot{\gamma}(t) \approx 0$.

The fault estimation error dynamics is given by

$$\dot{\tilde{f}}(t) = \dot{\gamma}(t) - \dot{\hat{\gamma}}(t) = -\sum_{i=1}^{r} \mu_i(\xi(t))\phi_i C_a e_a(t) \quad (42)$$

The equations (41) and (42) can be written

$$\begin{bmatrix} \dot{e}_a(t) \\ \dot{\tilde{f}}(t) \end{bmatrix} = \sum_{i=1}^{r} \mu_i(\xi(t)) \begin{bmatrix} N_i & H_i \\ -\phi_i C_a & 0 \end{bmatrix} \begin{bmatrix} e_a(t) \\ \tilde{f}(t) \end{bmatrix} \quad (43)$$

The estimation error (43) converges asymptotically towards zero if the matrices $\begin{bmatrix} N_i & H_i \\ -\phi_i C_a & 0 \end{bmatrix}$ are stable.

2) *Method of resolution:* Four steps are needed to determine the matrices of the multiple observer.

1. From (40e), we have

$$I = P + MC_a = \begin{bmatrix} \text{P} & M \end{bmatrix} \begin{bmatrix} I_1 \\ C_a \end{bmatrix} \qquad (44)$$

I_1 is an identity matrix of full rank.

Then, one obtains:

$$\begin{bmatrix} \text{P} & M \end{bmatrix} = \begin{bmatrix} I_1 \\ C_a \end{bmatrix}^+ \qquad (45)$$

with $\begin{bmatrix} I_1 \\ C_a \end{bmatrix}^+$ is the pseudo-inverse of $\begin{bmatrix} I_1 \\ C_a \end{bmatrix}$.

2. By determining P, one deduces

$G_i = PB_{ai}$

$H_i = PD_{ai}$

3. To obtain the gains K_i and ϕ_i, estimation errors are rewritten in the following form

$$\begin{bmatrix} \dot{e}_a(t) \\ \dot{\tilde{f}}(t) \end{bmatrix} = \sum_{i=1}^{r} \mu_i(\xi(t))(\bar{A}_i - \bar{K}_i \bar{C}) \begin{bmatrix} e_a(t) \\ \tilde{f}(t) \end{bmatrix} \qquad (46)$$

Then, we obtain

$$\dot{f}_a(t) = \sum_{i=1}^{r} \mu_i(\xi(t))(\bar{A}_i - \bar{K}_i \bar{C}) f_a(t) \qquad (47)$$

with

$$\bar{A}_i = \begin{pmatrix} PA_{ai} & H_i \\ 0 & 0 \end{pmatrix}, \quad \bar{K}_i = \begin{pmatrix} K_i \\ \phi_i \end{pmatrix}, \quad f_a(t) = \begin{pmatrix} e_a(t) \\ \tilde{f}(t) \end{pmatrix},$$

$$\bar{\text{C}} = \begin{pmatrix} \text{C}_\text{a} & 0 \end{pmatrix}.$$

The convergence condition of the estimation error is obtained by using the Lyapunov function $V(t)$

$$V(t) = f_a^T(t) X f_a(t), \qquad \text{X>0} \quad \text{X=X}^\text{T} \qquad (48)$$

Its time derivative is defined by

$$\dot{V}(t) = \dot{f}_a^T(t) X f_a(t) + f_a^T(t) X \dot{f}_a(t) \qquad (49)$$

The equation (49) leads to

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t)) \left[f_a^T(t)(\bar{A}_{ai}^T X + X\bar{A}_{ai}) f_a(t) \right] \\ &- \sum_{i=1}^{r} \mu_i(\xi(t)) \left[f_a^T(t)(\bar{C}^T \bar{K}_i^T X + X\bar{K}_i \bar{C}) f_a(t) \right] \end{aligned} \qquad (50)$$

Obviously, $\dot{V}(t) < 0$ holds if

$$\bar{A}_{ai}^T X + X\bar{A}_{ai} - \bar{C}^T \bar{K}_i^T X - X\bar{K}_i \bar{C} < 0, \forall i \in \{1, ..., r\} \qquad (51)$$

The conditions (51) are nonlinear with respect to the variables K_i and X. In order to resolve them with the classical LMI approaches, the change of variables $W_i = X\bar{K}_i$ is used.

Due to this change, (51) becomes:

$$\bar{A}_{ai}^T X + X\bar{A}_{ai} - \bar{C}^T W_i^T - W_i\bar{C} < 0, \forall i \in \{1,...,r\} \qquad (52)$$

The gain of the observer is determined via the resolution of $\bar{K}_i = X^{-1}W_i$.

4. The other matrices are deduced from (40a) and (40d).

To improve the performances of the multiple observer, its dynamics is selected in a manner which is appreciably faster than that of the multiple model. So, it is necessary to fix the eingenvalues of submodels in the left half complex plan. The inequality (52) becomes:

$$\bar{A}_{ai}^T X + X\bar{A}_{ai} - \bar{C}^T W_i^T - W_i\bar{C} + 2\alpha X < 0, \forall i \in \{1,...,r\} \qquad (53)$$

Theorem 3: The estimation errors between the Takagi-Sugeno model (36) and its observer (38) converge toward zero, if there exist a symmetric positive definite matrix $X = X^T > 0$ and matrices $W_i = X\bar{K}_i$ such that the following conditions hold $\forall i \in \{1,...,r\}$:

$$\bar{A}_{ai}{}^T X + X\bar{A}_{ai} - \bar{C}^T W_i^T - W_i\bar{C} + 2\alpha X < 0 \qquad (54)$$

3) *Simulation example:* Let us consider the multiple the multiple model, made up of two local models and involving four states and four outputs

$$A_1 = \begin{pmatrix} -2 & 3 & 0.4 & 0.95 \\ -0.5 & 0.25 & 0.4 & 1 \\ -3 & -0.2 & -1.5 & -0.5 \\ -0.3 & -0.25 & -0.6 & -1.5 \end{pmatrix}, \quad B_1 = \begin{pmatrix} 1 \\ 2 \\ -3 \\ 1 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0.1 & -0.3 & -0.5 & 0.1 \\ -0.4 & -5 & 1 & 4 \\ 3 & -0.5 & -5 & -0.7 \\ -0.6 & -2 & 1 & -0.2 \end{pmatrix}, \quad B_2 = \begin{pmatrix} 3 \\ 2 \\ 4 \\ 5 \end{pmatrix},$$

$$\mathrm{E}_1 = \begin{pmatrix} 5 & 3 \\ 1 & 6 \\ 4 & 4 \\ 7 & 2 \end{pmatrix}, \quad \mathrm{E}_2 = \begin{pmatrix} 2 & 7 \\ 5 & 3 \\ 6 & 1 \\ 4 & 3 \end{pmatrix}, \quad \mathrm{D} = \begin{pmatrix} 5 & 7 \\ 2 & 3 \\ 1 & 2 \\ 6 & 2 \end{pmatrix},$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

One takes $\bar{A}_1 = 20 * I$ and $\bar{A}_2 = 15 * I$, where I is the identity matrix.

The system input is given in Figure (7). The actuator fault $d(t)$ is defined as follows:

$$\mathrm{d(t)} = \left[\mathrm{d}_1(\mathrm{t}) \quad \mathrm{d}_2(\mathrm{t}) \right]^T$$

with

$$d_1(t) = 0.5 * \sin(0.1\pi t),$$

$$d_2(t) = \begin{cases} 0.5, t \le 50s \\ 0.25, 50s < t \le 70s \\ 0, t > 70s \end{cases}$$

and the sensor fault $\bar{u}(t)$ is made up of two components

$$\bar{u}(t) = \left[\bar{u}_1(t) \quad \bar{u}_2(t) \right]^T$$

with

$$\bar{u}_1(t) = \begin{cases} 0, t \le 20s \\ 0.5, 20s < t \le 40s \\ 0, 40s < t \le 60s \\ 0.8, 60s < t \le 80s \\ 0, t > 80s \end{cases},$$

$$\bar{u}_2(t) = \begin{cases} 0, t \le 20s \\ 0.4\sin(\pi t), 20s < t \le 55s \\ 0, 55s < t \le 75s \\ 1.5, t > 75s \end{cases}$$

Simulation results are shown in Figures (8) to (11).

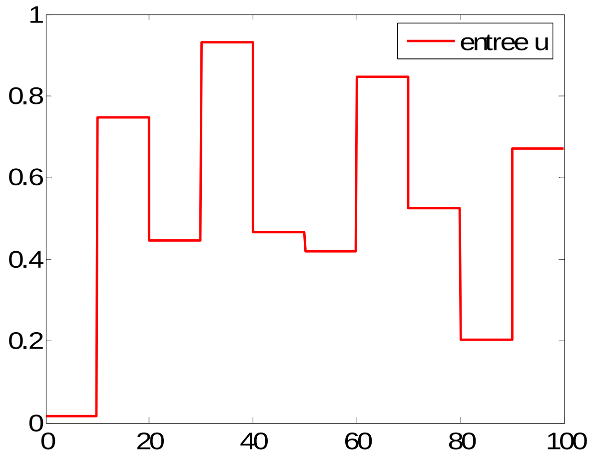

Fig.7 The known input u(t)

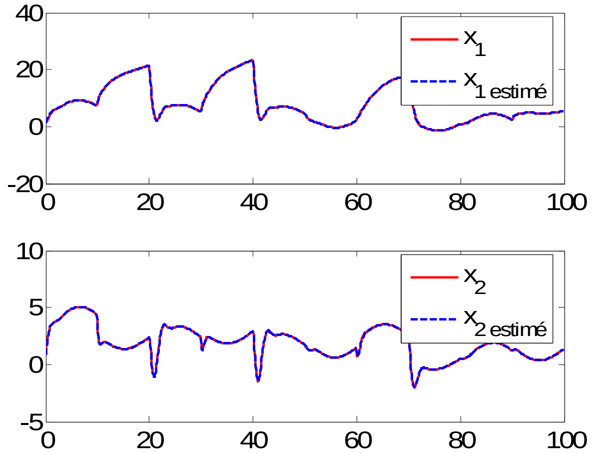

Fig.8 States and their estimates

System states and their estimates are depicted in Figures (8) and (9). Figure (10) visualizes the actuator fault and its estimation. The sensor fault is presented in Figure (11). The time evolution of the estimation errors are depicted in Figures (12) and (13).

The proposed observer provides good estimates of the system state. Indeed, this method allows to estimate well the faults affecting the system even in the case of time varying faults.

IV. CONCLUSION

The goal of this paper is to design a proportional integral observer with unknown inputs for nonlinear system represented in a Takagi-Sugeno form, submitted to actuator and sensor faults. A nonlinear mathematical transformation is considered in order to conceive an augmented system in which the sensor faults appear as unknown inputs. The stability is studied by Lyapunov theory and LMI constraints are provided to design the gains matrices. Simulation results are provided and show that the designed observer is able to estimate simultaneously actuator and sensor faults.

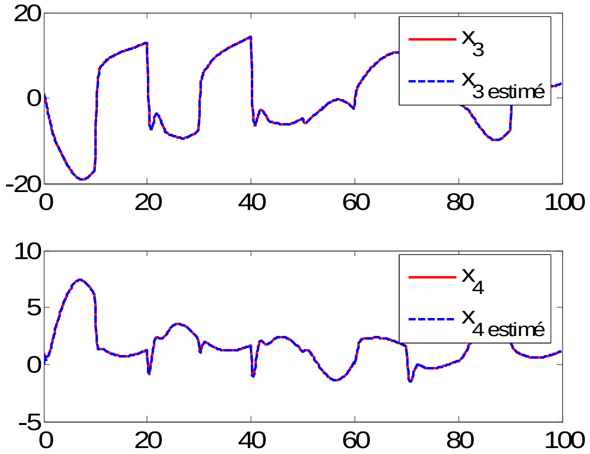

Fig.9 States and their estimates

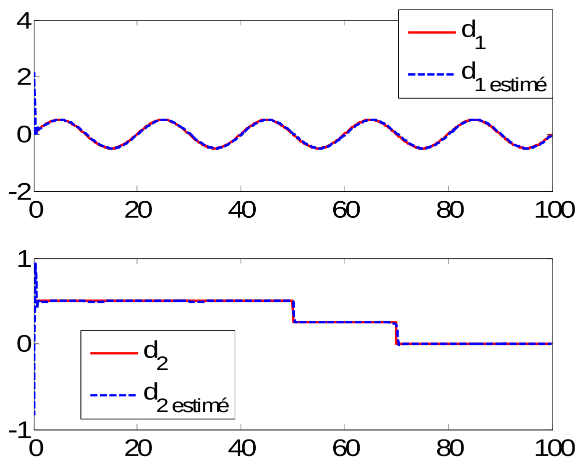

Fig.10 Actuator faults and their estimations

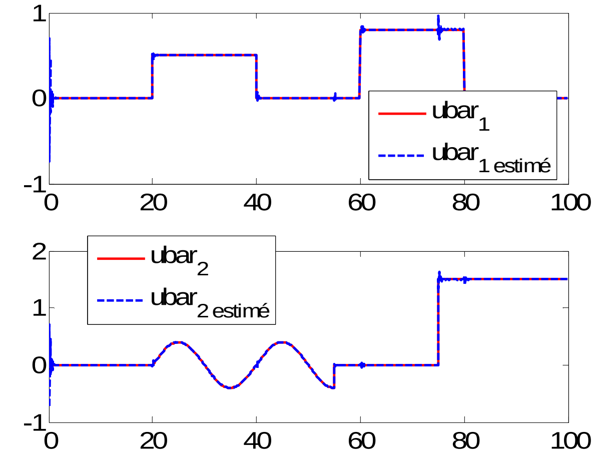

Fig.11 Sensor faults and their estimations

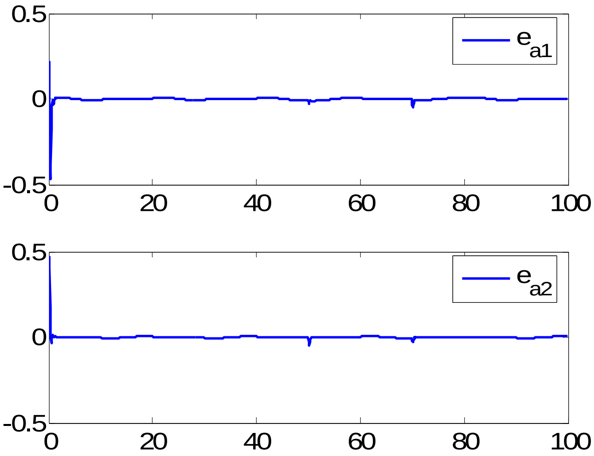

Fig.12 State estimation errors x_1 and x_2

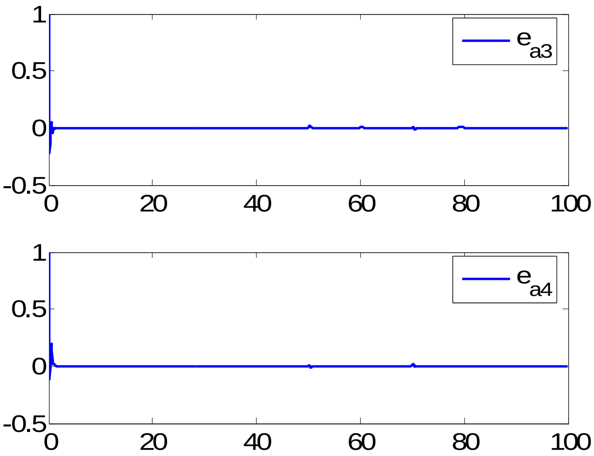

Fig.13 State estimation errors x_3 and x_4

REFERENCES

[1] Luenberger D.G., An introduction to observers, IEEE Transactions on Automatic Control, 1971, 16(6), 596-602.

[2] Darouach M., Zasadzinski M.,SXu S. J. , Full-order observers for linear systems with unknown inputs, IEEE Transactions on Automatic Control, 1994, 39(3), 606-609.

[3] Ichalal D., Marx B., Mammar S., Maquin D., Ragot J, Observer for Lipschitz nonlinear systems: Mean Value Theorem and Sector Nonlinearity Transformation, 2012 IEEE Multi-Conference on Systems and Control, MSC 2012, Dubrovnik, Croatia, October 3-5, 2012.

[4] Chadli M., Akhenak A., Ragot J., Maquin D., On the design of observer for unknown inputs fuzzy models, International Journal of Automation and Control, 2008, 2(1), 113-125.

[5] Murray-Smith R., T.A. Johansen, Multiple model approaches to modelling and control, Taylor and Francis, London, 1997.

[6] JamelW., Bouguila N., Khedher A. and Benothman K., Observer design for nonlinear systems represented by Takagi-Sugeno models,WSEAS Transactions on Systems, Issue 7, Volume 9, pp. 804- 813, July 2010.

[7] Johansen T.A., Shorten R. and Murray-Smith R., Fuzzy identification of systems and its application to moddeling and control, IEEE Transactions on Systems Man and Cybernetics-part C, 1985, 15, 116-132.

[8] Filev D., Fuzzy modeling of complex systems. International Journal of Approximate Reasoning, 1991, 5(3), 281-290.

[9] Gregorcic G., Lightbody G., Control of highly nonlinear processes using self-tuning control and multiple/local model approaches, IEEE International Conference on Intelligent Engineering Systems, INESˇS2000, Portoroz, Slovenia, September 17-19, 2000.

[10] Bezzaoucha S., Marx B., Maquin D., Ragot J. On the unknown input observer design: a decoupling class approach. 9th IEEE International Conference on Control & Automation, ICCA11, Santiago, Chile, December 19-21, 2011.

[11] Bezzaoucha S., Marx B., Maquin D., Ragot J. On the unknown input observer design : a decoupling class approach with application to sensor fault diagnosis. 1st edition of the International Conference on Automation and Mechatronics, CIAM’2011, Oran Algeria, November 22-24, 2011.

[12] Akhenak A., Chadli M., Ragot J., Maquin D., Design of sliding mode unknown input observer for uncertain Takagi-Sugeno model, 15th Mediterranean Conference on Control and Automation, MED’07, Athens, Greece, June 27-29, 2007.

[13] Ichalal D.,Marx B., Ragot J. andMaquin D., Nonlinear observer based fault tolerant control for nonlinear systems, 8th IFAC Symposium on Fault Detection, Supervision and Safety for Technical Processes, Safeprocess 2012, Mexico, August 27-28, 2012.

[14] Edwards C., A comparison of sliding mode and unknown input observers for fault reconstruction, 43rd IEEE Conference on Decision and Control, CDCˇS04, Atlantis, Paradise Island, Bahamas, December 14-17, 2004.

[15] Khedher A., Ben Othman K., Maquin D. and Benrejeb M., Adaptive observer for fault estimation in nonlinear systems described by a Takagi-Sugeno model, 18th Mediterranean Conference on Control and Automation, MED’10, June 24-26, Marrakech, Morroco, 2010.

[16] Khedher A., Benothman K., Maquin D. and Benrejeb M., State and sensor faults estimation via a proportional integral observer, 6th International multi-conference on Systems, Signals and Devices SSDˇS09 March 23-26, Djerba, Tunisia, 2009.

[17] Hamdi H., Rodrigues M., Mechmeche C., Benhadj-braiek N., Synthèse d’un observateur proportionnel intégral à entrées inconnues pour les systèmes singuliers représentés par des multi-modèles, Sixième Conférence Internationale Francophone d’Automatique, CIFA, Nancy, France, 2010.

[18] Khedher A., Ben Othman K., Benrejeb M. ”Active fault tolerant control (FTC) design for Takagi-Sugeno fuzzy systems with weighting functions depending on the FTC”. International Journal of Computer Science Issues, 2011, 8(3).

[19] Khedher A., Ben Othman K., Maquin D. and Benrejeb M., Sensor fault estimation for nonlinear systems described by Takagi-Sugeno models, Interntional Journal: Transaction on System, Signal and Devices, Issues on Systems, Analysis and Automatic Control, November 2011, 6(1), 49-66.