Application of fuzzy-AHP extent analysis to determine the relative importance of risk factors in operative mortality after Coronary Artery Bypass surgery

[1]Mahyar Taghizadeh Nouei*, Ali Vahidian Kamyad[1], Somayeh Ghazalbash[2], MahmoodReza Sarzaeem[3]

[1]Department of Applied Mathematics, School of Mathematical Sciences
Ferdowsi University of Mashhad, International Campus
Mashhad, Iran
m.nouei@yahoo.ca

[2]Research Development Center
Shariati Hospital, Tehran University of Medical Sciences (TUMS)
Tehran, Iran

[3]Department of Cardiovascular Surgery
Shariati Hospital, Cardiac Surgery and Transplantation Research Center (CTRC)
Tehran University of Medical Sciences (TUMS)
Tehran, Iran

***Abstract*—The goal of this study is to recognize most important risk factors associated with surgical mortality in patient who underwent CABG by the integration of fuzzy concept and analytical hierarchy process method to represent pairwise comparison of odd ratios and overcome ambiguities involved in the statistical data. A literature search from 1980 to January 2013 using the MEDLINE and Science Direct database is performed and data of the reported predictors were extracted. A fuzzy AHP model for comparing the relative importance of risk factors was developed. Moreover, fuzzy clustering method is applied to classify calculated weight of risk factors. The result indicated that advanced age (over 70 years), sever left ventricular ejection fraction (LVEF<30%), emergency state of patient, elevated creatinin (above 2 mg over dL) and reoperation have the highest calculated weight among other predictors. In addition to, other variables were identified to be contributing risk factors to operative mortality after CABG, although they have not reached the level of importance of core risk factors. This study also, showed that the importance of risk factors varied by geographic region. We conducted that fuzzy clustering and FAHP has successfully detected strongest risk factors to predict mortality rate after CABG and showed the power of the engineering tools in health area. Furthermore, developed model, as a decision support tool, can be helpful for surgeons to determine appropriate technique for better management of individual patient before surgery as well as to provide pertinent information to develop novel scoring model according to importance of risk factors in different regions.**

Key words: Fuzzy Analytic Hierarchy Process; Fuzzy Clustering; Risk Factors; CABG; Mortality

I. INTRODUCTION

Risk management, defined as the identification, assessment, and prioritization of risks followed by coordinated and economical application of resources to minimize, monitor, and control the probability and/or impact of unfortunate events, proved that to be essential to the health care industry [1]. Risk assessment is increasingly seen as standard practice to monitor and evaluate surgical performance with allowing the physician to define the possibility of adverse outcomes in individuals in a variety of situations by developing risk prediction models for post-operative adverse outcomes. So, Preoperative risk assessment can be an effective method of quality assurance.

Cardiac events account for more than half of the deaths after surgery in the United States, and are associated with substantial treatment costs [2].Therefore the need for choosing effective, safe and reliable methods of treatment is felt more [3]. Despite technological advancements, open-heart operations still carry a risk of mortality and morbidity and it is difficult to decide about appropriate protocols of treatment. To aid in the selection of patients for cardiac surgery, several risk scoring systems have been developed during the last decades [4]. Coronary artery bypass graft (CABG) is the most common type of open-heart surgery in the world, owing to improvements in surgical techniques, medications and patient care and it accounts for a significant portion of the total health care expenditure as well as more resources expended in cardiovascular medicine than any other single procedure [5];[6]. Therefore most of the methods developed to stratify cardiac risk were focused on this kind of surgery. Nowhere in the field of biosciences is the need for tools to deal with uncertainty more critical than in medicine and epidemiology[7]. The descriptions of the uncertainties in the risk analysis confirm the pertinence of the fuzzy methodologies. There are different applications, where the statistical methods and fuzzy technologies are combined or compared to achieve better results [8].The use of newer approaches, such as fuzzy logic seems to better address the challenge of increasing complexity predisposing factors linked to the occurrence of mortality events data after open heart surgery. The use of fuzzy sets to describe the risk factors and fuzzy-based decision techniques to help incorporate inherent imprecision, uncertainties and subjectivity of available data, as well as to propagate these attributes throughout the model, yield more realistic results [9]. Fuzzy environment is able to indicate the ambiguous risk factors in an acceptable form. In spite of its potential in dealing with uncertainties and vagueness, very few works applying fuzzy logic concept in epidemiological problems has been presented so far [7]. The goal of this study is to recognize most important risk factors associated with surgical mortality in patient who underwent CABG by the integration of fuzzy concept and analytical hierarchy process method to represent pairwise comparison of odd ratios and overcome ambiguities involved in the statistical data. Moreover, a fuzzy clustering method is applied to classify risk factors. To the best of our knowledge, to build up a fuzzy AHP model for comparing the importance of predictors of early mortality after CABG is the novel contribution and the problem with its specific characteristics is not reported in the literature.

II. MATERIALS & METHODS

A. *Data collection*

We performed a literature search from 1980 to January 2013 using the MEDLINE and Science Direct database because of comprehensive nature of these databases. Language restriction was enforced and non–English-language articles were not translated. The reference lists of all selected publications were checked to retrieve relevant publications which had not been found with the computerized search.

In the first step of screening the articles, only studies that developed a new scoring model were considered. For this research, it was required that studies reported on risk models to be used to estimate the risk of early mortality for CABG surgery, either with or without concomitant procedure. Moreover, prevalence of patients undergoing isolated CABG had to be reported more than 60 percent. In addition to, we included models exclusively focused on adults and had been presented the association (Odd Ratios with corresponding 95% Confidence Interval) of predictor factors with the outcome. More information of selected papers is summarized in Table 1.

Table 1. Summary of selected risk scoring models

Scoring code	Reference	Region	No. Of Patient	No. Of Center
1	QMMI Model [10]	USA	9498	12
2	JACVSD Model [11]	Japan	7133	97
3	Pitkanen et al [12]	Finland	5413	1
4	Amphiascore [13]	Netherlands	7282	1
5	Toronto II [14]	Canada	7491	2
6	NYS II [15]	USA	16120	1
7	Carosella et al [16]	Argentina	4698	4
8	NYS III [17]	USA	10148	1
9	Zheng et al [18]	China	9838	17
10	THIScore [19]	USA	5281	1
11	AusSCORE [20]	Australia	7709	1

The study quality of each publication was evaluated by use of standard assessment checklist developed base on theoretical considerations and methodological aspects, comprises 5 categories, study population, treatment, outcome, prognostic factors and data presentation, and includes items on validity, precision of method and clinical aspect of study design. The checklist and some additional explanation are provided at [21].The results of the quality assessment are presented in Table2.The positive scores on each item were summed and the result are attributed to study quality score.

Data extracted of the prognostic studies including information about scoring systems (study population number and related characteristics, start and end time of data collection, Year of publication), outcome measures, the type of procedure, measure of C-index and multivariate association calculated between predictor factors and outcome in terms of Odds Ratios (OR) with 95% confidence intervals (CI).

Table 2. The result attributed to study quality score.

Scoring code	Weight of scoring model (%)
1	76.9%
2	84.6%
3	76.9%
4	76.9%
5	69.2%
6	84.6%
7	76.9%
8	92.3%
9	69.2%
10	84.6%
11	92.3%

B. Data Analysis

The Analytic Hierarchy Process (AHP), via providing a plausible framework, is a popular decision making technique that has proven to be applicable for complex decisions involving many factors for prioritizing them among multi-criteria. This methodology has been used in various settings to make decisions[22]. Moreover, AHP has seen widespread applications across numerous fields in health care and medical decision making [23]. The conventional AHP method does not take into account the uncertainty and vagueness involved in fuzzy decision-making environment. So, to deal with vagueness, fuzzy version of AHP should be used in spite of its complexity. In fuzzy AHP, triangular fuzzy numbers are used to express the preference in the pairwise comparisons. A triangular fuzzy number (TFN) is the special class of fuzzy number whose membership is defined by three real numbers, expressed as (l, m, u). There are the several procedures to attain the priorities in FAHP such as fuzzy least square method, geometric mean method, Synthetic extend analysis, Mikhailov's fuzzy preference programming and two-stage logarithmic programming [24]. In this work, Chang's extent analysis methods[25] is utilized.

Taking into consideration the purpose and method of this study, criteria and factors were identified. The risk scoring models, listed in table 2, were the criteria in this decision model while the significant risk factors of mortality after CABG, mentioned in related model, such as age, gender, poor LVEF and so on were the alternatives. TFNs are used to represent pairwise comparison of odd ratios in order to capture the vagueness.

The steps of Chang's extent analysis [25] can be detailed as follows:

Step 1: Construct the Pairwise Matrix

By using odd ratio's confidence interval of risk factors as triangular fuzzy numbers (TFNs), via pairwise comparison, the fuzzy evaluation matrix is constructed. Assuming p risk factors and q scoring models, the pairwise comparison of factor i with factor j yields a square matrix A_{p*p} , where ${a_{ij}}^h = ({l_{ij}}^h, {m_{ij}}^h, {u_{ij}}^h)$, denotes the comparative importance of risk factor i with respect to risk factor j for h^{th} model and calculated as follow:

Consider two TFNs, a_i *and* $a_j, a_i = (l_i, m_i, u_i)$ *and* $a_j = (l_j, m_j, u_j)$, *So* ,

$$a_{ij} = (l_i/u_j\, , m_i/m_j\, , u_i/l_j). \tag{1}$$

Step 2: Define the value of fuzzy synthetic extent

The value of fuzzy synthetic extent with respect to ith risk factor is defined as:

$$\widetilde{S_i} = \sum_{j=1}^{p} \widetilde{a_{ij}}^h \otimes \left[\sum_{i=1}^{p}\sum_{j=1}^{p} \widetilde{a_{ij}}^h\right]^{-1}, \ \ i = 1, \dots, p \ \ and\ , h = 1, \dots, q \tag{2}$$

Where,

$$\left[\sum_{i=1}^{p}\sum_{j=1}^{p} \widetilde{a_{ij}}^h\right]^{-1} = [(1/\sum_{i=1}^{p}\sum_{j=1}^{p} u_{ij}), (1/\sum_{i=1}^{p}\sum_{j=1}^{p} m_{ij}), (1/\sum_{i=1}^{p}\sum_{j=1}^{p} l_{ij})] \tag{3}$$

And,

$$[\ \sum_{j=1}^{p} \widetilde{a_{ij}}^h\ ] = [\ \sum_{j=1}^{p} {l_{ij}}^h, \sum_{j=1}^{p} {m_{ij}}^h, \sum_{j=1}^{p} {u_{ij}}^h] \tag{4}$$

Step 3: Define the degree of possibility of $(\widetilde{\mathbf{S_i}} \gg \widetilde{\mathbf{S_j}})$

The degree of possibility $\widetilde{\mathbf{S_i}} = (l_i, m_i, u_i) \gg \widetilde{S_j} = (l_j, m_j, u_j)$ is defined as:

$$V(\tilde{S}_i \gg \tilde{S}_j) = \sup[\min(\mu_{\tilde{S}_i}(x), \mu_{\tilde{S}_j}(y))] \;, x \gg y \quad (5)$$

Since $\tilde{S}_i$ and $\tilde{S}_j$ are convex fuzzy numbers, these can equivalently be expressed as follows:

$$V(\tilde{S}_i \gg \tilde{S}_j) = \begin{cases} 1 & if\ m_i > m_j \\ hgt\,(\tilde{S}_i \cap \tilde{S}_j) = \mu_{\tilde{S}_j}(d) & Otherwise \end{cases} \quad (6)$$

Where d is the ordinate of the highest intersection point D between $\mu_{\tilde{S}_j}$, and $\mu_{\tilde{S}_i}$ (see Fig. 1).

When $\widetilde{S_i} = (l_i, m_i, u_i)\ and\ \widetilde{S_j} = (l_j, m_j, u_j)$, the ordinate of D is given by Eq. (7)

$$V(\tilde{S}_i \gg \tilde{S}_j) = hgt\,(\tilde{S}_i \cap \tilde{S}_j) = (l_i - u_j)/[(m_j - u_j) - (m_i - l_i)] \quad (7)$$

To compare $\tilde{S}_i$ and $\tilde{S}_j$, we need both the values of $V(\tilde{S}_i \gg \tilde{S}_j)$ and $V(\tilde{S}_j \gg \tilde{S}_i)$.

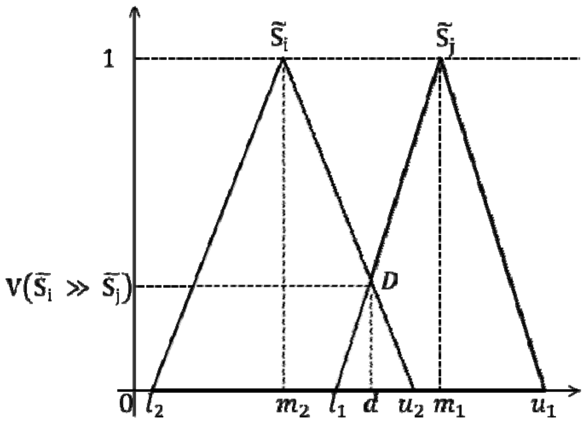

Figure 1. The intersection between $\tilde{S}_i$ and $\tilde{S}_j$.

This step is calculated for each scoring model ($h = 1, \ldots, q$).

Step 4: Define the degree of possibility $(\tilde{S} \gg \widetilde{S_1}, \ldots, \widetilde{S_k})$

The degree possibility for a convex fuzzy number to be greater than k convex fuzzy numbers can be defined by:

$$V(\tilde{S} \gg \widetilde{S_1}, \widetilde{S_2}, \ldots, \widetilde{S_k}) = V[(\tilde{S} \gg \widetilde{S_1})\ and\ (\tilde{S} \gg \widetilde{S_2})\ and \ldots and\ (\tilde{S} \gg \widetilde{S_k})]$$
$$= min\, V(\tilde{S} \gg \widetilde{S_i}), \quad i = 1,2, \ldots, k. \quad (8)$$

Assume that,

$$\acute{d}^h(A_i) = \min V(\tilde{S}_i \gg \widetilde{S_k}), for\ k = 1,2, \ldots, p\ ; k \neq i, and, h = 1, \ldots, q \quad (9)$$

So, the weight vector of risk factors in h^{th} scoring model matrix is given by:

$$\acute{W}^h = (\acute{d}^h(A_1), \acute{d}^h(A_2), \ldots, \acute{d}^h(A_p))^T, h = 1,2, \ldots, q \quad (10)$$

Via normalization, we get the normalized weight vector, denoted by:

$$W^h = (d^h(A_1), d^h(A_2), \ldots, d^h(A_p))^T, h = 1,2, \ldots, q \quad (11)$$

Where W is a non-fuzzy number.

Step 5: Combine the weights derived in step 4 and the weight of risk scoring models to obtain overall rate

Finally, we compute the overall composite weight of each risk factor based on the weight of each scoring models. The overall weight, P_i, is just normalization of linear combination between the weights derived in step 4 ,$d^h(A_i)$, and the normalized weight of scoring model , K^h ,developed based on appraisal results. (See Table 2)

$$P_i = \sum_{h=1}^{q} d^h(A_i) * K^h, i = 1,2, \ldots, p \quad (12)$$

To facilitate interpretation and comparison of the results, a computerized database was prepared and FAHP method was implemented by visual basic language.

The results analyzed by FAHP method (P_i) classified into 3 levels (core, level 1and level 2) by using Fuzzy C-means clustering method in order to reflect their importance for prediction of operative death after CABG. Computations of C-means method were carried out in R software version (2.15.2) using package “e1701-1.6-1”.

III. RESULTS

As mentioned earlier, important risk factors clustered in 3 levels. The results of clustering and FAHP method are depicted in Fig. 2. Age over 80 years, sever left ventricular ejection fraction (LVEF<30%), age between 75 to 80, emergency state of patient, age between 70 to 75, elevated creatinin (above 2 mg/dL) and reoperation , as is evident from table 4, have the highest calculated weight among other variables respectively, So, they are considered as core variables. In addition to, comorbidities such as Chronic Obstructive Pulmonary Disease (COPD) and peripheral arterial disease (PVD) were concluded to be of less prognostic importance than core variable. Other variables such as age lower than 70 years, moderately LVEF, female gender, acute Myocardial Infraction (MI), NYHA class (III and IV), use of Intra-aortic balloon pump (IABP) and urgent surgery were identified to be contributing risk factors to operative mortality after CABG, although they have not reached the level of importance of core risk factors.

Furthermore, our results showed that presence of Diabetes Mellitus (DM), family history of Congestive Heart Failure (CHF), Pulmonary Hypertension, Left Main Coronary Artery Stenosis(LMCAS), arrhythmia and other variables mentioned in table 4 were not quite as important in predicting short term mortality for patients undergoing CABG as core and level one variables. Some prognostic factors for example aortic valve stenosis [11] were only reported once. Hence, hard evidence for this has not yet been found and the evidence on their predictive value remains inconclusive.

A sample computation of Chang's extent analysis methods is presented in Appendix1. Moreover, the values of cluster center, Membership matrix, and the value of objective function calculated in different iteration is summarize in Appendix2.

IV. DISCUSSION AND CONCLUSION

The aim of this study was to identify most important variables which can help to predict operative mortality before performing the CABG surgery. Although several scoring model were developed to predict mortality after CABG, few published studies have developed to define and prioritize the importance of related risk factors.

The Working Group Panel on the Collaborative CABG Database Project has categorized 44 clinical variables into 7 core, 13 level 1 and 24 level 2 variables, to reflect their relative importance in determining short-term mortality after CABG. This group has identified and proposed uniform definitions for a list of 7 core variables (i.e., age, gender, acuity of operation, LVEF, previous operation, left main coronary artery disease and number of diseased coronary arteries) that they consider must be present in any database of patients undergoing CABG [26]. Similarly, Tu and associates have suggested a limited set of six core variables (age, gender, emergency operation, previous CABG or redo surgery, LVEF and left main disease) appear to be sufficient for fairly comparing hospital risk-adjusted mortality rates after CABG in Ontario [27].Moreover, these researchers as well as Hannan and associates believed that left ventricular ejection fraction, reoperation, and left main disease have an important impact on hospital risk-adjusted mortality rates and that these factors should be part of any risk adjustment model for assessing the short-term results of CABG [27];[28].

Preoperative risk factors such as advanced age, reoperation, poor LVEF and emergency surgery identified as core risk factor for early mortality as has been reported by our study. However, Left main coronary artery disease, In this work, was only reported by Toronto II scoring model [14] and the evidence on its relative importance remains questionable. In contrast, female gender was presented in several scoring models but the importance of this predictor was lower than another risk factors.

Our results proved that Creatinin level unambiguously related to operative mortality, although this predictor is considered as level one variable by mentioned group. But, Ranucci and associates included serum Creatinine were highly statistically significant predictor of early mortality after CABG for elective patient. Beside, these researchers concluded the model limited to this predictor as well as advanced age and poor LVEF had an accuracy equivalent to or better compared with more complex risk scores [29]. Therefore, our results do not conflict with presented works. Another aspect of conclusion in this work that has not been well studied is whether the patient's geographic region may impact surgical outcome. The importance of risk factors may be varied by geographic regions because of different patient demographics and clinical profile. For example, relative to importance of risk factors in the American scoring, redo surgery in the Europe and Asia appeared to have lower weight but slightly greater percentages of emergency surgery, and renal insufficiency, namely, elevated serum Creatinin. This study also noted significantly higher weight of Diabetes Mellitus in American systems compared with this in the other regions. In addition to, the analysis demonstrated that the importance of COPD was statistically higher in the Asian scoring model and marginally higher in American ones.

Consequently, Our results suggests that the factors lists in Fig. 2 should be part of any risk stratification models because they have an important impact on early mortality rates for patient undergoing CABG in different regions. Also we conducted that fuzzy clustering and FAHP, as engineering tools, and statistics, as a branch of mathematics, has successfully detected strongest risk factors to predict mortality rate after CABG and showed the power of the engineering tools in health area.

This study could be extended to weight identified risk factors of CABG morbidity or extended length of stay in ICU or hospital. Furthermore, predictors of adverse outcome following valve surgery (include aortic, mitral, Tricuspid and multiple valve surgery), Surgery on thoracic aorta, Heart Transplantation or cardiac intervention such as PCI can be prioritized in the same way.

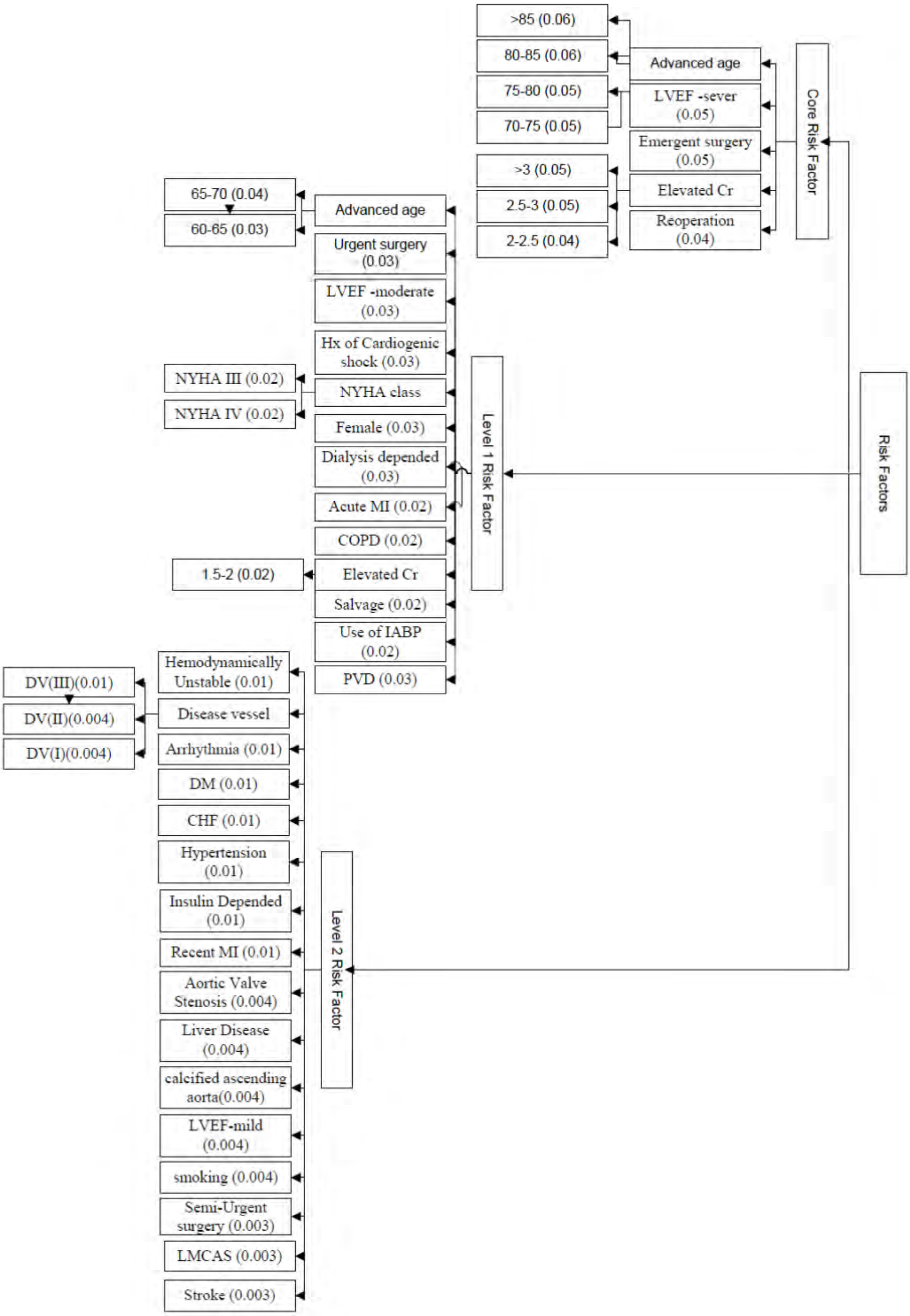

Figure 2. Core, Level 1 and Level 2 risk factors (number(s) in parentheses show the weight of risk factor calculated by FAHP)

ACKNOWLEDGMENT

The authors would like to acknowledge Dr. Zarey H, cardiac resident of Shariati Hospital, affiliated to Tehran University of Medical Sciences, TUMS, for their valuable cooperation and help in this study. We are grateful to Dr. Esfehani F, Epidemiologist Consultant for development research center as well as to Dr.Hakemi MS, Department of Nephrology, Shariati Hospital, TUMS, for providing their suggestions and helps.

REFERENCES

[1] Hubbard D. W, The Failure of Risk Management: Why It's Broken and How to Fix It. 2009, John Wiley & Sons.
[2] Ali M. J, et al., Reports of Investigation: ACC/AHA guidelines as predictors of postoperative cardiac outcomes. Canadian Journal of Anesthesia, 1999. 47(1): p. 10-19.
[3] Mirmohammad S. M, et al., Evaluation of accuracy of Euroscore risk model in prediction of perioperative mortality after coronary bypass graft surgery in Isfahan. Journal of Research in Medical Sciences, 2011. 16(6).
[4] Nilsson J, et al., Comparison of 19 pre-operative risk stratification models in open-heart surgery. European Heart Journal, 2006. 27: p. 867-874.
[5] Eagle K. A, et al., ACC/AHA Guidelines for Coronary Artery Bypass Graft Surgery: Executive Summary and Recommendations : A Report of the American College of Cardiology/American Heart Association Task Force on Practice Guidelines (Committee to Revise the 1991 Guidelines for Coronary Artery Bypass Graft Surgery). American Heart Association, 1999. 100: p. 1464-1480.
[6] Magovern J. A, et al., A Model That Predicts Morbidity and Mortality After Coronary Artery Bypass Graft Surgery. J Am Coil Cardiol, 1996. 28: p. 1147-1153.
[7] Massad E., et al., Fuzzy epidemics. Artificial Intelligence in Medicine, 2003. 29(3): p. 241-259.
[8] TAKÁCS M., Extended Fuzzy Methods in Risk Management, in 14th WSEAS International Conference on APPLIED MATHEMATICS. 2009, WSEAS Press: Puerto De La Cruz, Tenerife, Canary Islands, Spain
[9] Takács M., Multilevel Fuzzy Approach to the Risk and Disaster Management. Acta Polytechnica Hungarica, 2010. 7(4).
[10] Fortescue E.B, Kahn k, and Bates D. W, Development and Validation of a Clinical Prediction Rule for Major Adverse Outcomes in Coronary Bypass Grafting. American Journal of Cardiology, 2001. 88: p. 1251–1258.
[11] Motomura N, et al., First Report on 30-day and Operative Mortality in Risk Model of Isolated Coronary Artery Bypass Grafting in Japan. Ann Thorac Surg, 2008. 86: p. 1866-1872.
[12] PitkaÈnen O, et al., Intra-institutional prediction of outcome after cardiac surgery: comparison between a locally derived model and the EuroSCORE. European Journal of Cardio-thoracic Surgery, 2000. 18: p. 703-710.
[13] Huijskes R.V.H.P, Rosseel P.M.J, and Tijssen J.G.P, Outcome prediction in coronary artery bypass grafting and valve surgery in the Netherlands: development of the Amphiascore and its comparison with the Euroscore. European Journal of Cardio-thoracic Surgery, 2003. 24: p. 741-749.
[14] Ivanov J, Tu J. V, and N.C. D, Ready-Made, Recalibrated, or Remodeled? : Issues in the Use of Risk Indexes for Assessing Mortality After Coronary Artery Bypass Graft Surgery. American Heart Association, 1999. 99: p. 2098-2104.
[15] Hannan E. L, et al., Risk Stratification of In-Hospital Mortality for Coronary Artery Bypass Graft Surgery. Journal of the American College of Cardiology, 2006. 47(3): p. 661– 8.
[16] Carosella V. C, et al., The first Latin-American risk stratification system for cardiac surgery: can be used as a graphic pocket-card score. Interactive CardioVascular and Thoracic Surgery, 2009. 9: p. 203–208.
[17] Hannan E. L, et al., The New York Risk Score for In-Hospital and 30-Day Mortality for Coronary Artery Bypass Graft Surgery. The Annals of Thoracic Surgery, 2012. 95: p. 46-52.
[18] Zheng Z, et al., Risk factors and in-hospital mortality in Chinese patients undergoing coronary artery bypass grafting: Analysis of a large multi-institutional Chinese database. Acquired Cardiovascular Disease, 2012. 144: p. 355-359.
[19] Madan P, et al., Risk-prediction models for mortality after coronary artery bypass surgery:Application to individual patients. International Journal of Cardiology, 2011. 149: p. 227-231.
[20] Reid Ch, et al., An Australian risk prediction model for 30-day mortality after isolated coronary artery bypass: The AusSCORE. Acquired Cardiovascular Disease, 2009. 138: p. 904-910.
[21] Nouei M. T., Kamyad A. V., and Ghazalbash S., Fuzzy Clustering Analysis of Risk Factors for Mortality after CABG Based on AHP 2013.unpublished
[22] Saaty T. L., Decision making with the analytic hierarchy process. Int. J. Services Sciences, 2008. 1(1): p. 83-98.
[23] Liberatore M. J and N.R. L, The analytic hierarchy process in medical and health care decision making: A literature review. European Journal of Operational Research, 2008. 189: p. 194-207.
[24] Kabir G. and Akhtar H. A., Comparative Analysis of AHP and Fuzzy AHP Models for Multicriteria Inventory Classification. International Journal of Fuzzy Logic Systems (IJFLS), 2011. 1(1).
[25] Chang D. Y., Applications of the extent analysis method on fuzzy AHP. European Journal of Operational Research, 1996. 95(3): p. 649-655.
[26] Jones R. H, et al., Identification of Preoperative Variables Needed For Risk Adjustment of Short-Term Mortality After Coronary Artery Bypass Graft Surgery. Journal of the American College of Cardiology, 1996. 15: p. 1478-87.
[27] TU J. V, SYKORA K, and NAYLOR C. D, Assessing the Outcomes of Coronary Artery Bypass Graft Surgery: How Many Risk Factors Are Enough? Journal of the American College of Cardiology, 1997. 30(5): p. 1317-23.
[28] Hannan E. L, et al., Clinical Versus Administrative Data Bases for CABG Surgery: Does it Matter? Medical Care, 1992. 30(10).
[29] Ranucci M, et al., Risk of Assessing Mortality Risk in Elective Cardiac Operations : Age, Creatinine,Ejection Fraction, and the Law of Parsimony. American Heart Association, 2009. 119: p. 3053-3061.

APPENDIX

Appendix 1- A sample computation of Chang's extent analysis methods

Step 1: Pairwise Matrix for i=1(Sever LVEF) and h=1 (AusSCORE) are calculated by using Equation 1.

Risk Factor i	**Risk Factor j**	**Lower**	**Middle**	**Upper**
LVEF -sever	urgent surgery	0.433566434	1.05	4.985075
LVEF -sever	emergent salvage	0.198294243	0.607229	3.650273
LVEF -sever	re operation-first	0.37804878	1.100437	6.242991
LVEF -sever	PVD	0.525423729	1.194313	5.301587
LVEF -sever	LVEF-moderate	0.488188976	1.155963	5.344
LVEF -sever	LVEF -sever	0.278443114	1	3.591398
LVEF -sever	Age 60-65	0.369781312	1.194313	7.505618
LVEF -sever	Age 65-70	0.369781312	1.194313	7.505618
LVEF -sever	Age 70-75	0.235145386	0.707865	4.175
LVEF -sever	Age 75-80	0.235145386	0.707865	4.175
LVEF -sever	Age 80-85	0.124165554	0.446018	3.13615
LVEF -sever	Age>85	0.124165554	0.446018	3.13615
LVEF -sever	NYHA class III	0.486910995	1.205742	5.808696
LVEF -sever	NYHA class IV	0.305921053	0.807692	3.690608

Step 2: The value of fuzzy synthetic extent for for i=1,…, 14 and h=1 (AusSCORE) are calculated by using Equation 2,3,and 4.

Risk Factor	$\tilde{S}_1$:Middle	$\tilde{S}_1$: Lower	$\tilde{S}_1$: Upper
urgent surgery	0.055172414	0.003264035	0.860432381
emergent salvage	0.095402299	0.004457599	1.881318353
reoperation	0.052643678	0.002606356	0.986789584
PVD	0.048505747	0.003069167	0.71000714
LVEF-moderate	0.050114943	0.003044808	0.764160227
LVEF -sever	0.057931034	0.004530675	1.339787484
Age 60-65	0.048505747	0.002167904	1.008851953
Age 65-70	0.048505747	0.002167904	1.008851953
Age 70-75	0.08183908	0.003897355	1.58648488
Age 75-80	0.08183908	0.003897355	1.58648488
Age 80-85	0.129885057	0.005188353	3.004493489
Age>85	0.129885057	0.005188353	3.004493489
NYHA class III	0.048045977	0.002801224	0.766165896
NYHA class IV	0.071724138	0.004408882	1.219447291

Step 3: Fuzzy synthetic degree values for i=1, j=1,…,14 and h=1 are calculated by using Equation 6 and 7.

Risk Factor i	**Risk Factor j**	**Fuzzy synthetic degree values**
LVEF -sever	emergent salvage	0.972704521
LVEF -sever	reoperation	1
LVEF -sever	peripheral arterial disorder	1
LVEF -sever	LVEF-moderate	1
LVEF -sever	LVEF -sever	1
LVEF -sever	Age 60-65	1
LVEF -sever	Age 65-70	1
LVEF -sever	Age 70-75	0.982417945
LVEF -sever	Age 75-80	0.982417945
LVEF -sever	Age 80-85	0.948843723
LVEF -sever	Age>85	0.948843723
LVEF -sever	NYHA class III	1
LVEF -sever	NYHA class IV	0.989776614

Step 4: Degree of possibility for i=1 and h=1 calculated by using Equation 9 and 10

Risk Factor	Min Fuzzy synthetic degree values	Normalized weight
LVEF -sever	0.948843723	0.071734819

Step 5: Overall composite weight for i=1 and h=1,…, 11 are calculated by using Equation 12

Scoring Model	Risk Factor	Normalized weight of risk factor	Normalized weight of Scoring
AusSCORE	LVEF -sever	0.071734819	0.10436454
QMMI	LVEF -sever	0.048980181	0.08695161
JACVSD	LVEF -sever	0.042890418	0.09565807
Pitkanen et al	LVEF -sever	0.044178926	0.08695161
Amphiascore	LVEF -sever	0.062680979	0.08695161
Toronto II	LVEF -sever	0.117425719	0.07824514
NYS II	LVEF -sever	0.088623875	0.09565807
Carosella et al.	LVEF -sever	0.04491257	0.08695161
NYS III	LVEF -sever	0.063547092	0.10436454

Overall weight of LVEF = Normalized weight of risk factor* Normalized weight of Scoring= 0.053342796

Appendix 2- The values of cluster center, Membership matrix, the value of objective function in C-mean clustering

Membership matrix of c-means clustering algorithm

	1	2	3		1	2	3
[1,]	6.54E-02	2.68E-02	9.08E-01	[24,]	6.06E-01	3.56E-01	3.78E-02
[2,]	6.54E-02	2.68E-02	9.08E-01	[25,]	4.92E-01	4.71E-01	3.72E-02
[3,]	8.37E-03	2.91E-03	9.89E-01	[26,]	3.21E-01	6.48E-01	3.10E-02
[4,]	7.56E-03	2.62E-03	9.90E-01	[27,]	2.96E-01	6.75E-01	2.95E-02
[5,]	2.34E-03	7.83E-04	9.97E-01	[28,]	2.13E-01	7.63E-01	2.38E-02
[6,]	1.25E-02	3.64E-03	9.84E-01	[29,]	4.87E-02	9.44E-01	7.21E-03
[7,]	1.63E-02	4.68E-03	9.79E-01	[30,]	1.87E-02	9.78E-01	3.04E-03
[8,]	6.80E-02	1.74E-02	9.15E-01	[31,]	5.36E-04	9.99E-01	9.85E-05
[9,]	1.27E-01	2.99E-02	8.43E-01	[32,]	4.80E-04	9.99E-01	9.28E-05
[10,]	1.63E-01	3.65E-02	8.00E-01	[33,]	1.84E-03	9.98E-01	3.65E-04
[11,]	6.50E-01	7.49E-02	2.75E-01	[34,]	2.27E-03	9.97E-01	4.51E-04
[12,]	8.79E-01	4.56E-02	7.53E-02	[35,]	2.97E-03	9.96E-01	5.96E-04
[13,]	9.22E-01	3.32E-02	4.43E-02	[36,]	3.30E-03	9.96E-01	6.66E-04
[14,]	9.25E-01	3.25E-02	4.27E-02	[37,]	3.55E-03	9.96E-01	7.16E-04
[15,]	9.81E-01	9.95E-03	8.67E-03	[38,]	4.36E-03	9.95E-01	8.87E-04
[16,]	9.97E-01	1.92E-03	1.33E-03	[39,]	5.30E-03	9.94E-01	1.09E-03
[17,]	9.98E-01	9.99E-04	6.60E-04	[40,]	6.35E-03	9.92E-01	1.31E-03
[18,]	1.00E+00	2.04E-04	1.26E-04	[41,]	7.62E-03	9.91E-01	1.59E-03
[19,]	9.97E-01	1.84E-03	9.35E-04	[42,]	8.01E-03	9.90E-01	1.68E-03
[20,]	9.82E-01	1.29E-02	5.24E-03	[43,]	1.00E-02	9.88E-01	2.12E-03
[21,]	9.46E-01	4.13E-02	1.30E-02	[44,]	1.22E-02	9.85E-01	2.61E-03
[22,]	9.08E-01	7.33E-02	1.91E-02	[45,]	1.66E-02	9.80E-01	3.64E-03
[23,]	7.79E-01	1.89E-01	3.18E-02				

Custer centroids in c-mean clustering algorithm

[1]	[2]	[3]
0.025254588	0.005680969	0.050758896

Objective function values in c-mean clustering algorithm

Iteration:	Error:	Iteration:	Error:
1,	7.55E-05	8,	1.73E-05
2,	5.09E-05	9,	1.73E-05
3,	2.59E-05	10,	1.73E-05
4,	1.80E-05	11,	1.73E-05
5,	1.74E-05	12,	1.73E-05
6,	1.73E-05	13,	1.73E-05
7,	1.73E-05	14,	1.73E-05