Implementation of Distributed Algorithm with Network Coding for Multicast Network

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Abstract-In today's practical networks, end-to-end information delivery is performed by routing. Network coding generalizes routing by allowing a node to generate output data by mixing (i.e., computing certain functions of) its received data. Network coding techniques are used to find the minimum cost in given network. In wire line network, solving for the optimal coding subgraphs in network coding is equivalent to finding the optimal routing scheme in a multi-commodity flow problem. Multicast is an important factor for the communication in wireless network. This problem is also known as NP-complete. This paper focuses on the solution for the above problem and provides the analytical framework as well as distributed algorithm in multicast session. A set of node based distributed algorithm are designed at sources node and virtual at intermediate node.

Keyword-Network coding, multi-commodity flow problem, distributed algorithm, wireless networks.

I. INTRODUCTION

In wireline networks routing has long been an important technique for optimizing data transmission from sources to destinations. Optimal data routing in a network can be often understood as a multi-commodity flow problem. In any network , each node function as a switch for passing the data ,but make no change into data content ,This situation can be treated as a multi-commodity flow problem(MFP),[1] where data stream are treated as a commodities identified by their different destinations. The network coding extends the functionality of network nodes from traditional routing. Network coding increased the throughput,[2]network robustness,[3] and the efficiency of the network[4]. Network coding techniques have been applied to wireline networks, the performance gains of network is also increase in wireless network also. Although our solution is general enough to fit into a variety of network to achieve different optimization goals, such as overlay network, multihop wireless networks etc.

Moreover, multicast plays an essential role in the most popular applications supported by these new networks, such as overlay multicast, P2P content distribution, wireless sensor network etc. In internet each node attempts to send each packet over a route that minimizes that packet`s delay with no regard to other packet`s delays. With the aid of network coding, we are able to formulate the optimal multicast routing problem in the fashion of multicommodity flow problem. The major contribution of this work is an optimal distributed solution. In this solution, each node makes its own routing decision based on periodic updating information from neighboring nodes.

It is proved by Ahlswede et al.[5] that with network coding, the achievable throughput of a multicast session can be acquired by running max-flow algorithm from the source to each individual receiver, then choosing the minimal result. [6]prove the same result using algebraic approach, further shows that the above result can be obtained by running linear coding.[7] are the first to propose a practical network coding solution.

II. PRELIMINARIES

A. Network Coding

Fig.1.1 show that the famous example of butterfly network [5] for which coding in the interior of the network is necessary in order to achieve maximum possible multicast rate.

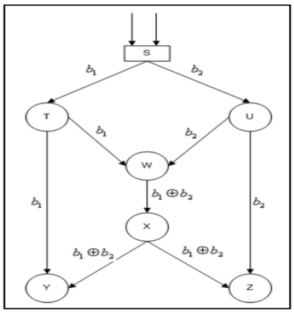

Fig.1.1:Butterfly Network

In this network we multicast two bits from the source node S to Y and Z. Node W derives from the received b_1 and b_2 the exclusive-OR bit $b_1 \oplus b_2$.The channel from W to X transmits $b_1 \oplus b_2$,which is then replicated at X for passing on to Y and Z. Then the node Y receives b_1 and $b_1 \oplus b_2$, from which the bit b_2 can be decoded. Similarly, the node Z decodes the bit b_1 from the received bits b_2 and $b_1 \oplus b_2$.In this way, all the nine channels in the network are used exactly once. The derivation of the exclusive-OR bit is a simple form of coding. If the same communication objective is to be achieved simply by bit replication at the intermediate nodes without coding, at least one channel in the network must be used twice, so that the total number of channel usage will be at least ten. Thus coding offers the potential advantage of minimizing both latency and energy consumption, and at the same time maximizing the bit rate.

B. Network Model

We consider a *n*-node network, where the nodes are represented as $N = \{1,2,\ldots,n\}$. Let L be the set of links, denoted as $L = \{(i;k) \mid$ a link goes from i to $k\}$. Each link $(i; k)$ is associated with a capacity *Cik*. There are a set of multicast sessions M. For each session $m \in M$, it has a sender $S(m)$, and a set of receivers $R(m)$. Let $\eta_i^m(j) \ge 0$, 0 be the traffic of session m, in bits/s, generating at node i and destined for node j (data sink). $\eta_i^m(j) \ge 0$ only if node $i = S(m)$, and $j \in R(m)$, i.e., if node i is the sender of session m, and node j is one of the receivers of m. We also define node flow $t_i^m(j)$ to be the total traffic of session m at node i destined for node j. $t_i^m(j)$ includes both $\eta_i^m(j)$ and the traffic from other nodes that is routed through i to destination j. Finally, $\phi_{ik}^m(j)$ is the fraction of the node flow $t_i^m(j)$ routed over link $(i; k)$. It is always true that $\phi_{ik}^m(j) = 0$ if $(i; k) \notin L$ (no traffic can be routed through non-existent link), or $i = j$ (traffic that has reached its destination is not sent back into the network). Also, node i must route its entire node flow $t_i^m(j)$ through all links, i.e.[1]

$$\sum \phi_{ik}^m(j) = 1, \forall_{i,j} \in N, \forall_m \in M \qquad (1)$$

Now we express the relation of above notations as follows:

$$t_i^m(j) = \eta_i^m(j) + \sum_{l \in N} t_l^m(j)\phi_{li}^m(j), \forall_{i,j} \in N, \forall_m \in M \qquad (2)$$

Eq. (2) expresses flow conservation: for a given multicast session *s*, the traffic into a node for a given destination is equal to the traffic out of it for the same destination.

III. MINIMUM COST MULTICAST

In this section we consider the problem of minimum cost with network coding for a subgraph for multicast session. Our framework provides a feasible set of network coding subgraph for every receiver of the multicast session.

Let the network supporting the multicast session, represented as the directed graph G=(N,E). Let $F_{ij}(m)$ denote network coded transmission rate of session m traffic on link (i,j). The flow rate of subsession f represent the part of $F_{ij}(m)$ that is relevant for receiver $\omega \in W(m)$. Thus the vector f($\omega; m$) = ($f_{ij}(\omega; m)$) form the coding subgraph[10] for the pair $(s(m), \omega)$.The flow rate of a session and its subsession are related as follows.

$F_{ij}(m) = \max_{\omega \in W(m)} f_{ij}(\omega; m)$ (1)

The total flow rate on the link (i,j) is

$F_{ij} = \sum_{m \in M} F_{ij}(m)$.

For all $\omega \in W(m)$, $f_{ij}(\omega; m) \geq$ 0 and

$$\sum_{j \in O_i} f_{ij}(\omega; m)$$

$$= \begin{cases} r_m \\ 0 \\ \sum_{j \in I_i} f_{ji}(\omega; m) \blacktriangle t_i(\omega; m). \end{cases}$$

The flow of any subsession follows the same conservation constraints as a unicast session in traditional routed networks; it can be optimized by a routing methodology. The main difference between the present problem and the traditional problem is that the session flow $F_{ij}(m)$ is the maximum of the subsession flows $f_{ij}(\omega; m)$. For the optimality of a multicast scheme, first consider the utility function $U_m(r_m)$ with each session $m \in M$. Assume that session m`s maximum rate demand is R_m bits/s and $U_m(r_m)$ is strictly increasing ,concave, and twice continuously differentiable in $r_m \in [0, R_m]$.

IV. DISTRIBUTED ALGORITHM

A distributed solution is used for optimizing the configuration of network coding in wireless networks [8][9]. Following are step for the distributed algorithm .A scaled gradient projection method is used to design a set of node based primal algorithm. This algorithm iteratively finds the minimum cost subgraph. To achieve minimum cost multicast, the coding subgraph is optimized with the power control schemes at the physical layer. These entire algorithms are distributed in the sense that network nodes can separately update their control variable after obtaining a limited number of control messages from their neighboring nodes. The scaled gradient projection algorithm is appropriate for providing a distributed solution. These algorithm include two kinds of routing algorithm implemented at the source node and intermediate nodes respectively.

Minimum cost subgraph

We consider finding the minimum-cost network coding subgraphs for a single multicast session with elastic rate demand. This procedure, followed by a network code designed specifically for the derived subgraphs, provides the optimal network configuration for the multicast session. Suppose that the directed graph $G = (N,A)$ models the network. Let *fij* and *cij* be the cost function and the capacity, respectively, of link (i, j). We assume that *fij* is convex and monotonically increasing and that *cij* is non-negative. Then the optimal cost for a rate R multicast connection from source node s to sink nodes T that is asymptotically-achievable with network coding is given by the following optimization problem.

$$minimize \sum_{(i,j) \in A}' f_{ij}(z_{ij})$$

$$subject\ to z_{ij} \geq x_{ij}^{(t)}, \forall (i,j) \in A, t \in T$$

$$\sum_{\{j | (i,j) \in A\}}' x_{ij}^{(t)} - \sum_{\{j | (j,i) \in A\}}' x_{ji}^{(t)} = \sigma_i^{(t)}, \quad (1)$$

$$\forall_i \subset N, t \subset T, c_{ij} \geq x_{ij}^{(t)} \geq 0, \forall (i,j) \subset A, t \subset T,$$

$$\sigma_i^{(t)} = \begin{cases} R & if\ i = s, \\ -R & if\ i = t, \\ 0 & otherwise \end{cases}$$

Node based routing variables and optimality condition.

To finding the optimal coding subgraphs is equivalent to solving for the minimum-cost flow distribution. The problem can therefore be tackled with an optimal routing methodology. To enable each node to independently adjust its virtual flow values, we adopt the *routing variables* .For source node *s*, they are defined as

$$\phi_{ss'} = \frac{F_{ss'}}{R}, \phi_{sj}(\omega) = \frac{f_{sj}(\omega)}{R}, \forall_{\omega} \in W \ and \ j \in O(s),$$

and for intermediate node, $i \in \mathcal{N} \setminus \{s, w\}$ they are defined as

$$\phi_{ij}(\omega) = \frac{f_{ij}}{t_i(\omega)}, \forall_{\omega} \in W \ and \ j \in O(i),$$

These newly defined variables are subject to node-based simplex constraints

Node based distributed congestion control and routing algorithm

After obtaining the optimality conditions, we come to the question of how individual nodes can adjust their local routing variables to find the optimal coding subgraphs. we adapt this technique to design algorithms for adjusting all virtual sessions' routing configurations to find the minimum-cost coding subgraphs. These distributed algorithms are used at the source node and intermediate nodes, respectively. Our scheme uses a different technique for computing the scaling matrices and step sizes. This scheme allows us to guarantee the convergence of the algorithms from all initial conditions.

Source Node Congestion Control/Routing Algorithm (CR)

This algorithm is implemented at the source node s(m) of every session m$\in$M. It adjusts the routing variables on all the outgoing links of s(m) and for all subsessions w$\in$W(m).We therefore call it the Congestion Control/Routing Algorithm (CR) algorithm. For conciseness we suppress the session index m and use the short hand notation $\varphi_s(\omega) = (\varphi_{sj}(\omega))_{j \in \varphi}$. At the kth iteration , the feasible set of vector $\varphi_s = (\varphi_m, (\varphi_s(\omega))_{\omega \in W(m)})$ is

$$F^k_{\phi_s} = \{\phi_s \geq 0 : \phi_{ss'} + \phi_s(\omega)' \cdot 1 = 1 \text{ and } \phi_{sj}(\omega) = 0, \forall_j \in B^k_s(\omega), \omega \in W\},$$

Where ` denote the vector transpose and $B^k_s(\omega)$ is the blocked node set of node s relative to subsession ω at iteration k^3 .

A. Intermediate Node Routing Algorithm (RT)

Consider any session m$\in$M and for brevity, omit the index m. Relative to a subsession ω,an intermediate node i changes the allocation of the subsession`s traffic on its outing links by adjusting its current routing vector $\phi^k_i(\omega) = (\phi^k_{ij}(\omega))_{j \in \phi_i}$ within the feasible set

$$F^k_{\phi_s} = \{\phi_s \geq 0 : \phi_{ss'} + \phi_s(\omega)' \cdot 1 = 1 \text{ and } \phi_{sj}(\omega) = 0, \forall_j \in B^k_s(\omega), \omega \in W\},$$

Similar to CR, it has a scaled gradient projection form

$$\mathcal{F}^k_{\phi_s} = \{\phi_s \geq 0 : \phi_{ss'} + \phi_s(w)' \cdot 1 = 1 \text{ and } \phi_{sj}(w) = 0, \forall j \in B^k_s(w), w \in W\},$$

$$\phi^{k+1}_i(w) = RT(\phi^k_i(w)) = \left[\phi^k_i(w) - (M^k_i(w))^{-1} \cdot \delta\phi^k_i(w)\right]^+_{M^k_i(w)}.$$

Where $M^k_i(w)$

is chosen to be the diagonal matrix.

A. *Marginal Cost Exchange Protocol*

In order to let each node acquire the necessary information $\partial\phi_i(\omega)$ to implement either *CR*4 or *RT*, protocols for exchanging control messages must be developed. The rules for propagating the marginal cost information are specified. Before iterating its local algorithm, node *i* collects local measures $\frac{\partial D_{ij}}{\partial f_{ij}(\omega;m)}$ and inquires its next-hop neighbors $j \in \sigma_i$ for their marginal costs $\frac{\partial D}{\partial r_j(\omega;m)}$ with respect to the adjusted session(s) *w*. It then evaluates the terms $\delta\phi_{ij}(\omega;m)$

B. Convergence of CR and RT algorithms

The scaled gradient projection method seeks to reduce the objective value with each iteration. Because the update direction at every iteration is opposite to the gradient (scaled by a positive definite matrix) with respect to the adjusted variables, it is a descent direction. However, reduction of the objective cost is guaranteed only when appropriate scaling matrices are used. In this subsection, we specify such matrices for *CR* and *RT*, respectively. It turns out that the scaling matrix at each node *i* depends on the number of nodes in its downstream node set $DN_i^k(\omega)$ relative to session ω`s flow at current iteration *k*. For convenience, introduce notations [11][12].

$$AN_i^k(\omega) \equiv \vartheta(i) \setminus B_i^k(\omega) \text{and} AN_i^k \equiv \bigcup_{\omega} AN_i^k(\omega).$$

V. CONCLUSION

This paper presents a general solution for optimal multicast routing. We show that with the aid of network coding, the intractable optimal multicast routing problem becomes tractable. We further show that this problem can be solved in an entirely distributed way by presenting distributed routing algorithm. Using the scaled gradient projection method, we designed a set of node based congestion control and routing algorithm that are proved to achieve the minimum cost multicast configuration in networks. Thus the cut bound 2.0 is achieved by this network coding solution.

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