STUDY OF THE BEHAVIOR MODELS BASED ON PROBABILITY AND TIME BY USING MARKOV PROCESS AND TRANSITION MATRIX.

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Abstract:

This paper suggests an approach to software system architecture specification based on behavior models. The behavior of the system is defined as an event occurring probability and time. The behavior of the system over a time of stationary Markov process is completely characterized by the one step transition matrix, the matrix of instantaneous transition rates. Given the element of the appropriative matrix, it is possible to calculate probability of event of the process.

Keyword:

software architecture, behavior models, transition matrix, Markov process.

1.Introduction

Many scholars have developed various behavior models by using number of parameters and developed mathematical models to establish the behavior of the system. Auguston [5] approach to formal software system architecture specification is based on behavior models. The behavior of the system is defined as set of events with two basic relations precedence and inclusion [5].The concept event attribute supports a continuous architecture refinement [2,4,5] up to executable design implementation models.

In this paper an attempt is made to show how the system is processed based on the time and probability using Markov process and transitions matrix.
Given any two states e_i and e_j, if the probability $p^{(n)}_{ij}>0$ for some 'n', then there is a positive probability of reaching the e_j starting from e_i in 'n' steps. In that case the state e_j is said to be accessible from the state e_i, if e_i and e_j are accessible from each other, then we say e_i communicate with e_j. If every state in a Markov chain is accessible from every other state possible in different number of transitions , then the chain and the corresponding transitions matrix[1,14] is said to be irreducible (communicating chain).

If 'c' is a set of states such that number of states outside 'c' can be reached from any state in 'c', then 'c' is a set said to be closed. Thus, if c is a closed set and if e_i, e_c, e_j and c, then $p_{ij} = 0$. In that case, $p_{ij}^{(2)}= \Sigma_k p_{ik}$ when $p_j =0$. And more generally, $p_{ij}^{(n)}=0$, if $n\geq 1$,so that no state outside 'c' can be reached from any state inside 'c' in any number of transitions. A closed set may contain one or more states. If a closed set contain only one state, then it is called an absorbing state. So it follows that a Markov chain [10,12] and the corresponding transition matrix are irreducible if every state can be reached from every other state i.e. all states communicate with each other.

2. Transition state

Starting from any state e_j, whether the system ever returns with certainty to the same state is an important question. If so one may ask how long does it take on the average for that event to happen? To analyze the question, we first generalize the event that first returns to origin. Connection with random walk model [6] and define $f_{ij}^{(n)}$ to be the probability that starting from state e_i, the chain reaches the state e_j for the first in 'n' steps.

$$f_{ij}^{(n)} = p[x_n = e_j, x_n \neq e_j, 0 < m < n / x_0 = e_i] \qquad \cdots\cdots(1)$$

Where, $f_{ij}^{(n)}$ represents the first passage probability from e_i to e_j in n steps, and $p_{ij}^{(n)}$ represents the probability of reaching e_j from e_i in 'n' steps, but not necessarily for the first time. It is easy to establish a relation between $f_{ij}^{(n)}$ and $p_{ij}^{(n)}$ by arguing as in starting from e_i the state e_j can be reached for the first time at the rth step with probability $f_{ij}^{(n)}$, $r \leq n$, and again in the remaining (n-r) steps with probability $p_{ij}^{(n-r)}$ for, $1 \leq r \leq n$. Summing over all these mutually exclusive possibilities, we obtain a key relation;

$$p_{ij}^{(n)} = \sum_{r=1}^{n} f_{ij}^{(n)} P_{ij}^{(n-r)} \quad (n \geq 1) \quad \cdots\cdots(2)$$

Here, $f_{ij}^{(0)}=0$, $P_{ij}^{(0)}=1$, $P_{ij}^{(0)}=0$, $i \neq j$ and $f_{ij}^{(1)} = p_{ij}$. Let $p_{ij}(z)$ and $f_{ij}(z)$ represent the movement generating function of sequences ($P_{ij}^{(n)}$) and ($f_{ij}^{(n)}$) respectively. Then moment generating function,

$$P_{ij}(z) = \sum_{n=0}^{\infty} p_{ij}^{(n)} Z^n$$

$$= P_{ij}^{(0)} + \sum_{n=1}^{\infty} \sum_{r=1}^{n} f_{ij}^{(r)} P_{ij}^{(n-r)} Z^n$$

$$= P_{ij}^{(0)} + \sum_{r=1}^{\infty} f_{ij}^{(r)} Z^{(r)} \sum_{k=0}^{\infty} P_{jj}^{(k)} Z^k$$

$$= P_{ij}^{(0)} + F_{ij}^{(z)} P_{ij}^{(z)}$$

Where,

$$F_{ij}^{(z)} = \sum_{n=1}^{\infty} f_{ij}^{(n)} Z^n$$

In particular for i=j, we obtain the useful relation.

$$P_{ii}(z) = 1 + F_{ii}(z) P_{ii}(z) \text{ (or)}$$

$$P_{ii}(z) = \frac{1}{1 - F_{ii}(z)}$$

Clearly, $f_{ij} = \sum_{n=1}^{\infty} f_{ij}^{(n)} = F_{ij}(1)$ represents the first passage probability [1,12,14] that starting from state e_i, the system will sooner or later will pass through state ej. Thus $f_{ij}<1$ always, and when $f_{ij}=1$, the sequence.

3. Classification of states: - Now it has been assumed that the processes have whatever property are necessary to make the methods work. We must consider what these properties are and develop the terminology to distinguish between Markov chains that have the properties and those that do not.

The method of raising a transition matrix to higher powers to obtain n-steps transition probabilities or state probability will always work, so also will the recursive formula for the first passage and first return probabilities [1,11,14]. However, the linear equation methods for obtaining steady state, probabilities and mean first passage time may fail. In fact the steady state, first passage probabilities or both, may not exit for certain kinds to processes.

To get an indication of what can happen, consider the process characterized by the following transition matrix [5] and the corresponding figure 1.

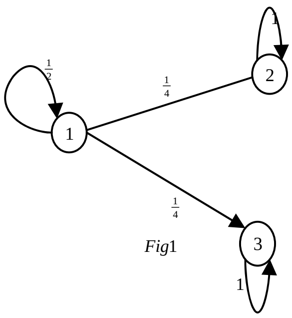

*Fig*1

If the process starts in the first state it will either stay in that state with probability $\frac{1}{2}$, or change to either second state or third state, each with probability $\frac{1}{4}$. Once the process is entered to either the second or third state, the process will remain in that state forever. Hence, if the initial state is the second state the n-step transition probability for the transition state 1 or 3 is zero for all n. Similarly, if the initial state is 3, then $p_{31}^{(n)}$ and $p_{32}^{(n)}$ are zero for all n. If the process begins in state one, then it may stay in that state for a while, but ultimately it must enter either 2 or 3 and remain there forever, by symmetry, one may conclude that the process is equally likely to end up in state 2 or 3 so as n goes to infinity.

$$P_{11}^{(n)} \to 0$$
$$P_{12}^{(n)} \to 0$$
$$P_{1j}^{(n)} \to 0$$

This discussion has been heuristic, but the conclusion can be verified by raising p to higher powers [1,14,15], for example,

$$P^{(2)} = P^2 = \begin{bmatrix} \frac{1}{4} & \frac{3}{8} & \frac{3}{8} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^{(3)} = P^3 = \begin{bmatrix} \frac{1}{8} & \frac{7}{16} & \frac{7}{16} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P^{(4)} = P^4 = \begin{bmatrix} \frac{1}{32} & \frac{31}{64} & \frac{31}{64} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

If we persist in these calculations we can see that the limiting matrix is

$$\underset{m\to\infty}{lt}\, p^{(n)} = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This verifies our heuristic discussion. The rows of $p^{(n)}$ do not become identical. Since the behavior of the process [5,7,15] as 'n' goes to infinity depends, on which states the process started in. Just to see what happens, we should try to obtain steady-state probabilities using the linear equation method [13] when p is matrix given above.

The mean first passage times are also of questionable meaning. From the example above, it is clear that passage from state 2 to state 3 can never occur. So also, passage from 1 to 2 might not ever occur. In such cases, there can not exit any finite mean number of transitions required to take the process from i to j.
Again, we should try the linear equation method any way, just to see what happens.

What we need now is the terminology and criteria to distinguish between processes of the kind just examined. There are a whole series of words and accompanying concepts to be introduced. For the most part, the words are the suggestive and the concepts, intuitive. First, we consider some words to describe the structure of process [3,5], in term of how states relate to one another. Later we will develop term to describe individual states.

One state j is reachable from another i if there exists some sequence of possible transition which would take the process from state i to state j. In the previous process, every state was reachable from state 1, but only state 2 was reachable from 2, and only state 3 from 3.Two states communicate if each is reachable from the other. No two states communicate at a same time in the process.In terms of the transition diagram, state j is reachable from i if there is a walk from point i to point j. Two states communicate if there are walks going in both directions.

A close set of states is a set such that no state outside the set is reachable from any state in the set, that is, once a closed set is entered, it can not be left using the transition diagram. A rough method to determine whether a set is closed is to encircle the points in the set and see if there are any arrows penetrating the boundary in an out ward direction. Notice that finding close set by this method depends somewhat on our ability to perceive them and perhaps on the way the diagram is drawn. Consider these questions, it is always possible to encircle the points of a closed set with just one circle, and how do we handle a diagram like the one in figure 2.
If single state form a closed, as in the process treated earlier, the state is called an absorbing state.

A minimal closed set [10,12] is one which has no proper closed state. Every state in a minimal closed set communicates with every other state in the set. For this reason the minimal closed sets are some time called communicating classes.

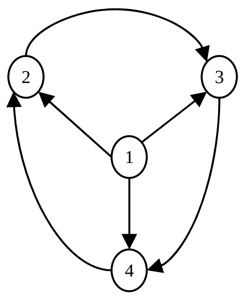

Fig 2

A Transition Diagram

If all states of a process belong to single communicating classes, the process is set to be irreducible. Other wise the process is reducible.

To illustrate a number of these concepts consider the following transition matrix:

$$p = \begin{bmatrix} \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & \frac{1}{5} & 0 & 0 & 0 \\ & \frac{1}{2} & & & & & & \\ & & \frac{1}{2} & & & & & \\ & & & \frac{1}{3} & \frac{1}{3} & & & \frac{1}{3} \\ & & & \frac{1}{3} & \frac{1}{3} & & & \frac{1}{3} \\ & & & & & 1 & & \\ & & \frac{1}{2} & & & & \frac{1}{2} & \\ & & & \frac{1}{3} & \frac{1}{3} & & & \frac{1}{3} \end{bmatrix}$$

It is a bit difficult to see the structure of the process in this form, but figure 3, reveals it clearly (the probilities are omitted for clarity) that states 3 and 7 form a minimal closed set, as do the states 4,5 and 8. Either of these closed sets can be entered from state 1, but not from state 2 or 6. States 2 and 6 forms a closed set once entered from state 1. The process cannot leave the set, but they do not form a minimal closed set. State 6 alone is a closed set, in fact, it is an absorbing state [9,14]. (Notice that an absorbing state is recognizable by the “1” on the diagonal). In summary, the minimal closed sets are [2], [3, 8] and [7,13,15]. States 1 and 2 are “left over” and do not belong to any minimal closed set.

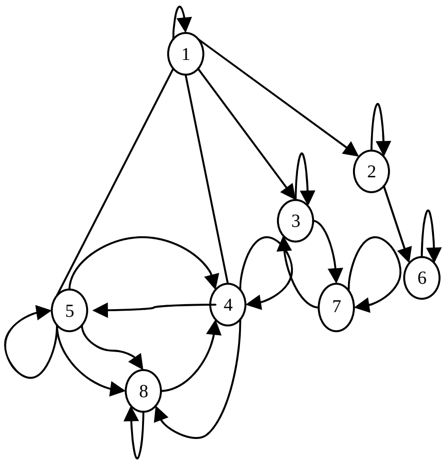

Fig. 3

A Transition Diagram

A state belonging to a minimal closed subset is called recurrent, the other left over are called transient. These terms make sense when we realize that a closed set could never again occur, once a recurrent state occurs [1,10,16]. It indicates that the process has entered the minimal closed set to which that state belongs, then it is certain to go on recurring from time to time. Notice that it is not necessary that a recurrent state ever occur because it could belong to a minimal closed set which the process never enters. But once it does occur it will necessarily reoccur.

If the above process is reducible, then the transition matrix can be written in the form;

P=

p_1			
	p_2		
	.	. . . . p_c	
Q_1	Q_2	$\dots Q_c$	Q_{c+1}

Where p_i is a square sub matrix containing the probabilities governing the transitions within the i^{th} minimal closed set of states. Q_{c+1} is a square sub matrix of probabilities of transitions among the transient state, and other Q_i are matrices (not necessarily square) containing probabilities of transitions from one transient state to another state within the i^{th} minimal closed set. Of course, to accomplish this expression of

P may require the recording of rows and columns. In the above example, appropriate recording of states yields the following transition matrix;

$$p = \begin{bmatrix} 1 & & & & & & & \\ & \frac{1}{2} & \frac{1}{2} & & & & & \\ & \frac{1}{2} & \frac{1}{2} & & & & & \\ & & & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & \\ & & & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & \\ & & & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & & \\ 0 & \frac{1}{5} & 0 & \frac{1}{5} & \frac{1}{5} & 0 & \frac{1}{5} & \frac{1}{5} \\ \frac{1}{3} & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}$$

There are definite advantages in expressing the transition matrix of a reducible process in this way. One of the minimal closed sets becomes visibly distinguishable. More importantly the powers of the transition matrix assume a predictable form. To see what happens, when square such a matrix in partitioned form.

$$P^2 = \begin{bmatrix} P_1 & 0 & 0 & 0 \\ 0 & P2 & 0 & 0 \\ 0 & 0 & P_3 & 0 \\ Q_1 & Q_2 & Q_3 & Q_4 \end{bmatrix} \begin{bmatrix} P_1 & 0 & 0 & 0 \\ 0 & P2 & 0 & 0 \\ 0 & 0 & P_3 & 0 \\ Q_1 & Q_2 & Q_3 & Q_4 \end{bmatrix}$$

$$\begin{bmatrix} P_1^2 & 0 & 0 & 0 \\ 0 & P_2^2 & 0 & 0 \\ 0 & 0 & P_3^2 & 0 \\ Q_1P_1 + Q_4Q_1 & Q_2P_2 + Q_4Q_2 & Q_3P_3 + Q_4Q_3 & Q_4^2 \end{bmatrix}$$

In this case it can be noticed that the product retains the same partitioned format and that matrices along the diagonal are just the squares of the diagonal matrices of P with higher powers. We may verify that the processes with similar properties, which are complicated, then p^n can be constructed by raising p_1, p_2.....p_c, and q_{c+1} to the n^{th} power separately. The minimal closed set behaves almost as if they were unrelated irreducible Markov chains [12]. Intuitively, once the process is within a closed set, there are still questions to be answered, such as when will the process enter one of the minimal closed sets and which minimal closed set will it enter.

If a process is reducible, it contains one or more minimal closed set (or communicating classes) of recurrent states, plus some transient states. If itt is irreducible, (i.e. does not contain a proper subset which is a minimal closed set), one might assume that all states must be recurrent. Although this is usually the case, it is technically possible for all states to be transient. A transition matrix having this property is given below.

$$P = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & . & . & . \\ 0 & \frac{1}{2} & \frac{1}{2} & 0 & 0 & . & . & . \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} & 0 & . & . & . \\ 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \end{bmatrix}$$

The dots indicate that the matrix is infinite. No state of this process, once left, can occur again. There are many closed sets, in fact, an infinite number. But there is no minimal closed set. Consequently, all states are transient. The oddity derives from the nature of infinity. In any process having an infinite number of states, at least one would have to be recurrent, so if a process is finite and irreducible, every state is recurrent.

Whether a state is transient or recurrent depends only on the structure of the process, not on values of transition probabilities. We can, however, characterize the categories in terms of probabilities. Recall that the first return probability $f_{ii}^{(n)}$ [1,11] gives the probability that state i is visited for the first time after n steps. Consequently the sum $\sum_{n=1}^{\infty} f_{ii}^{(n)}$ gives the probability that state i is ever revisited. Let $f_i = \sum_{n=1}^{\infty} f_{ij}^{(n)}$, and call it the probability of recurrence of state i then it can be shown that state i is recurrent if and only if $f_i=1$, and is transient if and only if $f_i<1$.

If the process is in a transient state i at time 0. The probability that it will ever occupy that state again f_i which is less than 1, if it does occupy state i again, then the probability that it will return once again is still f_i, each time it returns to i, there is a positive probability, $1-f_i$, that it will never happen again. Thus, the process may return some infinite number of times to a transient state, but eventually it will return no more over an infinite period of time, the probability of being in any transient state goes to zero.

If the process is in a recurrent state at time 0, it is certain that the state will occur again, because $f_i=1$ once it does it is certain to occur once again and so on. Over an infinite period of time, a recurrent state which has occurred once will occur on infinite number of time.

On the other hand, peculiar anomaly can occur for a recurrent state to have infinite mean recurrence time mathematically as; $m_{ii} = \sum_{n=1}^{\infty} n f_{ii}^{(n)}$

It is possible that the infinite series do not converge, even though the infinite series sums to1.

$$f_i = \sum_{n=1}^{\infty} f_{ii}^{(n)}$$

A recurrent state having infinite mean recurrence time is called a null. State over an infinite period of time, null state occurs so infrequently that even though an infinite number of times the probability of being in one falls to zero. Null states are of great practical use; but because they can occur, they must be considered when categorizing states. Recurrent states having finite mean recurrence time, the usual ones, are called non null or positive, another anomaly occasionally shown up in applications. For discussions purpose consider the following transition matrix:

$$\begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

The states now fall into two classes, [4,5] and [3,15] which are such that as process alternates between the two sets if the process is in state 1 or 2, then at the next step it must be in state 3 or 4 . At the following step it must be in state 1 or 2 and so on. In other words, state 1 if it occurs at step 0 could occur again only at step 2, 4, 6, 8, 10 and so on. A state which can occur only at step m, 2m and 3m and so on, where m is some integer greater than 1 is called a periodic state of period m. Therefore, all of the states of the above example are periodic of period 2. A state for which no such m, greater than 1 exists is called a periodic.

A state i is periodic if all walk in the transition starting from i, and returning i are of length m, or 2m and so on where m is greater than 1.If we can find a walk of length 1 (i.e. a loop) or if we can find two walks which have relatively prime length [12] (i.e. there is no other than one which divides both of them) we are assured that the state is a periodic. Furthermore, periodic is a class property, that is, if one state of a communicating class periodic of period 'm' then they all are periodic. So if you can determine that one state is a periodic, we know that the other states of the same class are too.

The trouble with periodic states, so for as steady-state probability are concerned, is that the limit of transitions probability function do not exit. To see this, square the transition matrix of the above process;

$$P^2 = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$P^3 = \begin{bmatrix} 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 \end{bmatrix}$$

This is the same as P

$$P^n = p \quad \text{If n is odd}$$
$$= P^2 \quad \text{If n is even}$$

The transitions probabilities all continue to alternate between the two values 0 and 1/2. They do not approach any limits. Because the difficulty is more theoretical than practical. The subject of periodic states will not be pursued further in this next. If we should encounter a problem with then, they are treated in [10].

Now that we have words to describe the exceptions. We may consider a process for which the steady-state probability [11] may be found by the method of solving the linear equations discussed earlier. A state which is not transiting, not periodic and not null is called an ergodic state [1]. An irreducible process, consisting of ergodic state is called as ergodic process. It can be shown that an ergodic process possesses unique steady state distribution which is independent of the initial state and is given by unique solutions to the linear equations [1,12]

$$\Pi = \Pi_p$$

$$\Pi_j = 1$$

The theorem assures that solutions to the equations always exit which are unique, and has the property to qualify as the probability distributions.

There is no need for a method to determine limiting probability transient or null states. The probability of being in such a state tends to zero periodic state do not posses steady state probabilities as we have defined them. All other states are ergodic and are covered by the linear equations method provided that the process is irreducible. If it is not, we will have to know the probability that the minimal closed set to which a state belongs is ever entered the class will behave like an irreducible process.

4. Conclusion and future discussion:

In this paper we have show that the behavior of the system is defined as a process occurring event probability and time. The Markov process plays a fundamental role in the behavior of the system representing the event probability. Useful analytical results have been obtained for making other assumptions in the behavior of the system. It is also possible to estimate throughput as more than one event reach. At the same time it also calculates the probability of event. An architecture model can be tested and applied for model checking, so as to verify and to validate the system model at the early stages of the design.

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