Applying moving boundary to compute the alluvium transport and testing the results by remote sensing

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***Abstract*— To compute the alluvium transport, we use a current model based on a 2D finite-difference grid and a sediment transport model. The first one gives the velocity distribution on the surface of water body and in the case of transient analysis, the velocity distribution is computed at each computational time step. This velocity distribution will be taken as the input for the second model. There are two important problems solved in this research: using moving boundary to get more correct results and testing the results by remote sensing. The models were used to compute the alluvium transport in Ca Mau coastal zone. The resonableness in the transport trend of alluvium shows that those models are confident.**

Keywords- alluvium transport; current model; moving boundary.

I. INTRODUCTION

The alluvium transport at estuaries affects much to our economy, ecological environment. Alluviums make the ability of flood drainage decreases.

In the world, there are many research groups implemented this project. Most of them used Defant and Hansen methods [1, 7] at Atlantic, Pacific. Some of them used ADI method to solve the problem [2, 3, 4, 6, 9, 10]. The strength of those models is that the software is easy to use because they simplified the equations by removing many parameters. Unfortunately it is also their limit. The accuracy is low. Therefore, the main objectives for the development of these models are:

- Improving the accuracy of the available models by applying moving boundary, and
- Testing the results by remote sensing.

The remainder of this paper is following, section 2 presents detailed expositions for the models. The experiment results are undertaken in section 3. Section 4 focuses on future developments of the model. Finally, section 5 gives conclusions of the study.

II. METHODOLOGY

A. Current model

The model solves the depth averaged 2D shallow water equations. They are equations of floods, ocean tides and storm surges. They are derived by using the hypotheses of vertically uniform horizontal velocity and negligible vertical acceleration (i.e. hydrostatic pressure distribution). The assumptions are valid where the wave length is much greater than the depth of water. In the case of ocean tide the equations are applicable everywhere.

The 2D equations in the horizontal plane are describable by the following partial differential equations of mass continuity and momentum conservation in the X and Y directions for an in-plan Cartesian coordinate frame of reference.

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}-fv+g\frac{\partial \varepsilon}{\partial x}+gu\frac{\sqrt{u^2+v^2}}{(h+\varepsilon)C^2}-\frac{\tau_x}{h+\varepsilon}=f_x(t)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+fu+g\frac{\partial \varepsilon}{\partial y}+gv\frac{\sqrt{u^2+v^2}}{(h+\varepsilon)C^2}-\frac{\tau_y}{h+\varepsilon}=f_y(t)$$

$$\frac{\partial \varepsilon}{\partial t}+\frac{\partial}{\partial x}(h+\varepsilon)u+\frac{\partial}{\partial y}(h+\varepsilon)v=0$$

where:

u, v: depth averaged velocity components in X and Y directions [m/s]

ε : water surface elevation [m]

h+ε : depth of water [m]
t: time [s]
x, y: distance in X and Y directions [m]
C: Chezy coefficient
f: Coriolis force coefficient
τ_x τ_y : horizontal diffusion of momentum coefficient in X and Y directions
$f_x(t)$, $f_y(t)$: sum of components of external forces in X and Y directions

B. Alluvium transport model

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=\frac{1}{H}\frac{\partial}{\partial x}\left(HK_x\frac{\partial C}{\partial x}\right)+\frac{1}{H}\frac{\partial}{\partial y}\left(HK_y\frac{\partial C}{\partial y}\right)+\frac{S}{H}$$

where:

C	: depth averaged concentration [kg/m^3].
u, v	: depth averaged velocity components in X and Y directions [m/s]
K_x, K_y	: diffusion coefficients in the X and Y directions [m^2/s]
H	: relative depth [m], $H = h + \zeta$
S	: source of sediment particles [g/m^2.s]

To compute K_x, K_y, we use Elder formula:

$$K_x = 5.93\sqrt{g}h|u|C^{-1}$$
$$K_y = 5.93\sqrt{g}h|v|C^{-1}$$

where:

C	: Chezy coefficient [m$^{0.5}$/s]

C. Algorithm

All of the models use an alternating direction implicit (ADI) finite different method to solve the problems. The method involves two stages. In each stage, a tri-diagonal matrix for the computational domain is built to solve the values:

$$\begin{bmatrix} B_1 & C_1 & & & & & \\ A_2 & B_2 & C_2 & & & & \\ & A_3 & B_3 & C_3 & & & \\ & & \cdots & & & & \\ & & & A_i & B_i & C_i & \\ & & & & \cdots & & \\ & & & & A_{I-1} & B_{I-1} & C_{I-1} \\ & & & & & A_I & B_I \end{bmatrix} * \begin{bmatrix} C_1^{t+1/2} \\ C_2^{t+1/2} \\ C_3^{t+1/2} \\ \vdots \\ C_i^{t+1/2} \\ \vdots \\ C_{I-1}^{t+1/2} \\ C_I^{t+1/2} \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ \vdots \\ D_i \\ \vdots \\ D_{I-1} \\ D_I \end{bmatrix}$$

There are millions of nodes to compute the values. There are also millions of equations to be solved. The tri-diagonal matrix method will increase the speed of computing because values out of the diagonals are 0.

- **Initial conditions**

At time t = 0:

u = 0, v = 0, ε = 0

C(x,y,0) = C_0(x,y) or C(x,y,0)= constant

- **Boundary conditions**
  - *For the current model*

- Compute tidal components

$$\varsigma = \sum_{i=1}^{N} A_i \sin(\omega_i t + \varphi_i)$$
, or

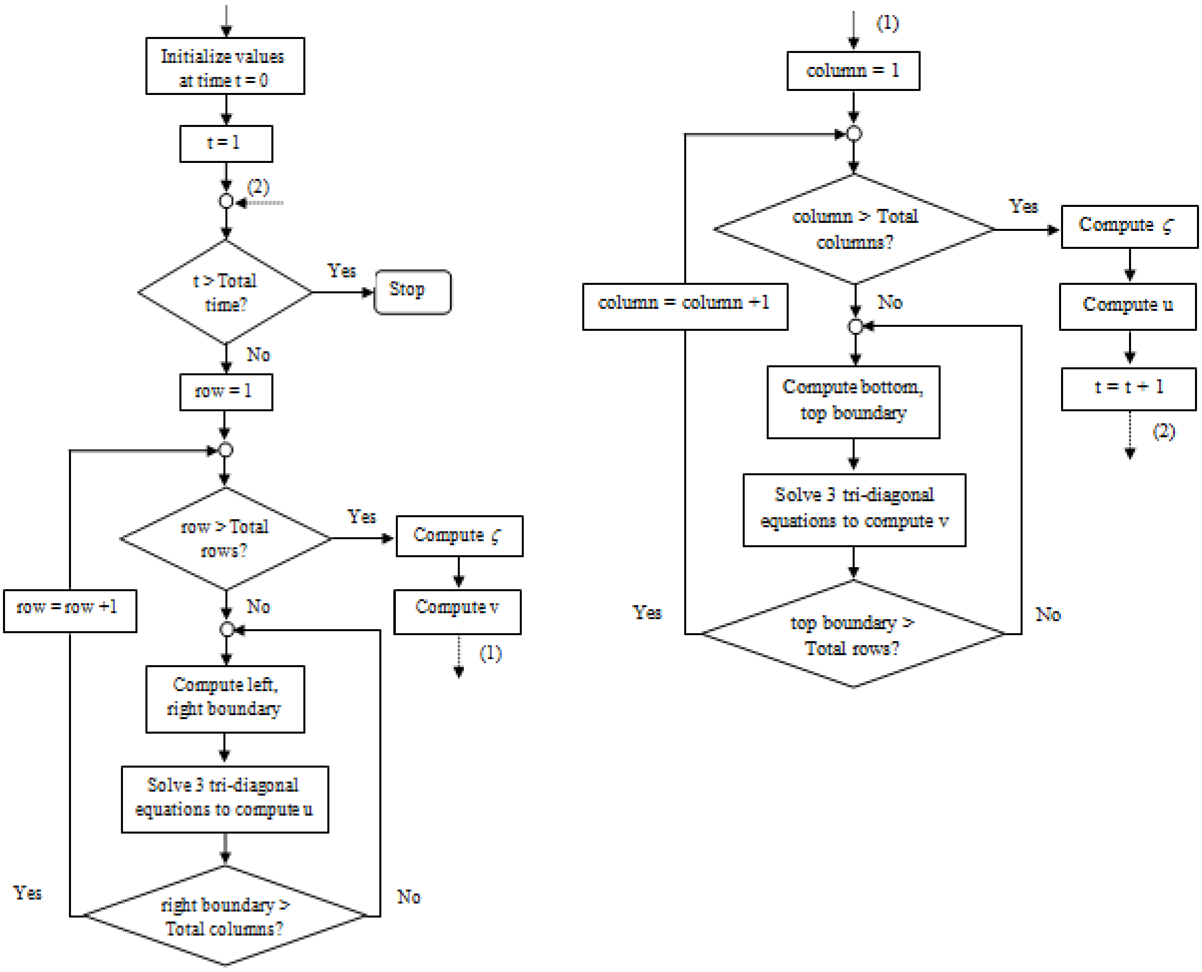
- Compute Q = U * W

Then, compute the velocities at the boundaries.

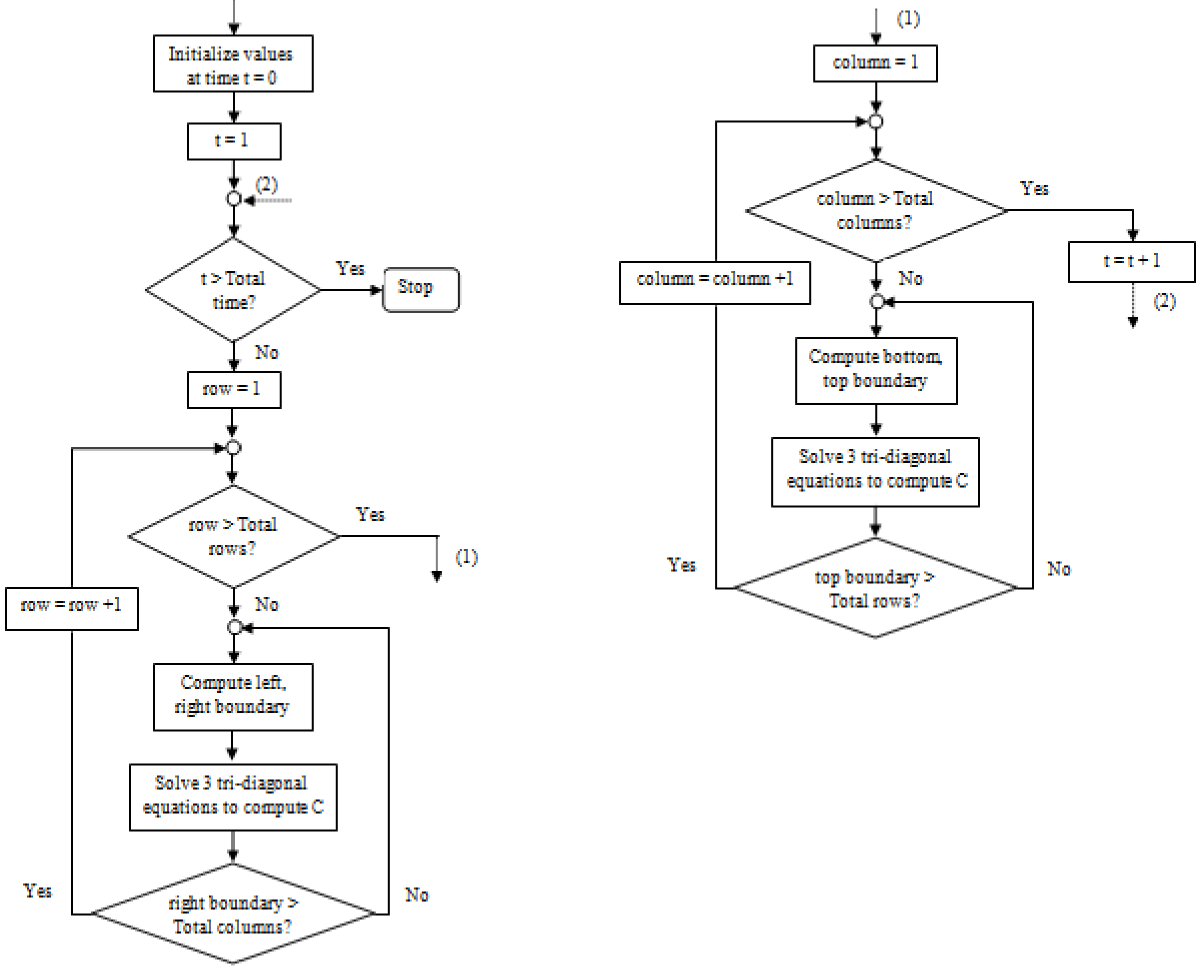
  - *For the alluvium transport model*

- Solid boundary: $\frac{\partial C}{\partial n} = 0$
- Liquid boundary:
    - Water flow runs from outside to the domain: $C = C_b(t)$
    - Water flow runs out of the domain: $\frac{\partial^2 C}{\partial x^2} = 0$

- **Diagram**
    - Compute the current



- Compute the alluvium transport



D. *Moving boundary*

In tidal fall, the surface area decreases, the shallow area appears and not be inundated. Then, we will not calculate the tidal features in these areas. Limit calculation will be moved by water withdrawal of tide.

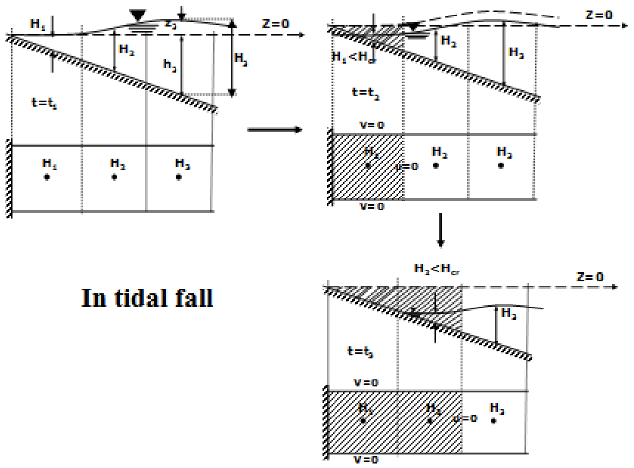


Figure 1. Boundary in tidal fall

In tidal rise, these shallow areas will be gradually submerged in water. Limit calculation will be gradually enlarged as the tidal water level increases.

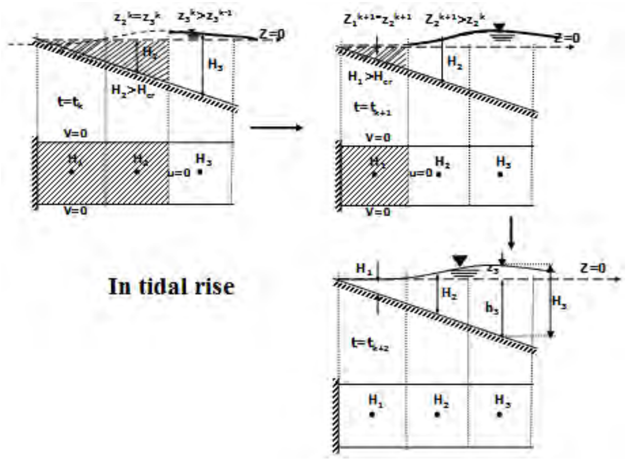


Figure 2. Boundary in tidal rise

Let "liquid cell" cell involving in the calculation domain. Solid cells do not participate in this area.

In tidal fall (corresponding to the time t), the water withdraws from the calculation box. The cells having the depth H (the terrain depth + water variation) under Hcr (depth limit) then become solid cells (not participate in the calculation domain at this time). For a liquid cell being surrounded by the solid cells, its velocity will be 0. The box having more than two liquid cells is kept on calculating.

In tidal rise (we know this by comparing the water level of the liquid cell next to this cell, the box is worth at the time the water level greater than the value at the time before), the water will overflow the solid cells, then the water level H is greater than Hcr. These cells will have the value of water as the value of the cell next to them (with H> Hcr).

III. EXPERIMENTAL RESULTS

A. Applying the models in Ca Mau coastal zone

The study area (W=61 km; L=88 km) is computed under the following conditions:

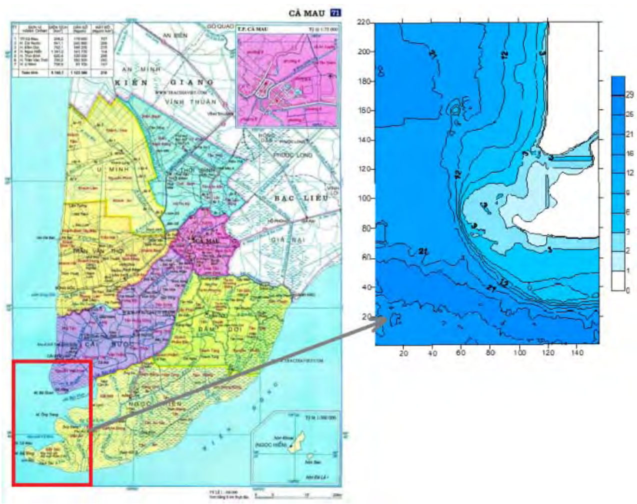


Figure 3. Map of the study area

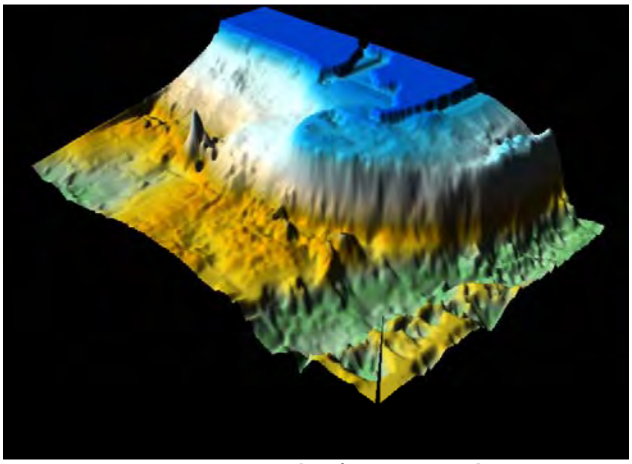


Figure 4. 3D Topography of Ca Mau coastal zone

- Boundary condition the for current model

On the open boundary, the water levels are given by computing tidal components as shown in table 1 and 2. The parameters of these tidal components are from [11, 12, 13].

TABLE 1. TIDAL CHARACTERITICS AT EASTERN SEA

No.	Name of tidal components	Amplitude (m)	Phase (rad)
1	M_2	0.72	0.59
2	N_2	0.15	0.08
3	S_2	0.3	1.3
4	K_2	0.08	1.3
5	K_1	0.59	5.4
6	O_1	0.42	4.6
7	P_1	0.19	5.4
8	Q_1	0.01	4.2
9	M_4	0.01	4.8
10	MS_4	0.01	5.8
11	M_6	0.004	2.6

TABLE 2. TIDAL CHARACTERITICS AT THAI LAN BAY

No.	Name of tidal components	Amplitude (m)	Phase (rad)
1	M_2	0.15	1.35
2	N_2	0.15	0.08
3	S_2	0.12	1.35
4	K_2	0.08	1.3
5	K_1	0.38	0.18
6	O_1	0.25	1.8
7	P_1	0.49	5.4
8	Q_1	0.07	4.2
9	M_1	0.08	3.5

- Wind: Southern West direction, 4.2m/s.
- Alluvium concentration at boundaries: 0.0001g/ml

B. The results when applying moving boundary

To know clearly about the difference between the results of applying moving boundary and those of non applying moving boundary, we show the graph of water surface elevation of the point near the boundary (see Figure 5) and the total wet area in tidal rise and tidal fall.

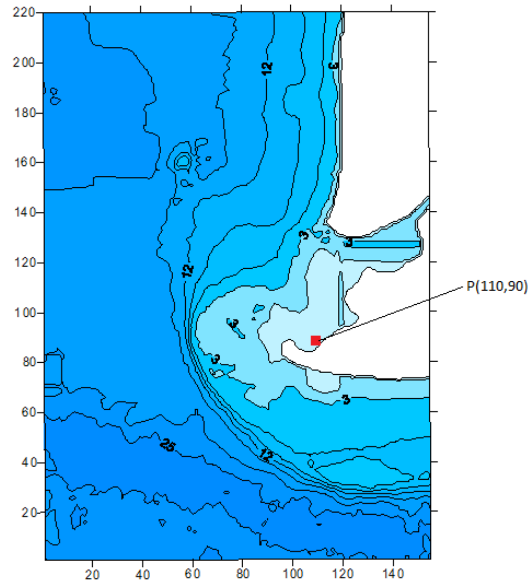


Figure 5. Point to get the water surface elevation

We can see the graph of water surface elevation of the point P in Figure 6.

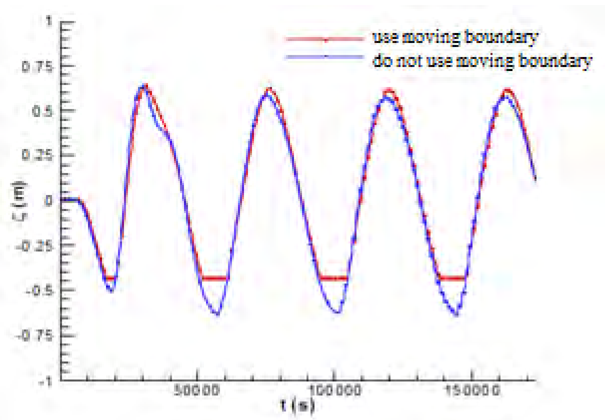


Figure 6. Water surface elevation at point P

From Figure 6, we see that in case we do not compute the moving boundary, the graph of water surface elevation is steady. Otherwise, in the front of the oscillation, the graph is similar to that of non computing moving boundary but in the end of the oscillation, the elevation is zero. This is reasonable because in tidal fall, the elevation of the point at the boundary is zero.

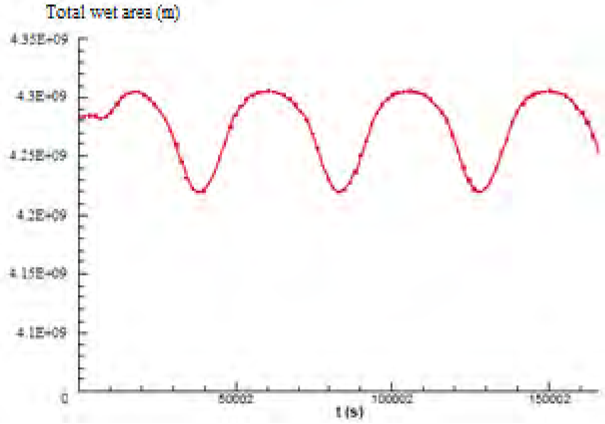


Figure 7. Water surface area in tidal rise and tidal fall

When the tide changes, the boundary is also changed. Therefore the total wet surface area is increased in tidal rise and decreased in tidal fall.

Figure 8 and Figure 9 show the changes of the boundary caused by the changes of the tide.

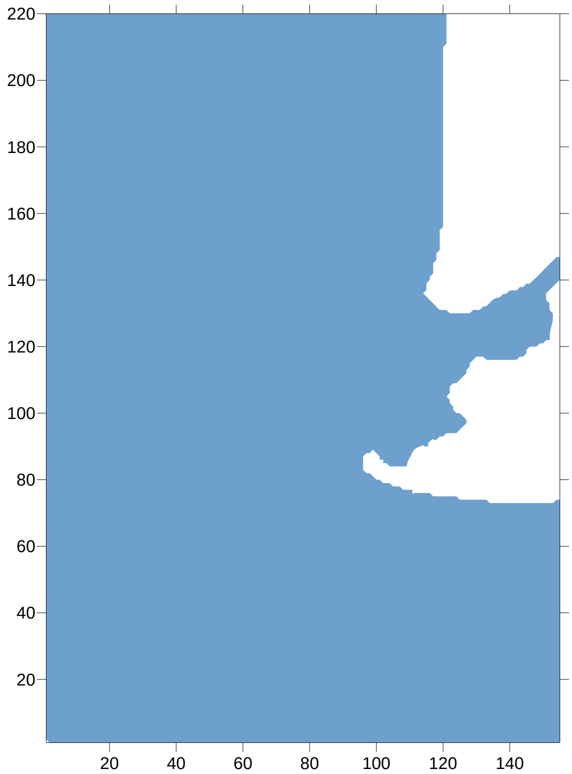


Figure 8. Boundary in tidal rise

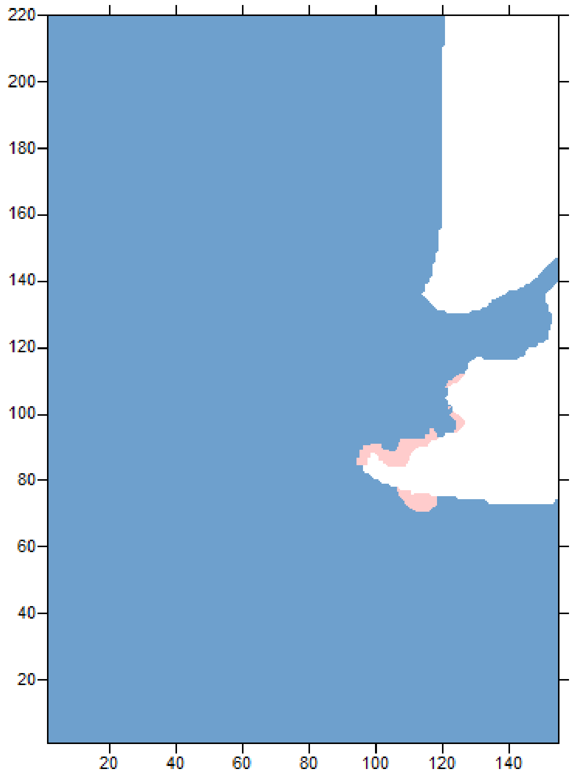


Figure 9. Boundary in tidal fall

The difference between Figure 8 and Figure 9 shows that there are up to 4 kilometers in X direction and about 2 kilometers in Y direction embedded into water in tidal rise. If $\Delta x = \Delta y = 300$m, there are up to 13 nodes in X direction and about 6 nodes in Y direction to be computed. The number of nodes to compute is significant and it will affect the result a lot.

- **Alluvium transportation**

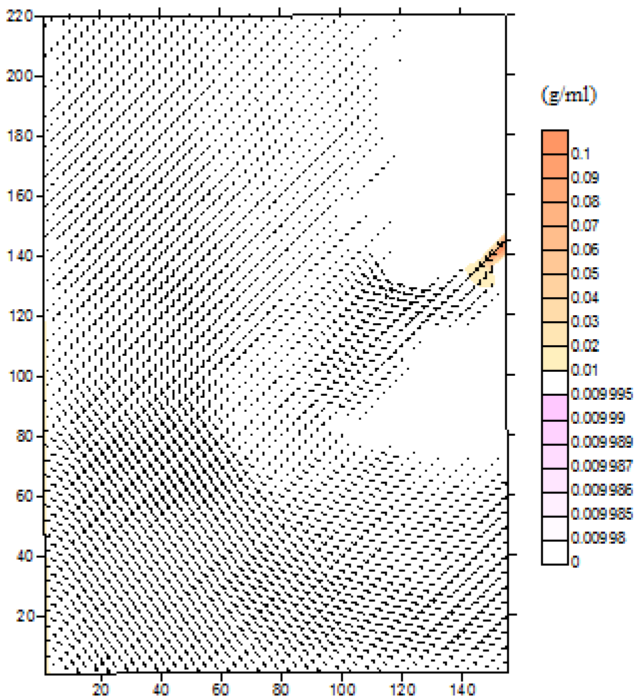


Figure 10. Alluvium transportation after 1 month and 15 days

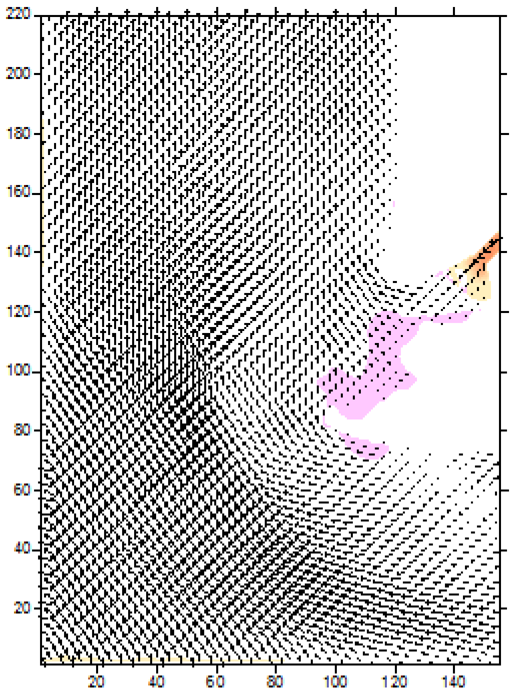


Figure 11. Alluvium transportation after 1 month, 15 days and 6 hours

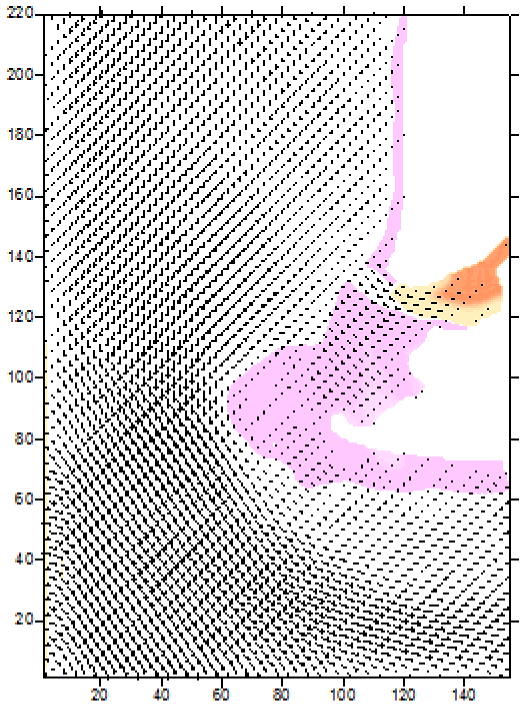


Figure 12. Alluvium transportation after 1 month, 15 days and 12 hours

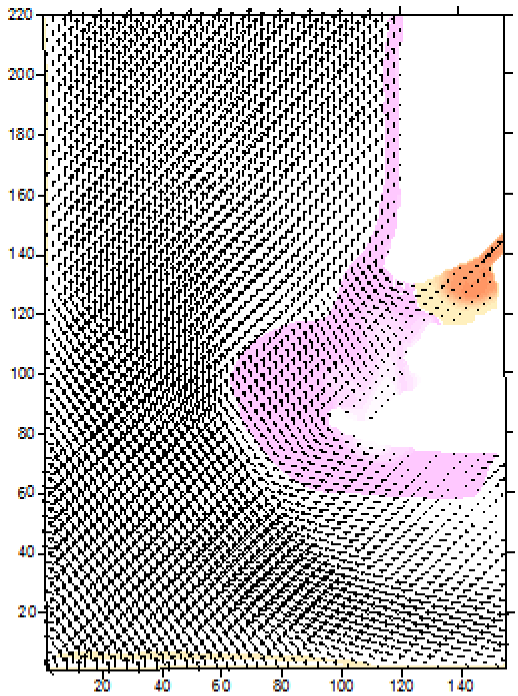


Figure 13. Alluvium transportation after 1 month, 15 days and 18 hours

- **Evaluate:**

At Bay Hap estuary, there is a heavy alluvium transportation. In tidal rise, the alluvium runs from the estuary to the ocean. In tidal fall, it runs following with the water flow to the reverse direction.

In reality, Bay Hap estuary is a place filled with a lot of alluvium. The models give good results in this area.

C. Testing the models

Scientists often test the current model first. Then they test the alluvium transport with the transport in reality.

One way to test the alluvium transport is to test the result by remote sensing. If the transport computed by the models and by remote sensing are similar, the models are reliable.

Figure 14, Figure 15, Figure 16 and Figure 17 show the alluvium transport computed by the models and by remote sensing.

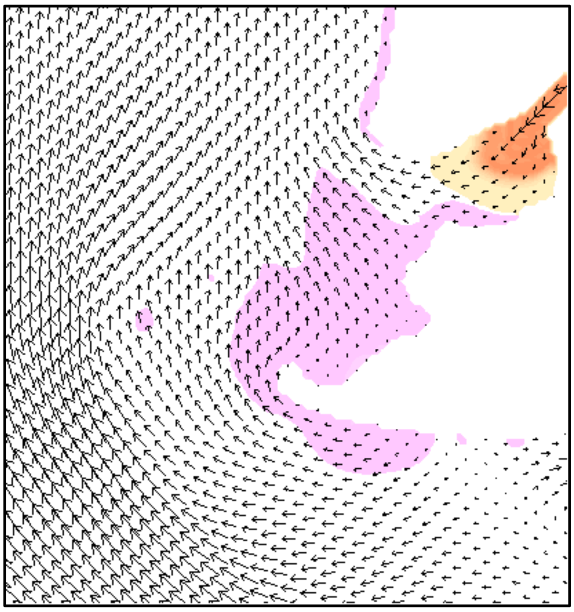


Figure 14. Alluvium transportation computed by the models in tidal rise

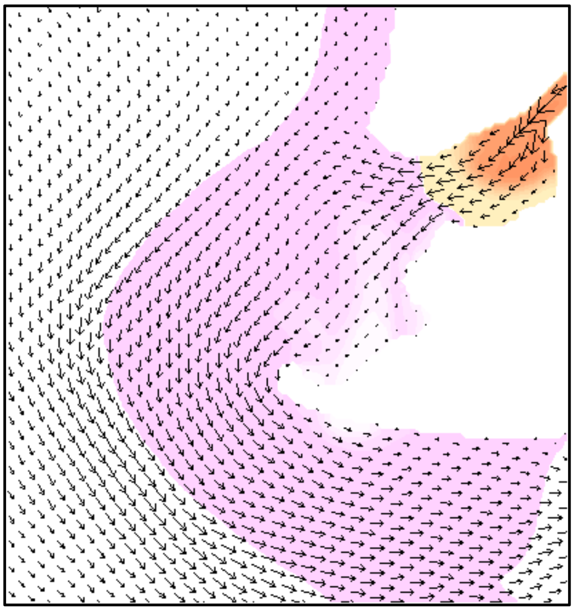


Figure 15. Alluvium transportation computed by the models in tidal fall

Figure 16. Alluvium transportation computed by remote sensing in tidal rise

Figure 17. Alluvium transportation computed by remote sensing in tidal fall

Figure 14 and Figure 15 show that in tidal rise, the direction of the alluvium transport is the same as that of the tide. In tidal fall, the direction is reverse. The alluvium transports in Figure 16 and Figure 17 are similar to those of the models. Therefore, the models are reliable.

IV. FUTURE DEVELOPMENTS

The alluvium transport may cause accretion or erosion. The next research will be studying the accretion, erosion at estuaries based on the alluvium transport.

V. CONCLUSIONS

This paper presents models to compute the alluvium transport. In this study, we compute the changes of boundary in tidal rise and in tidal fall so that the accuracy will be increased. The models were tested by remote sensing and the results show that the alluvium transport trend is suitable with reality.

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