Fuzzy Optimal Solution to Fuzzy Transportation Problem: A New Approach

S. Mohanaselvi
Department of Mathematics
SRM University
Kattankulathur, Chennai - 603203, INDIA
E-mail: mohanaselvi.s@ktr.srmuniv.ac.in

K. Ganesan
Department of Mathematics
SRM University
Kattankulathur, Chennai - 603203, INDIA
E-mail: ganesan.k@ktr.srmuniv.ac.in, hod.maths@ktr.srmuniv.ac.in

Abstract — **In this paper we propose a new algorithm for the initial fuzzy feasible solution to a fully fuzzy transportation problem. Then by using fuzzy version of modified distribution method, we obtain the fuzzy optimal solution for the fully fuzzy transportation problem without converting to a classical transportation problem. A numerical example is provided to illustrate the proposed algorithm. It can be seen that the proposed algorithm gives a better fuzzy optimal solution to the given fuzzy transportation problem.**

Keywords-Fuzzy transportation problem; Triangular fuzzy number; Fuzzy arithmetic; Optimal solution.

AMS classification: 90C08,90C90

INTRODUCTION

Transportation problem is a special class of linear programming problem which deals with the distribution of single commodity from various sources of supply to various destinations of demand in such a manner that the total transportation cost is minimized. In order to solve a transportation problem, the decision parameters such as availability, requirement and the unit transportation cost of the model must be fixed at crisp values. But in real life applications supply, demand and unit transportation cost may be uncertain due to several factors. These imprecise data may be represented by fuzzy numbers. The idea of fuzzy set was introduced by Zadeh [17] in 1965. Bellmann and Zadeh [2] proposed the concept of decision making in fuzzy environment. After this pioneering work many authors have studied fuzzy linear programming problem techniques such as S.C. Fang [5], H. Rommelfanger [13] and H. Tanaka [15]etc.

Fuzzy transportation problem is a transportation problem whose decision parameters are fuzzy numbers. The objective of the fuzzy transportation problem is to determine the transportation schedule that minimizes the total fuzzy transportation cost while satisfying the availability and requirement limits. Chanas et al [4] developed a method for solving fuzzy transportation problems by applying the parametric programming technique using the Bellman–Zadeh criterion [2]. Chanas and Kuchta [3] proposed a method for solving a fuzzy transportation problem by converting the given problem to a bicriterial transportation problem with crisp objective function which provides only crisp solution to the given problem. Liu and Kao [8] proposed a new method for the solution of the fuzzy transportation problem by using the Zadeh's extension principle. Using parametric approach, Nagoorgani and Abdul Razak [10] obtained a fuzzy solution for a two stage Fuzzy Transportation problem with trapezoidal fuzzy numbers. Omar et. al [11] also proposed a parametric approach for solving transportation problem under fuzziness. Pandian and Natarajan [12] proposed a fuzzy zero point method to find the fuzzy optimal solution of fuzzy transportation problems. Since the fuzzy transportation problem is a special class of fuzzy linear programming problem, the straight forward method is to apply the existing fuzzy linear programming techniques to solve the fuzzy transportation problem. But most of the existing techniques provide only crisp solution for fuzzy transportation problem. In general, the authors have transformed the given fuzzy transportation problem in one or more crisp transportation problems and then obtained the crisp optimal solution. In this paper, we propose a new algorithm to find the initial fuzzy feasible solution to a fuzzy transportation problem without converting to a classical transportation problem. It is important to note that the proposed algorithm avoid degeneracy and provides the fuzzy optimal solution quickly for the given fuzzy transportation problem.

The rest of the paper is organized as follows. In section 2, we recall the basic concepts and the results of triangular fuzzy number and their arithmetic operations. In section 3, we introduce the fuzzy transportation problem with triangular fuzzy numbers and related results. In section 4, we propose a new algorithm to find the initial fuzzy feasible solution for the given fuzzy transportation problem and obtained the fuzzy optimal solution by applying the fuzzy version of MODI method. A numerical example is also provided to illustrate the theory developed in this paper.

PRELIMINARIES

Definition: 2.1

A fuzzy set $\tilde{a}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\tilde{a}: R \to [0,1]$ has the following characteristics:

(i) $\tilde{a}$ is convex,

i.e. $\tilde{a}\{\lambda x_1 + (1-\lambda)x_2\} \geq \min\{\tilde{a}(x_1), \tilde{a}(x_2)\}$, for all $x_1, x_2 \in R$ and $\lambda \in [0,1]$

(ii) $\tilde{a}$ is normal i.e. there exists an $x \in R$ such that $\tilde{a}(x) = 1$

(iii) $\tilde{a}$ is piecewise continuous.

Definition: 2.2

A fuzzy number $\tilde{a}$ on R is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\tilde{a}: R \to [0,1]$ has the following characteristics:

$$\tilde{a}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \leq x \leq a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \leq x \leq a_3 \\ 0, & \text{elsewhere} \end{cases}$$

We denote this triangular fuzzy number by $\tilde{a} = (a_1, a_2, a_3)$. We use $F(R)$ to denote the set of all triangular fuzzy numbers.

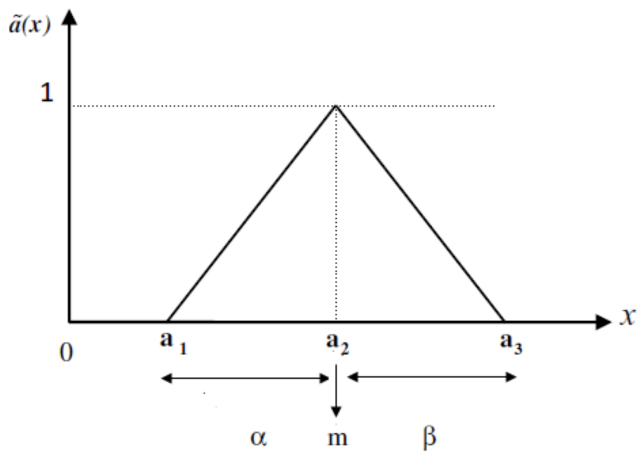

Fig 1. Triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

Also if $m = a_2$ represents the modal value or midpoint, $\alpha = (a_2 - a_1)$ represents the left spread and $\beta = (a_3 - a_2)$ represents the right spread of the triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$, then the triangular fuzzy number $\tilde{a}$ can be represented by the triplet $\tilde{a} = (\alpha, m, \beta)$. i.e. $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

Definition: 2.3 A triangular fuzzy number $\tilde{a} \in F(R)$ can also be represented as a pair $\tilde{a} = (\underline{a}, \overline{a})$ of functions $\underline{a}(r)$ and $\overline{a}(r)$ for $0 \le r \le 1$ which satisfies the following requirements:

(i). $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.

(ii). $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function.

(iii). $\underline{a}(r) \le \overline{a}(r)$, $0 \le r \le 1$

Definition: 2.4 For an arbitrary triangular fuzzy number $\tilde{a} = (\underline{a}, \overline{a})$, the number $a_0 = \left(\frac{\underline{a}(1) + \overline{a}(1)}{2}\right)$ is said to be a location index number of $\tilde{a}$. The two non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$, $a^* = (\overline{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index function respectively. Hence every triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ can also be represented by $\tilde{a} = (a_0, a_*, a^*)$

2.1 Ranking of triangular Fuzzy Numbers

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari [1] proposed a new ranking method based on the left and the right spreads at some α -levels of fuzzy numbers.

For an arbitrary triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$ with parametric form $\tilde{a} = (\underline{a}(r), \overline{a}(r))$, we define the magnitude of the triangular fuzzy number $\tilde{a}$ by

$$\begin{aligned} Mag(\tilde{a}) &= \frac{1}{2}\left(\int_0^1 (\underline{a} + \overline{a} + a_0)\ f(r)\ dr\right) \\ &= \frac{1}{2}\left(\int_0^1 (a^* + 4a_0 - a_*)\ f(r)\ dr\right). \end{aligned}$$

where the function $f(r)$ is a non-negative and increasing function on [0,1] with $f(0) = 0$, $f(1) = 1$ and $\int_0^1 f(r)\ dr = \frac{1}{2}$. The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r) = r$.

Hence $Mag(\tilde{a}) = \left(\frac{a^* + 4a_0 - a_*}{4}\right) = \left(\frac{\underline{a} + \overline{a} + a_0}{4}\right)$.

The magnitude of a triangular fuzzy number $\tilde{a}$ synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. Mag ($\tilde{a}$) is used to rank fuzzy numbers. The larger Mag($\tilde{a}$), the larger fuzzy number.

For any two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ in F(R), we define the ranking of $\tilde{a}$ and $\tilde{b}$ by comparing the Mag ($\tilde{a}$) and Mag ($\tilde{b}$) on R as follows:

(i). $\tilde{a} \succeq \tilde{b}$ if and only if Mag($\tilde{a}$) $\ge$ Mag($\tilde{b}$)

(ii). $\tilde{a} \preceq \tilde{b}$ if and only if Mag($\tilde{a}$) $\le$ Mag($\tilde{b}$)

(iii). $\tilde{a} \approx \tilde{b}$ if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$)

Definition: 2.5
A triangular fuzzy number $\tilde{a} = \left(a_0, a_*, a^*\right)$ is said to be symmetric if and only if $a_* = a^*$.

Definition: 2.6
A triangular fuzzy number $\tilde{a} = \left(a_0, a_*, a^*\right)$ is said to be non-negative if and only if Mag($\tilde{a}$) $\geq$ 0 and is denoted by $\tilde{a} \succeq \tilde{0}$. Further if Mag($\tilde{a}$) > 0, then $\tilde{a} = \left(a_0, a_*, a^*\right)$ is said to be a positive fuzzy number and is denoted by $\tilde{a} \succ \tilde{0}$.

Definition: 2.7
Two triangular fuzzy numbers $\tilde{a} = \left(a_0, a_*, a^*\right)$ and $\tilde{b} = \left(b_0, b_*, b^*\right)$ in F(R) are said to be equivalent if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$).That is $\tilde{a} \approx \tilde{b}$ if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$).Two triangular fuzzy numbers $\tilde{a} = \left(a_0, a_*, a^*\right)$ and $\tilde{b} = \left(b_0, b_*, b^*\right)$ in F(R) are said to be equal if and only if $a_0 = b_0, a_* = b_*, a^* = b^*$. That is $\tilde{a} = \tilde{b}$ if and only if $a_0 = b_0, a_* = b_*, a^* = b^*$.

2.2 Arithmetic operation on triangular Fuzzy Numbers

Ming Ma et al. [9] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for a, b$\in$L we define $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

For arbitrary triangular fuzzy numbers $\tilde{a} = \left(a_0, a_*, a^*\right)$ and $\tilde{b} = \left(b_0, b_*, b^*\right)$ and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular fuzzy numbers are defined by $\tilde{a} * \tilde{b} = \left(a_0 * b_0, a_* \vee b_*, a^* \vee b^*\right)$. In particular for any two triangular fuzzy numbers $\tilde{a} = \left(a_0, a_*, a^*\right)$ and $\tilde{b} = \left(b_0, b_*, b^*\right)$, we define

(i). Addition: $\tilde{a} + \tilde{b} = \left(a_0, a_*, a^*\right) + \left(b_0, b_*, b^*\right) = \left(a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}\right)$

(ii). Subtraction: $\tilde{a} - \tilde{b} = \left(a_0, a_*, a^*\right) - \left(b_0, b_*, b^*\right) = \left(a_0 - b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}\right)$.

(iii). Multiplication: $\tilde{a} \times \tilde{b} = \left(a_0, a_*, a^*\right) \times \left(b_0, b_*, b^*\right) = \left(a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}\right)$.

(iv). Division: $\tilde{a} \div \tilde{b} = \left(a_0, a_*, a^*\right) \div \left(b_0, b_*, b^*\right) = \left(a_0 \div b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\}\right)$.

FUZZY TRANSPORTATION PROBLEM

Consider a fuzzy transportation with m sources and n destinations with triangular fuzzy numbers. Let $\tilde{a}_i$ $\left(\tilde{a}_i \succeq \tilde{0}\right)$ be the fuzzy availability at source i and $\tilde{b}_j$ $\left(\tilde{b}_j \succeq \tilde{0}\right)$ be the fuzzy requirement at destination j. Let $\tilde{c}_{ij}$ be the fuzzy unit transportation cost from source i to destination j. Let $\tilde{x}_{ij}$ denote the number of fuzzy units to be transported from source i to destination j. Then the problem is to determine a feasible way of transporting the available amount at each source to satisfy the demand at each destination so that the total transportation cost is minimized.

The mathematical formulation of the fuzzy transportation problem whose parameters are triangular fuzzy numbers under the case that the total supply is equivalent to the total demand is given by:

$$
\begin{aligned}
&\min \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij} \\
&\text{subject to} \quad \sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i, \quad i = 1,2,...m \\
&\qquad\qquad \sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j, \quad j = 1,2,...n \\
&\qquad\qquad \sum_{i=1}^{m} \tilde{a}_i \approx \sum_{j=1}^{n} \tilde{b}_j, i = 1,2,...m \text{ and } j = 1,2,...n \\
&\qquad\qquad \text{and } \tilde{x}_{ij} \succeq \tilde{0}
\end{aligned} \tag{1}
$$

This problem can also be represented as follows:

Table 1: Fuzzy transportation problem

Destinations

		1	……..	n	Supply
Sources	1	$\tilde{c}_{11}$	……..	$\tilde{c}_{1n}$	$\tilde{a}_1$
	⋮	⋮		⋮	⋮
	m	$\tilde{c}_{m1}$	……..	$\tilde{c}_{mn}$	$\tilde{a}_m$
	Demand	$\tilde{b}_1$	……..	$\tilde{b}_n$	

Definition: 3.1

A set of non-negative allocations $\tilde{x}_{ij}$ which satisfies (in the sense equivalent) the row and the column restrictions is known as fuzzy feasible solution.

Definition: 3.2

A fuzzy feasible solution to a fuzzy transportation problem with m sources and n destinations is said to be a fuzzy basic feasible solution if the number of positive allocations are $(m+n-1)$. If the number of allocations in a fuzzy basic solution is less than $(m+n-1)$, it is called fuzzy degenerate basic feasible solution.

Definition: 3.3

A fuzzy feasible solution is said to be fuzzy optimal solution if it minimizes the total fuzzy transportation cost.

Theorem: 1 (Existence of fuzzy feasible solution)

The necessary and sufficient condition for the existence of a fuzzy feasible solution to the fuzzy transportation problem is $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$

Proof: (Necessary condition)

Let there exists a fuzzy feasible solution to the fuzzy transportation problem given in (1).

Then $\sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{x}_{ij} \approx \tilde{a}_i$ and $\sum_{j=1}^{n}\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{b}_j$. Therefore $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$.

(Sufficient condition)

Let us assume that $\sum_{i=1}^{m} \tilde{a}_i = \sum_{j=1}^{n} \tilde{b}_j$. We have to distribute the supply at the i-th source in proportion to the requirements of all destinations.

Let $\tilde{x}_{ij} = \tilde{\lambda}_i \tilde{b}_j$, where $\tilde{\lambda}_i$ is the proportionality factor for the i-th source. Since the supply must be completely distributed.

$$\sum_{i=1}^{m} \tilde{x}_{ij} \approx \tilde{\lambda}_i \sum_{j=1}^{n} \tilde{b}_j, \ .$$

Therefore $$\tilde{x}_{ij} = \tilde{\lambda}_i \tilde{b}_j = \frac{\tilde{a}_i}{\sum_{j=1}^{n} \tilde{b}_j}.\tilde{b}_j$$

$$\sum_{j=1}^{n} \tilde{x}_{ij} = \tilde{\lambda}_i \sum_{j=1}^{n} \tilde{b}_j = \frac{\tilde{a}_i}{\sum_{j=1}^{n} \tilde{b}_j}.\sum_{j=1}^{n} \tilde{b}_j = \tilde{a}_i$$

$$\sum_{i=1}^{m} \tilde{x}_{ij} = \frac{\tilde{b}_j}{\sum_{i=1}^{m} \tilde{a}_i}.\sum_{i=1}^{m} \tilde{a}_i = \tilde{b}_j$$

which shows that all the constraints are satisfied. Since $\tilde{a}_i$ and $\tilde{b}_j$ are positive, $\tilde{x}_{ij}$ determined must be all positive. Therefore the fuzzy transportation problem yields a fuzzy feasible solution.

A NEW ALGORITHM TO FIND THE INITIAL FUZZY FEASIBLE SOLUTION TO FUZZY TRANSPORTATION PROBLEM

An algorithm to find the initial fuzzy feasible solution to fuzzy transportation problem is presented as follows [14]:

Step 1: Represent each fuzzy data $\tilde{a} = (a_1, a_2, a_3)$ in terms of $\tilde{a} = \left(a_0, a_*, a^*\right)$.

Step 2: Now select the maximum cost cell $\tilde{c}_{ij}$ using the proposed ranking function.

Step 3: Choose the corresponding row of the maximum cost cell $\tilde{c}_{ij}$ and select the minimum cost cell.

Step 4: Allocate the maximum possible quantity to the minimum cost cell.

Step 5: Repeat step 3 and 4 for the column cell containing the maximum $\tilde{c}_{ij}$.

Step 6: In case of a tie, choose arbitrarily the cost.

Step7: If the row and column both deserves to be deleted put zero fuzzy number in the minimum cost cell of either row or column which are not yet deleted.

Step 8: Repeat step 3 to step 7 till all the allocations are completed.

4.1 Optimality condition to a fuzzy Transportation Problem

After determining the initial fuzzy basic feasible solution by the proposed algorithm, we have to test the current initial fuzzy basic feasible solution for optimality using fuzzy version of modified distribution method. Let $\tilde{u}_1, \tilde{u}_2, ..., \tilde{u}_m$ be the multipliers to the m constraints and let $\tilde{v}_1, \tilde{v}_2, ..., \tilde{v}_n$ be the multipliers to the n constraints. We can calculate $\tilde{u}_i$ and $\tilde{v}_j$ for the allocated cells using the relation $\tilde{c}_{ij} = \tilde{u}_i + \tilde{v}_j$ by setting a multiplier to zero which is associated with the row or column of the transportation table that contains the maximum number of allocated cells. The criterion for optimality is given by $\tilde{c}_{ij} - \left(\tilde{u}_i + \tilde{v}_j\right) \succeq \tilde{0}$ for all i and j for the unallocated cells fuzzy transportation table.

NUMERICAL EXAMPLE

Consider an example given in [6], a balanced fuzzy transportation problem in which all the decision parameters are triangular fuzzy numbers of the form (a, b, c).

Table 2: Balanced Fuzzy Transportation problem

	1	2	3	4	Supply
1	(-2,3,8)	(-2,3,8)	(-2,3,8)	(-1,1,4)	(0,3,6)
2	(4,9,16)	(4,8,12)	(2,5,8)	(1,4,7)	(2,7,13)
3	(2,7,13)	(0,5,10)	(0,5,10)	(4,8,12)	(2,5,8)
Demand	(1,4,7)	(0,3,5)	(1,4,7)	(2,4,8)	(4,15,27)

To apply the proposed algorithm and the fuzzy arithmetic, let us express all the triangular fuzzy numbers in the given problem based upon both location index and fuzziness index functions. That is in the form of $\tilde{a} = \left(a_0, a_*, a^*\right)$ we have,

Table 3: Balanced fuzzy transportation problem in which all the triangular fuzzy numbers are of the form $\left(\mathbf{a_0}, \mathbf{a_*}, \mathbf{a^*}\right)$.

	1	2	3	4	Supply
1	$(3,5-5r,5-5r)$	$(3,5-5r,5-5r)$	$(3,5-5r,5-5r)$	$(1,2-2r,3-3r)$	$(3,3-3r,3-3r)$
2	$(9,5-5r,7-7r)$	$(8,4-4r,4-4r)$	$(5,3-3r,3-3r)$	$(4,3-3r,3-3r)$	$(7,5-5r,6-6r)$
3	$(7,5-5r,6-6r)$	$(5,5-5r,5-5r)$	$(5,5-5r,5-5r)$	$(8,4-4r,4-4r)$	$(5,3-3r,3-3r)$
Demand	$(4,3-3r,3-3r)$	$(3,3-3r,2-2r)$	$(4,3-3r,3-3r)$	$(4,2-2r,4-4r)$	$(15,11-11r,12-12r)$

Applying the proposed new algorithm and the fuzzy arithmetic, the initial fuzzy basic feasible solution in terms of location index and fuzziness index is given by

Table 4: Initial fuzzy feasible solution in terms of location index and fuzziness index.

$(3,5-5r,5-5r)$ $(3,\ 3-3r,\ 3-3r)$	$(3,5-5r,5-5r)$	$(3,5-5r,5-5r)$	$(1,2-2r,3-3r)$
$(9,5-5r,7-7r)$	$(8,4-4r,4-4r)$	$(5,3-3r,3-3r)$ $(3,\ 5-5r,\ 6-6r)$	$(4,3-3r,3-3r)$ $(4,\ 2-2r,\ 4-4r)$
$(7,5-5r,6-6r)$ $(1,\ 5-5r,\ 6-6r)$	$(5,5-5r,5-5r)$ $(3,\ 5-5r,\ 2-2r)$	$(5,5-5r,5-5r)$ $(1,\ 5-5r,\ 6-6r)$	$(8,4-4r,4-4r)$

Hence the initial fuzzy transportation cost by the proposed method

$$= (3,5-5r,5-5r)\,(3,\ 3-3r,\ 3-3r) + (5,3-3r,3-3r)\,(3,\ 5-5r,\ 6-6r) + (4,3-3r,3-3r)\,(4,\ 2-2r,\ 4-4r)$$
$$+ (7,5-5r,6-6r)\,(1,\ 5-5r,\ 6-6r) + (5,5-5r,5-5r)\,(3,\ 5-5r,\ 2-2r) + (5,5-5r,5-5r)\,(1,\ 5-5r,\ 6-6r)$$

= (9, 5-5r, 5-5r) + (15, 5-5r, 6-6r) + (16, 3-3r, 4-4r) + (7, 5-5r, 6-6r) + (15, 5-5r, 5-5r) + (5, 5-5r, 6-6r)
= (9+15+16+7+15+5, 5-5r, 6-6r)
= Rs. (67, 5-5r, 6-6r)

By applying the fuzzy version of modified distribution method, it can be seen that the current initial fuzzy basic feasible solution is optimum.

Hence the fuzzy optimal solution in terms of location index and fuzziness index is given by

$$\tilde{x}_{11} = (3,\ 3\text{-}3r,\ 3\text{-}3r),\quad \tilde{x}_{23} = (3,\ 5-5r,\ 6-6r)$$
$$\tilde{x}_{24} = (4,\ 2-2r,\ 4-4r),\quad \tilde{x}_{31} = (1,\ 5-5r,\ 6-6r)$$
$$\tilde{x}_{32} = (3,\ 5-5r,\ 2-2r),\ \tilde{x}_{33} = (1,\ 5-5r,\ 6-6r),\ \text{where } 0 \le r \le 1.$$

The corresponding fuzzy optimal transportation cost is given by $\min \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij} = (67,5-5r,6-6r)$, where $0 \le r \le 1$ can be suitably chosen by the decision maker.

For $r = 0$, we have the fuzzy optimal solution in terms of location index and fuzziness index as

$$\tilde{x}_{11} = (3,\ 3,\ 3),\quad \tilde{x}_{23} = (3,\ 5,\ 6),\quad \tilde{x}_{24} = (4,\ 2,\ 4),$$
$$\tilde{x}_{31} = (1,\ 5,\ 6),\quad \tilde{x}_{32} = (3,\ 5,\ 2),\quad \tilde{x}_{33} = (1,\ 5,\ 6),$$

Hence the fuzzy optimal solution the given problem in the general form, i.e. in terms of (a, b, c) is

$\tilde{x}_{11} = (0,3,6)$, $\tilde{x}_{23} = (-2,3,9)$, $\tilde{x}_{24} = (2,4,8)$, $\tilde{x}_{31} = (-4,1,7)$, $\tilde{x}_{32} = (-2,3,5)$ and $\tilde{x}_{33} = (-4,1,7)$.

The corresponding fuzzy optimal transportation cost is given by in terms of (a, b, c) is $\min \tilde{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} \tilde{c}_{ij}\tilde{x}_{ij} = \text{Rs.}(62,\ 67,\ 73)$.

For the same problem, Fegad et al [6] have obtained the fuzzy optimal solution as: $\tilde{x}_{11} = (0,3,6)$, $\tilde{x}_{23} = (0,3,5)$, $\tilde{x}_{24} = (2,4,8)$, $\tilde{x}_{31} = (1,1,1)$, $\tilde{x}_{32} = (0,3,5)$ and $\tilde{x}_{33} = (1,1,2)$ and also, the minimum fuzzy transportation cost as Rs. [4, 67, 227]. It is important to note that the solution obtained by our algorithm is better than the solution obtained by them.

CONCLUSION

We have proposed a new algorithm for the fuzzy optimal solution to the given fuzzy transportation problem with triangular fuzzy numbers without converting the given problem into classical transportation problem. Using the new algorithm we represent the given fuzzy transportation problem in terms of left and right fuzziness index function of a fuzzy number and obtained an initial fuzzy feasible solution. By applying the fuzzy version of modified distribution method we have tested the optimality of the fuzzy feasible solution. Further, the fuzzy optimal solution obtained by the proposed algorithm is better than the fuzzy optimal solution obtained by the existing methods.

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AUTHORS PROFILE

S. Mohanaselvi is currently Assistant Professor, Department of Mathematics, Faculty of Engineering and Technology, SRM University. Currently she is pursuing PhD at SRM University under the guidance of Dr. K. Ganesan. She received her M. Phil Mathematics from Madurai Kamaraj University. Her research area includes Fuzzy optimization, Multi-objective decision making and interval analysis. She has presented her research articles in two international conferences. Her academic experience is about 9 years in teaching. She is currently working in the area of multi criteria decision making.

Dr. K. Ganesan is currently Professor and Head, Department of Mathematics, Faculty of Engineering and Technology, SRM University. He received his PhD in Mathematics from the University of Madras, Chennai. His academic experience is about 23 years in teaching. His research interest includes Fuzzy optimization and decision making, Interval Mathematics, Fuzzy Mathematics, Operation research, Differential equations etc. He published more than 20 research papers in international journals and conferences. He is a regular reviewer for various international journals published by Elsevier Science, Springer, World Scientific Publ Co Pte Ltd etc. He has organized many national conferences and workshops. He is currently working with a BRNS funded research project for Department of Atomic energy, Government of India. He is one of the authors of more 10 Mathematical books. He is guiding more than 6 students in this area.