Design and Analysis of Fuzzy Metagraph Based Data Structures

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Abstract— Fuzzy metagraph is an emerging technique used in the design of many information processing systems like transaction processing systems, decision support systems, and workflow systems. Very often, even a carefully chosen graph data structure could be improvised to provide more efficiency in terms of time complexity or space complexity or both. In this paper, a well-designed fuzzy metagraph is proposed and distinct matrices have been developed to reduce the time-complexity. Fuzzy Expert System (FES) integrated with the Fuzzy Metagraph can yield excellent state-of-the-art decisions under complex circumstances which other graph structures find it very difficult. Thus a user with this effective decision making system can make effective and quick decisions to solve the problem.

Keywords- Metagraph, Fuzzy Metagraph, Fuzzy Expert System (FES), Abstract Data Type (ADT).

I. INTRODUCTION

Metagraph is a graph-based hierarchical data structure in which every node is a set having one or more elements. It has all the properties of graphs. In a metagraph, there is a set to set mapping in place of a node to node as in a conventional graph structure. Metagraphs have lot of applications in the field of information processing systems, decision support systems, models management and the rule based management systems in which a single work consists of many information processing tasks to be performed by the humans or the machines. Metagraphs under such situations can provide a useful and comprehensive function for modeling by extending the features offered by the traditional graph structures i.e., digraphs, hypergraphs. Metagraph allows different components of the process to be represented both graphically and analytically.

In 1965 fuzzy logic was introduced by Zadeh as modification of classical set theory. Fuzzy set theory enables the processing of imprecise information by means of membership function. Fuzzy logic allows intermediate categories between notations such as true/false, hot/cold, black/white etc. as used in Boolean logic. In fuzzy system, values are indicated by a number in the range of 0 to 1 where 0 represents absolute falseness and 1 represents absolute truthfulness.

Fuzzy expert systems are well known to make decisions to problems that exhibit uncertainty resulting from inexactness, vagueness or subjectivity. Fuzzification is the process of converting crisp input to fuzzy value. Membership Functions are used to convert crisp inputs into fuzzy values. A Membership Function maps each element of the input to a membership grade (or membership value) between zero and one.

An Abstract Data Type (ADT) is a mathematical model of the data objects that make up a data type as well as the functions that operate on these objects. Fuzzy set ADT integrated with Fuzzy Metagraph perform different operations like union, intersection, disjoint subset union, Clustering and Searching. This method has reduced time complexity

The rest of the paper is organized as follows. Section 2 gives the related work. Section 3 deals with fuzzy metagraph based technique. Section 4 points out Fuzzy metagraph based data structures technique. Section 5 concludes the paper.

II. RELATED WORKS

Deepti Gaur, and Aditya Shastri have proposed a model for metagraph data structure. They have used to store data inside the computer memory either in the form of an Adjacency matrix or in an Adjacency list so it can be used efficiently [6] and they have proposed metagraph based substructure pattern mining technique. They have developed an algorithm which adapts the depth-first search strategy to mine frequent connected sub metagraph efficiently [7]. They have proposed fuzzy metagraph method of clustering to find the similar fuzzy nodes in a fuzzy metagraph. They have used T-norms (Triangular Norms) functions and join two or more T norms to cluster the fuzzy nodes [8]. They have proposed vague metagraph method, which is a graphical model that not only visualized the process of any system but also their formal analysis where the analysis will be accomplished by means of an algebraic representation of the graphical structure. The graphical structure has been represented by the adjacency and incidence matrix of a vague metagraph [9].

Pankaj Dashore, and Suresh Jain have used rule based system and fuzzy metagraph for real world application (Online Transaction such as banking, E commerce and share market,) to make correct decisions. It has been constructed by projection operation of a fuzzy metagraph to provide high level view that reduces the unnecessary details. The projection of fuzzy metagraph is more dominant because there is less number of edges [13], [14], [15].

Zheng-Hua Tan has proposed a Fuzzy Metagraph based knowledge representation. The Fuzzy Metagraph has been applied to fuzzy rule-based systems for knowledge representation and reasoning in the format of algebraic representation and Fuzzy Metagraph closure matrix [20].

III. METHODOLGY

This section briefly reviews fuzzy metagraph techniques. Basu and Blanning introduced the concept of metagraph [1]. A metagraph S = {X, E} is a graphical representation consisting of two tuples X and E. Here X is its generating set and E is the set of edges defined on generating sets. The set of elements X = {x1, x2, x3,…,xn} represents variables which occur as nodes and E = {e1, e2, e3, … en} occur as edges of a metagraph.

3.1 Fuzzy Metagraph

The concept of a fuzzy graph is the "fuzzification" of the crisp graphs using fuzzy sets. A fuzzy graph $\tilde{G}$ can be defined as a triple {X, $\tilde{X}$, $\tilde{E}$ }, where $\tilde{X}$ is a fuzzy set on X and $\tilde{E}$ is a fuzzy relation on X× X.

A fuzzy set $\tilde{X}$ on X is completely characterized by its membership function μ:X→ [0, 1] for each x ϵ X, μ(x) illustrates the truth value of the statement of x ∈ $\tilde{X}$. The fuzzy metagraph is the concept of Fuzzification of the crisp Metagraph using fuzzy generating set. Fuzzy generating set is the node set of all the elements of fuzzy metagraph [8, 20]. Consider a finite set X={x1, x2, x3,…, x $_n$}. A fuzzy metagraph is a triple $\tilde{S}$ ={X, $\tilde{X}$, $\tilde{E}$ } in which $\tilde{X}$ is a fuzzy set on X and $\tilde{E}$ is a fuzzy edge set { $\tilde{e}_m$,m=1, 2, 3,.. m}. Each component $\tilde{e}$ in $\tilde{E}$ is characterized by an ordered pair <$\tilde{V}_m$,$\tilde{W}_m$>. In the pair $\tilde{V}_m \subseteq \tilde{X}$ is the in-vertex of $\tilde{e}_m$ and $\tilde{W}_m \subseteq \tilde{W}$ is the out-vertex.

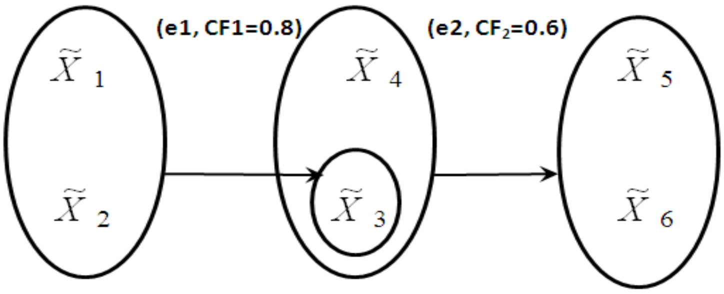

Fig .1 Fuzzy Metagraph

Often, the membership value of an edge is also called certainty factor (CF) of the edge. For simplicity, assign $\tilde{X}_i$ denoting (X_i, μ ($\tilde{X}_i$)) and $\tilde{e}_k$ denoting (e_k, CF_k). Figure 1 shows fuzzy metagraph whose element set is X = { $\tilde{X}_1$, $\tilde{X}_2$,..., $\tilde{X}_6$ } is known as fuzzy meta node and whose edge set consists of: $\tilde{e}_1$ = <{ $\tilde{X}_1$, $\tilde{X}_2$ }, { $\tilde{X}_3$ }> and $\tilde{e}_2$ = <{ $\tilde{X}_3$, $\tilde{X}_4$ }, { $\tilde{X}_5$, $\tilde{X}_6$ }> . The in-vertex and out-vertex of $\tilde{e}_1$ are { $\tilde{X}_1$, $\tilde{X}_2$} and { $\tilde{X}_3$}.

3.1.1 Adjacency Matrix of Fuzzy Metagraph

An example is the adjacency matrix, a square matrix with one row and one column for each element in the generating set. Each member of the matrix is a set of triples, one for each edge connecting the row element to the column element. The triples define the in vertex, out vertex, and the edge [20].

Table I. Adjacency matrix of Fuzzy Metagraph

	$\tilde{X}_3$	$\tilde{X}_5$	$\tilde{X}_6$
$\tilde{X}_1$	$<\tilde{X}_2, \varnothing, \tilde{e}_1>$	$\varnothing$	$\varnothing$
$\tilde{X}_2$	$<\tilde{X}_1, \varnothing, \tilde{e}_1>$	$\varnothing$	$\varnothing$
$\tilde{X}_3$	$\varnothing$	$<\tilde{X}_4, \tilde{X}_6, \tilde{e}_2>$	$<\tilde{X}_4, \tilde{X}_5, \tilde{e}_2>$
$\tilde{X}_4$	$\varnothing$	$<\tilde{X}_3, \tilde{X}_6, \tilde{e}_2>$	$<\tilde{X}_3, \tilde{X}_5, \tilde{e}_2>$

3.1.2 Adjacency List of a Fuzzy Metagraph

An adjacency list basically has *V* linked lists, with each corresponding linked list containing the elements that are adjacent to a particular vertex.

$\tilde{X}_1 \rightarrow <\tilde{X}_2, \varnothing, \tilde{e}_1>$, $\tilde{X}_2 \rightarrow <\tilde{X}_1, \varnothing, \tilde{e}_1>$, $\tilde{X}_3 \rightarrow <\tilde{X}_4, \tilde{X}_6, \tilde{e}_2> \rightarrow <\tilde{X}_4, \tilde{X}_5, \tilde{e}_2>$, $\tilde{X}_4 \rightarrow <\tilde{X}_3, \tilde{X}_6, \tilde{e}_2> \rightarrow <\tilde{X}_3, \tilde{X}_5, \tilde{e}_2>$.

IV. FUZZY METAGRAPH BASED DATA STRUCTURES AND TECHNIQUES

Different kinds of data structures are suited to different kinds of applications, and some are highly specialized to certain tasks. After the data structures are chosen, the algorithms to be used often become relatively obvious. A data structure is an actual implementation of a particular abstract data type (ADT). An ADT is a set of operation such as intersection, union, set difference and set complement. What is to be done is mentioned but how is to be done is not mentioned. According to Fuzzy Metagraph based ADT, to perform different operation like fuzzy set union, fuzzy set intersection, Searching, Sorting and clustering. Tractable Problems are solvable in polynomial (P) time. Intractable Problems are not solvable in polynomial time, Deterministic Polynomial (P) time: $O(n^2)$, $O(n^3)$, $O(1)$, $O(n \log n)$, $O(\log n)$, Non Deterministic polynomial (NP) time: $O(2^n)$, $O(n^n)$, $O(n!)$

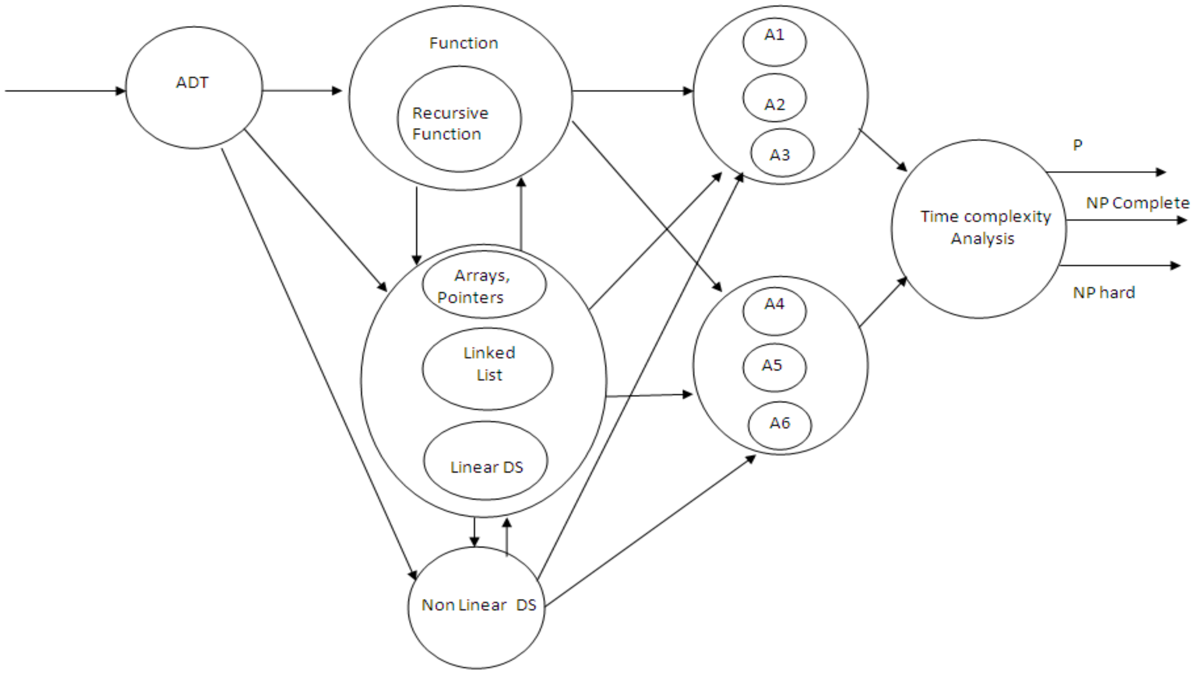

Fig .2 Data Structures is represented in Metagraph organization

Figure 2 shows the Data Structures (DS) represented in metagraph format, the set of algorithms design technique is A= {A1, A2, A3, A4, A5, A6}. Where A1 is Brute Force Algorithm, A2 is Divide and Conquer Algorithm, A3 is Transform and Conquer Algorithm, A4 is Decrease and Conquer, A5 is Greedy Algorithm and A6 is Dynamic Programming. This system will help users to make correct decision with very low risk for analysis the time complexity. Data structures are implemented by a programming language as data types. An optimization problem tries to find an optimal solution and decision problem tries to answer a yes/no question. Many problems will have decision and optimization versions Like Minimum Spanning Tree and Traveling salesman problem. Some problems are decidable, but intractable as they grow too large, we are unable to solve them in reasonable time. The connectivity properties of metagraph can be used to determine whether a specific collection of models is sufficient to calculate a set of target variables from a set of input variables, possibly under a set of assumptions.

Warshall algorithm is used for computing the transitive closure of the fuzzy metagraph. Reducing the edges by merging the vertices in the fuzzy metagraph based data structures. To identify the loops which are connected in graph, combine those vertices in to a single node by using union find algorithm. Strongly component Fuzzy Metagraph is DAG (Direct Acyclic Graph). If G has a cycle G=s1, s2, s3….sn then s=s1 U s2 U s3…U s_n will be strongly connected component. f: A → B Polynomial time reduction from language A to language B. The condition equivalent to YES maps to YES and NO maps to NO.

4.1 Distinct matrixes for Fuzzy Metagraph based data structures

Given a Fuzzy Metagraph $\tilde{S}=\{X, \tilde{X}, \tilde{E}\}$ and its adjacency matrix A is defined as an infinite sum, namely

$A^* = A + A^2 + A^3 + A^4 + \ldots\ldots + A^n, n \to \infty$

$$A^* = \sum_{n=1}^{\infty} A^n$$

The closure matrix A* for the Fuzzy Metagraph is formed by adding the successive powers of the adjacency matrices.

Table II. The Closure of Adjacency matrix of Fuzzy Metagraph

	$\tilde{X}_3$	$\tilde{X}_5$	$\tilde{X}_6$
$\tilde{X}_1$	$\langle \tilde{X}_2, \emptyset, \tilde{e}_1 \rangle$	$\langle\{\tilde{X}_2, \tilde{X}_4\}, \{\tilde{X}_3, \tilde{X}_6\}, \langle \tilde{e}_1, \tilde{e}_2\rangle\rangle$	$\langle\{\tilde{X}_2, \tilde{X}_4\}, \{\tilde{X}_3, \tilde{X}_5\}, \langle \tilde{e}_1, \tilde{e}_2\rangle\rangle$
$\tilde{X}_2$	$\langle \tilde{X}_1, \emptyset, \tilde{e}_1 \rangle$	$\langle\{\tilde{X}_1, \tilde{X}_4\}, \{\tilde{X}_3, \tilde{X}_6\}, \langle \tilde{e}_1, \tilde{e}_2\rangle\rangle$	$\langle\{\tilde{X}_1, \tilde{X}_4\}, \{\tilde{X}_3, \tilde{X}_6\}, \langle \tilde{e}_1, \tilde{e}_2\rangle\rangle$
$\tilde{X}_3$	$\emptyset$	$\langle \tilde{X}_4, \tilde{X}_6, \tilde{e}_2 \rangle$	$\langle \tilde{X}_4, \tilde{X}_5, \tilde{e}_2 \rangle$
$\tilde{X}_4$	$\emptyset$	$\langle \tilde{X}_3, \tilde{X}_6, \tilde{e}_2 \rangle$	$\langle \tilde{X}_3, \tilde{X}_5, \tilde{e}_2 \rangle$

The closure matrix of Fuzzy Metagraph in Figure 1 can be illustrated as Table II. The square, A^2, of the adjacency matrix of Fuzzy Metagraph in Figure 1 is given in Table III.

Table III. The Square of Adjacency matrix of Fuzzy Metagraph

	$\tilde{X}_5$	$\tilde{X}_6$
$\tilde{X}_1$	$\langle\{\tilde{X}_2, \tilde{X}_4\}, \{\tilde{X}_3, \tilde{X}_6\}, \langle \tilde{e}_1, \tilde{e}_2\rangle\rangle$	$\langle\{\tilde{X}_2, \tilde{X}_4\}, \{\tilde{X}_3, \tilde{X}_5\}, \langle \tilde{e}_1, \tilde{e}_2\rangle\rangle$
$\tilde{X}_2$	$\langle\{\tilde{X}_1, \tilde{X}_4\}, \{\tilde{X}_3, \tilde{X}_6\}, \langle \tilde{e}_1, \tilde{e}_2\rangle\rangle$	$\langle\{\tilde{X}_1, \tilde{X}_4\}, \{\tilde{X}_3, \tilde{X}_6\}, \langle \tilde{e}_1, \tilde{e}_2\rangle\rangle$

ALGORITHM Transitive closure of the Fuzzy Metagraph (A[1…n])

{

//Input: The adjacency matrix A of a fuzzy metagraph with n generating set.

//Output: the transitive closure of the fuzzy metagraph

$R^{(0)} = A$

for k=1 to n do

for i=1 to n do

for j=1 to n do

$R^{(k)}[i,j] = R^{(k-1)}[i,j]$ or ($R^{(k)}[i,k]$ and $R^{(k)}[k,j]$)

return $R^{(n)}$

}

4.2. Fuzzy Metagraph Based Searching Techniques

Decrease by a constant, Decrease by a constant factor and variable size decrease are some of the major variations of the Decrease and Conquer technique. BFS, DFS and Topological sort belong to the same Decrease and conquer Technique [5, 10]. Figure 3, shows an example of fuzzy metagraph. Let us consider the set A={ $\tilde{X}_1$, $\tilde{X}_2$}, B={ $\tilde{X}_3$, $\tilde{X}_4$}), C={ $\tilde{X}_5$} and D={ $\tilde{X}_6$, $\tilde{X}_7$}. When performing the Depth First Search, We construct a DFS tree. One metagraph has several DFS trees. The result of Fuzzy Metagraph based DFS (FMDFS) is A-B-D-C. A Breadth-First search (BFS) is another technique for traversing a finite undirected graph. The result of Fuzzy Metagraph based BFS (FMBFS) is A-B-C-D. Fuzzy metagraph traversal algorithm is based on the adjacency matrix and adjacency list of a graph. Thus these two techniques can find a lot of applications in the various fields [5, 10].

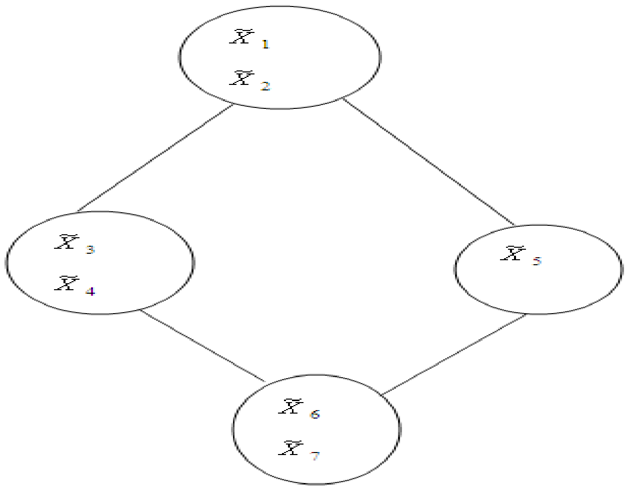

Fig. 3 An example of a Fuzzy Metagraph based traversal technique

```
Algorithm FMDFS (V)
{
Visited (V) =1;
for each vertex w adjacent from V do
  {
    if (Visited [w] = 0) then
      FMDFS (w);
   }
}

Algorithm FMBFS (V)
{
        Visited (V) =1;
        Initialize Queue Q to be empty
        loop
        for each vertex w adjacent from V do
        {
         Add (w, Q);
        Visited (W) =1;     // mark w as visited
        }
        if empty (Q) then return;
        Delete (V, Q);     // Delete from the Queue
        forever
}
```

Fuzzy Metagraph based BFS and DFS yield same efficiency in terms of adjacency matrix and adjacency list. For the adjacency matrix representation, the traversal time is in $O(|V^2|)$, and for the adjacency list representation , it is in $O(|V|+|E|)$ where $|V|$ and $|E|$ are the number of vertices and edges a graph respectively. A Fuzzy Expert System (FES) is simply an expert system that uses a collection of fuzzy membership functions and rules, instead of Boolean logic, to reason about data. Fuzzy expert system consists of Fuzzification, inference system, rule base, Defuzzification units. It has the capability to solve decision making problems for which no exact algorithm exists. Fuzzy Expert System (FES) integrated with the Fuzzy Metagraph can yield excellent state-of-the-art decisions under complex circumstances which other graph structures find it very difficult [3].

V. CONCLUSION

Metagraphs have a lot of applications in the field of information processing systems; decision support systems, transaction processing systems, and workflow systems. Distinct algorithms and matrixes have been developed for Fuzzy Metagraph based data structures. The graphical model not only visualized the process of any system but also their formal analysis where the analysis will be accomplished by means of an algebraic representation of the graphical structure. Data will be stored inside the computer memory either in the form of an Adjacency matrix or as an Adjacency list so that it can be used efficiently. Future works may be concentrated on the optimization techniques applied for Fuzzy Metagraph to enhance the performance of the system.

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