Fast Method for Two-dimensional Renyi's Entropy-based Thresholding

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Abstract—**Two-dimensional (2-D) thresholding can give a better segmentation than one-dimensional thresholding by taking the spatial correlation of the image. Unfortunately, the computational cost is an obstacle for the implementation of real-time image processing. In this paper, a fast method for 2-D Renyi's entropy-based thresholding, which is the generalized method of Shannon entropic method and the correlation entropic method, is proposed. In order to diminish the processing time required for calculating 2-D histogram, a fast scheme is introduced and the computational complexity is reduced from $O(W^2MN)$ to $O(MN)$. More importantly, based on the proposed method, the computational complexity for selecting the optimal threshold value is reduced from $O(L^4)$ to $O(L^2)$. The effectiveness of this method is illustrated by experimental results.**

Keywords– image segmentation;2-D histogram; Renyi's entropy; thresholding

I. INTRODUCTION

Image segmentation is a challenging and important task in many image processing and analysis systems such as automatic target recognition, document image analysis, medical image analysis, etc. One of the most simple and efficient techniques for segmentation is thresholding. The thresholding can be seen as an operation for converting a gray-level image to a bi-level or multi-level image. For the simplest case, the thresholding can be seen as an operation of converting a gray-level image into a binary image by selecting an appropriate threshold value. If the values of pixels are below the threshold, then the pixels belong to the object, and if above the threshold, then to the background, or vice versa [1].

Up to now, many thresholding techniques have been devoted to the threshold selection, and all of these methods can be primarily divided into two groups. One is global thresholding, where a single threshold value is computed from the gray-level histogram of the image for bi-level thresholding, and it is easy to extend this thresholding to multilevel thresholding. The other is local thresholding, where the input image is thresholded by using the local threshold values which are calculated from a certain size of sub-images within the whole image. In comparison with the later, the former approach is easy to implement and widely used in various image processing applications. Entropic thresholding approach, which is based on optimizing an entropic measure from information theory, is one of the most common approaches in global thresholding techniques [2]. Most entropic methods are derived from the method proposed by Pun [3]. In Pun's method, the entropic concept was introduced in threshold selection to obtain the optimal threshold value by maximizing the upper bound of the *a posteriori* entropy. In Ref. [4], Kapur et al. corrected and improved Pun's method by maximizing the sum of the two class entropies of the object and background. Later, Albuquerque et al. [5] generalized the method of Kapur et al. to Tsallis's entropy. Then, on the basis of the idea suggested in Ref. [4], Yen et al. [6] optimally segmented the image by maximizing the entropic correlation. And then, both of methods [4, 6] were extended to Renyi's entropy by Sahoo el al. [7]. Here, all these methods mentioned above are regarded as one-dimensional (1-D) entropic approaches due to the fact that their methods are based on the 1-D histogram. According to the references [4,7], the 1-D approaches have a drawback, which is that the same threshold value would be obtained for the different images with the same histogram. This is because the spatial correlation between the pixels is not taken into consideration.

In order to deal with the drawback of the 1-D entropic approaches and achieve a better thresholding result, many algorithms have been proposed by taking the spatial correlation into account. In Ref. [8], Abutaleb found an optimal threshold pair by maximizing the sum of the 2-D entropies. The 2-D entropies were computed from the 2-D histogram which was determined by using the gray value of the pixels and the local average gray value of the pixels. Later, Brink [9] improved Abutaleb's method by maximizing the smaller of two entropies of the background and object. And then, Sahoo et al. [10,11] proposed two thresholding methods using 2-D Renyi's entropy and 2-D Tsallis-Havrda-Charvat entropy one after another. In all these methods, entropies were computed from 2-D histogram. These methods are regarded as 2-D approaches. It is evident that compared with the 1-D approaches, the 2-D approaches can give a better thresholding result. However, the computational cost, which gives a rise of exponential increment with the image size and the total number of gray-levels, is an obstacle for the implementation of real-time image processing. For this reason, many fast methods were proposed for the different 2-D approaches. Wu et al. [12] presented a recursive method for Abutaleb's method [8] to reduce the computational complexity from $O(L^4)$ to $O(L^2)$, Gong et al. [13] proposed a fast method for Brink's method [9]. Recently, a fast algorithm was proposed by Tang et al. [14] for the 2-D Tsallis entropy thresholding method [11]. It also reduced the computational complexity to $O(L^2)$ from $O(L^4)$. According to our experiments and analysis, however, some of these methods could not give the same thresholding results as the original methods.

In this paper, we present a fast method for 2-D Renyi's entropy-based thresholding method of Sahoo et al. [10] by rewriting the criterion function for optimal threshold selection. In order to reduce the processing time for computing the 2-D histogram, a fast scheme is also presented. This paper is organized as follows: in section 2, the 2-D Renyi's entropy algorithm is briefly reviewed. Section 3 describes the proposed method. Section 4 reports the experimental results on some real-world images and discussions. We give some conclusions in Section 5.

II. 2-D Renyi's Entropy-based Thresholding

Similarly to the 2-D maximum entropy sum method of Abutaleb [8], Sahoo et al. [10] proposed a 2-D entropic thresholding technique using Renyi's entropy, and showed that Abutaleb's method is a special case of their method when order-α tends to 1. Here, we will give a review of this method.

Suppose that a digital image of size $M\times N$ has L grey levels denoted by $G = \{0, 1,..., L - 1\}$. Let $f(x, y)$ be the grey level of the pixel located at the spatial location (x, y). Then the image can be represented by $F = \{ f(x, y) \mid x = 0,1,2,...,M - 1,\ y = 0,1,2,...,N - 1\}$. At each point (x, y) of F, the average gray value of local neighborhood with the size $W\times W$ centered at the pixel (x,y) can be calculated by

$$g(x, y) = Round\left(\frac{1}{W^2} \sum_{j=-(W-1)/2}^{(W-1)/2} \sum_{i=-(W-1)/2}^{(W-1)/2} f(x+i, y+j) \right). \qquad .1$$

where $Round(\mu)$ denotes the integer part of μ. Then, using the pixel's gray value $f(x, y)$ and the average gray value $g(x,y)$ as a pair vector (t, s), we can obtain a normalized 2-D histogram from the frequency of occurrence of the pair vectors as follows:

$$h(i, j) = \frac{1}{MN} \sum_{x=0}^{M} \sum_{y=0}^{N} \delta(i, j).$$

where

$$\delta(i, j) = \begin{cases} 1, & f(x, y) = i \quad and \quad g(x, y) = j, \\ 0, & Otherwise. \end{cases}$$

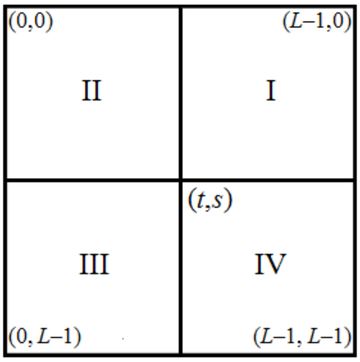


Figure 1. 2-D histogram

As shown in Fig.1, the 2-D histogram can be defined as a matrix with horizontal axes for the gray value and vertical axes for the local average gray value. When we assume the pair (t, s) as a threshold vector, the domain of the histogram is divided into four quadrants. Furthermore, these four can be classified into diagonal quadrants *II* and *IV*, and off-diagonal quadrants *I* and *III*. Based on the difference between the gray-levels and the local average gray-levels, the diagonal quadrants, which have small gray-level variations, are regarded as containing object and background, and the off-diagonal quadrants, which have large variations, are regarded as edges and noises ignored in calculation. According to the above assumptions, the *a posteriori* probability $P_{IV}(t,s)$ can be approximated as $P_{IV}(t,s) \approx 1 - P_{II}(t,s)$. Here, $P_{II}(t,s)$ and $P_{IV}(t,s)$ are computed by

$$P_{II}(t,s) = \sum_{i=0}^{t} \sum_{j=0}^{s} p(i,j)\,, \tag{4}$$

and

$$P_{IV}(t,s) = \sum_{i=t+1}^{L-1} \sum_{j=s+1}^{L-1} p(i,j)\,. \tag{5}$$

Then, two normalized *a posteriori* probabilities distributions of the object and background classes are obtained by using $P_{II}(t,s)$ and $P_{IV}(t,s)$. Using these two-probability distributions Sahoo et al. [10] defined the Renyi's entropies associated with the object and background classes as follows:

$$H_O^{\alpha}(t,s) = \frac{1}{1-\alpha} \ln \sum_{j=0}^{s} \sum_{i=0}^{t} \left(\frac{p(i,j)}{P_{II}(t,s)} \right)^{\alpha}, \tag{6}$$

$$H_B^{\alpha}(t,s) = \frac{1}{1-\alpha} \ln \sum_{j=s+1}^{L-1} \sum_{i=t+1}^{L-1} \left(\frac{p(i,j)}{P_{IV}(t,s)} \right)^{\alpha}, \tag{7}$$

where order-α is a positive real number and not equal to 1.

Then, following the idea suggested in Ref. [10], we can define the criterion function as

$$\psi^{\alpha}(t,s) = H_B^{\alpha}(t,s) + H_O^{\alpha}(t,s).$$

Finally, the optimal threshold pair ($t^*(\alpha)$, $s^*(\alpha)$) can be obtained by solving the following maximization problem:

$$(t^*(\alpha), s^*(\alpha)) = \arg \max_{(t,s) \in L \times L} \left\{ H_B^{\alpha}(t,s) + H_O^{\alpha}(t,s) \right\}.$$

According to the interpretation of the above method, the computation of $H_O(t, s)$ and $H_B(t, s)$ is the major part of this method, and takes the computational complexity of $O(L^2)$ for each pair of (t, s). Consequently, for the L^2 pairs of

(t, s), the computational complexity of the entire calculation is $O(L^4)$. It is conceivable that the process for finding the optimal threshold pair (t^*, s^*) from (9) is time-consuming, when compared to the 1-D entropic method.

III. PROPOSED METHOD

As mentioned in introduction, many recursive methods have been proposed for the different types of entropic thresholding methods. All these methods have the same goal, which is to get rid of the repeating calculations in the process of threshold selection, and the proposed method is no exception. However, the computation of a 2-D histogram is also a time-consuming process, especially for the large size of neighborhood as in consideration. To fasten the computational time, we first calculate the 2-D histogram by introducing the idea proposed in the adaptive thresholding method by Shafait et al. [15].

A. *Fast scheme for calculation of 2-D histogram*

For an input image F, first we define an integral image I, in which the gray value at a location (x, y) is equal to the cumulative sum of the gray values of all the pixels from (0,0) to (x, y) in the original image F. Therefore, we can write the gray value at location (x, y) as

$$I(x, y) = \sum_{i=0}^{x} \sum_{j=0}^{y} f(i, j) \tag{10}$$

Obviously, we can easily compute the integral image of any grey scale image in a single pass [15,16]. Once the integral image is computed, the local average $g(x, y)$ for any local window with the size of $W \times W$ can be simply calculated by using two addition and two subtraction operations instead of summing up all the pixel values within that local window as follows:

$$\begin{aligned} g(x, y) = {} & I(x+a, y+a) + I(x-a-1, y-a-1) \\ & - I(x+a, y-a-1) - I(x-a, y+a-1) \big/ W^2 \end{aligned} \tag{11}$$

where $a = W / 2$.

By using the local average $g(x, y)$ computed from the integral image, the 2-D histogram can be computed very efficiently, independent of the local window size. When this formula used in (2), the computational complexity for the calculation of the 2-D histogram can be reduced from $O(W^2MN)$ to $O(MN)$.

B. *Fast method for threshold selection*

In order to eliminate the unnecessary repetitions in the process of the computation for each threshold pair, the two entropies of $H_O\ (t, s)$ and $H_B\ (t, s)$ need to be rewritten to make a recursive formula. Equations (6) and (7) can be easily written as follows:

$$H_O^{\alpha}(t, s) = \frac{1}{1-\alpha}\left[\ln \sum_{j=0}^{s}\sum_{i=0}^{t}[p(i, j)]^{\alpha} - \alpha \ln P_{II}(t, s)\right]. \tag{12}$$

$$H_B^{\alpha}(t, s) = \frac{1}{1-\alpha}\left[\ln \sum_{j=s+1}^{L-1}\sum_{i=t+1}^{L-1}[p(i, j)]^{\alpha} - \alpha \ln P_{IV}(t, s)\right] \tag{13}$$

Then, the criterion function $\psi^{\alpha}(t, s)$ can be obtained by summing up (12) and (13) as follows:

$$\psi^{\alpha}(t,s) = \frac{1}{1-\alpha}\left[\ln G_A(t,s) G_B(t,s) - \alpha \ln P_{II}(t,s)(1 - P_{II}(t,s))\right]. \tag{14}$$

where

$$G_A(t,s) = \sum_{j=0}^{s}\sum_{i=0}^{t}\left[p(i,j)\right]^{\alpha} \;, \tag{15}$$

$$G_B(t,s) = \sum_{j=s+1}^{L-1}\sum_{i=t+1}^{L-1}\left[p(i,j)\right]^{\alpha} \;. \tag{16}$$

It is obvious that in order to obtain the total sum of 2-D Renyi's entropies from (14) three variables $G_A(t, s)$, $G_B(t,s)$ and $P_{II}(t, s)$ are required to be computed. Here, it is easy to find that $P_{II}(t+1, s)$ can be expressed in terms of $P_{II}(t, s)$ and new variable $P_s(t+1)$.

$$\begin{aligned} P_{II}(t+1,s) &= \sum_{j=0}^{s}\sum_{i=0}^{t+1} p(i,j) = \sum_{j=0}^{s}\sum_{i=0}^{t} p(i,j) + \sum_{j=0}^{s} p(t+1,j) \\ &= P_{II}(t,s) + P_s(t+1). \end{aligned} \tag{17}$$

The new variable $P_s(t+1)$ is also recursive as follows:

$$\begin{aligned} P_s(t+1) &= \sum_{j=0}^{s} p(t+1,j) = \sum_{j=0}^{s-1} p(t+1,j) + p(t+1,s) \\ &= P_{s-1}(t+1) + p(t+1,s), \end{aligned} \tag{18}$$

where additional term $p(t+1, s)$ is already known.

For a certain image, the variables $G_A(t, s)$ and $G_B(t, s)$ can be calculated before the searching process of the optimal threshold pair is carried out. To store their values, let us define an integral matrix $G(t,s)$, like the integral image in the calculation of the 2-D histogram. The value of the matrix at threshold pair (t, s) can be computed by

$$G(t,s) = \sum_{j=0}^{s}\sum_{i=0}^{t}\left[p(i,j)\right]^{\alpha} \;. \tag{19}$$

In the searching process, two variables $G_A(t, s)$ and $G_B(t, s)$ can be directly obtained by

$$G_A(t,s) = G(t,s) \;, \tag{20}$$

$$G_B(t,s) = G(L-1,L-1) + G(t,s) - G(L-1,s) - G(t,L-1) \;. \tag{21}$$

When the searching process of the optimal threshold value started from $t = 0$ and $s = 0$, $P_{II}(t+1,s)$ can be computed by adding $p(t+1, s)$ to $P_{II}(t, s)$, while $G_A(t, s)$ and $G_B(t, s)$ can be obtained by the integral matrix $G(t,s)$. Here, $P_{II}(t, s)$ is got from the last time computation. By this way, only one iteration is needed to get the sum total entropies at each pair (t,s) instead of the repeated calculations. Therefore, the computational complexity of the proposed method reduces from $O(L^4)$ to $O(L^2)$. When compared with the original method, evidently, the computational time is remarkably reduced. The algorithm of the proposed method is described in the following:

Input: A gray level image with $L = 256$

Step.1 Get the probability function $p(i, j)$ by computing the normalized 2-D histogram

Step.2 Calculate the integral matrix from (19) and initialize $P_{II}(0,0)$ as 0

Step.3 Begin the iteration for the gray value t from 0

Step.4 Initialize Ps as 0

Step.5 Begin the iteration for the local average s from 0

Step.6 Compute $G_A(t, s)$ and $G_B(t, s)$ from (20) and (21), respectively, and the following:

$Ps = Ps + p(t,s)$ and $P_{II}(t+1,s) = P_{II}(t,s) + Ps$

Step.7 Compute the total sum Renyi's entropy from (14), and save it and the corresponding threshold pair (t, s) if its value is larger than the last maximum value of it

Step.8 Go to Step.6 if $s < L$

Step.9 Go to Step.4 if $t < L$

Step.10 Threshold the image using the obtained optimal threshold value

Output: Binary image

IV. EXPERIMENTAL RESULTS AND DISCUSSIONS

In order to evaluate the performance of the proposed method, we tested a set of various real-world images and synthetic images. All the concerned experiments were implemented on Intel® Core™ i3 3.07 GHz with 2 GB RAM in Microsoft Visual Studio 2005. Four images, with gray-level L=256, were used here. They were “Kid.bmp”, “Flower.bmp”, “Pentagon.bmp” and “Mandrill.bmp” with the sizes of 600×610, 480×480, 1024×1025, and 512×512, respectively, shown in Figs.2 (a)-5(a). The first two were newly introduced and the last two were downloaded from [17]. All the original images have different types of gray-level histogram depicted in Figs.2 (b)-5(b).

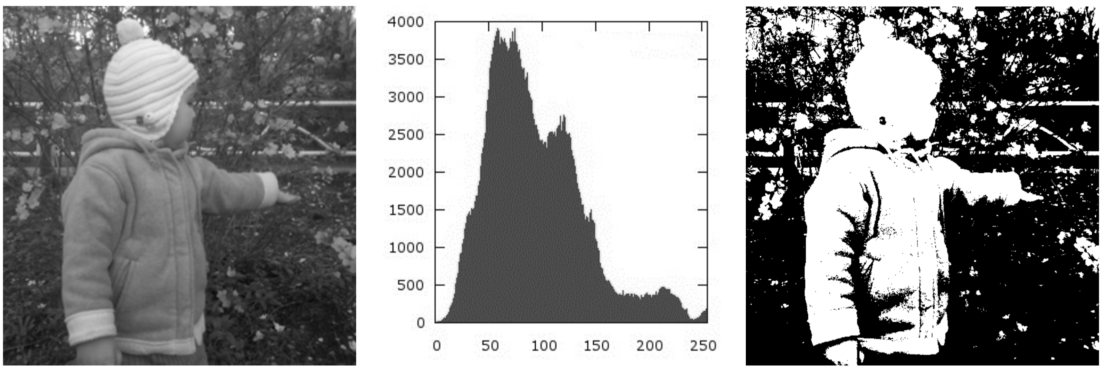
The experimental results by the proposed method are shown in Figs.2(c)-5(c) and their corresponding optimal threshold values are tabulated in TABLE I. The computational times of the 2-D Renyi’s entropy-based method and the proposed method for each image are listed in TABEL II. In these experiments, the size of local neighborhood is set as 3×3 and the order-α is chosen as 0.7 as described in Ref. [10]. It can be easily known from TABLE II that the proposed method is much faster than the original method.

TABLE I. THRESHOLD VALUES OBTAINED BY PROPOSED METHOD

Image	***Kid***	***Flower***	***Mandrill***	***Pentagon***
Threshold value	103	121	97	117

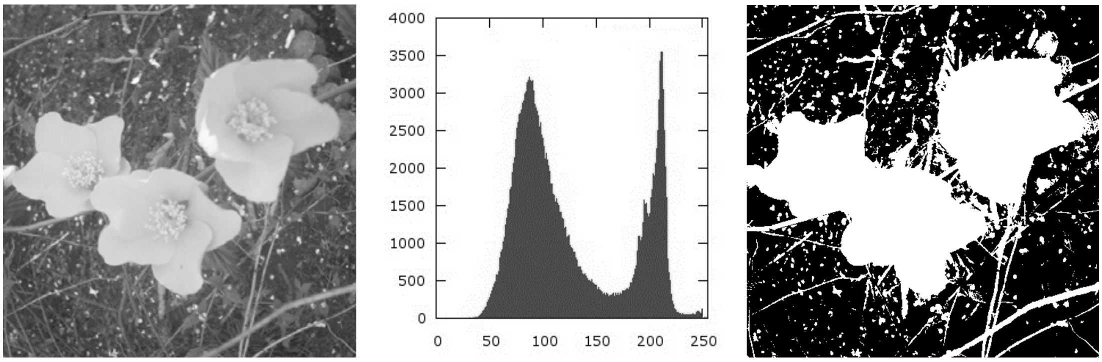
TABLE II. COMPARISON OF COMPUTATIONAL TIME OF TWO METHODS (SECS)

Image	***Kid***	***Flower***	***Mandrill***	***Pentagon***
Proposed method	0.055	0.054	0.053	0.137
Sahoo’s method [12]	148.572	149.164	150.043	182.514



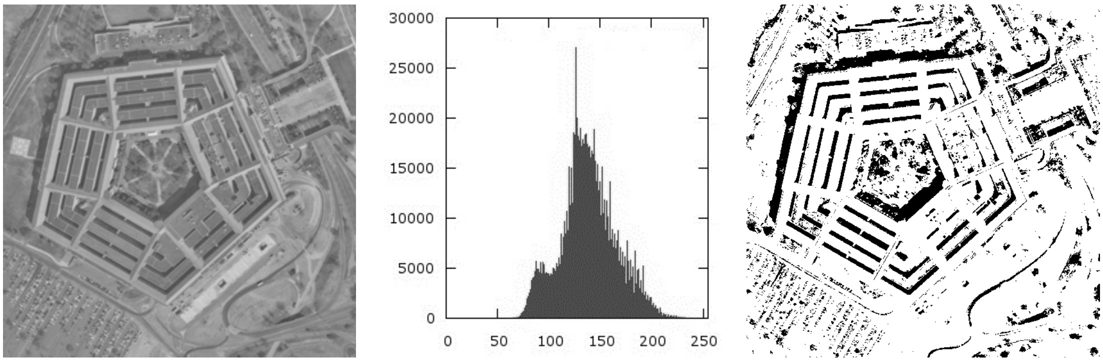
(a) (b) (c)

Figure 2. (a)Kid image, (b) histogram, (c) thresholded image.



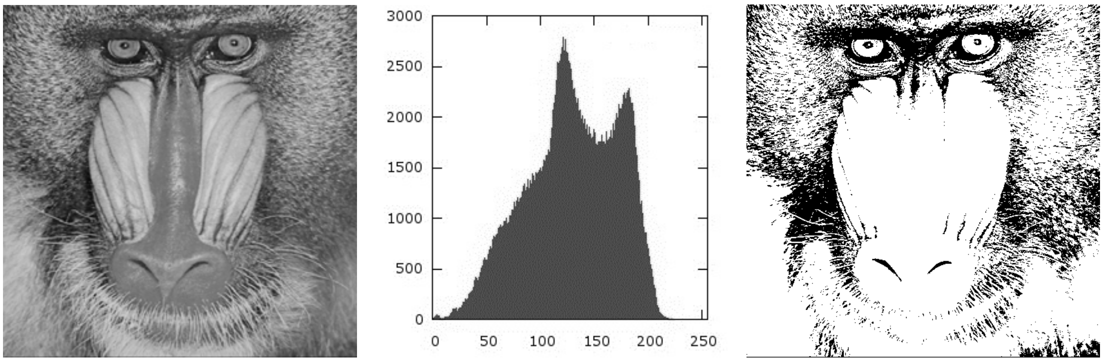
(a) (b) (c)

Figure 3. (a) Flower image, (b) histogram, (c) thresholded image.



(a) (b) (c)

Figure 4. (a) Pentagon image, (b) histogram, (c) thresholded image.



(a) (b) (c)

Figure 5. (a) Mandrill image, (b) histogram, (c) thresholded image.

V. CONCLUSIONS

A fast method for 2-D Renyi's entropy-based thresholding was presented. The experimental results showed that the proposed method can remarkably reduce the computational time and can yield the same threshold value as the original one. Compared with the original method, the fast method is much feasible for the implementation of a real-time processing system. It is notable that the presented modifications can be applied not only to the method by Sahoo

[10] to give the same threshold value but also to other 2-D thresholding methods, such as Abutaleb's [8], Brink's [9], Sahoo's [11], etc.

REFERENCES

[1] A. Yimit, Y. Hagihara, T. Miyoshi, Y. Hagihara, "A New Two-Dimensional Direction Histogram Based Entropic Thresholding," ICIG pp.106-110, Proceedings of 6th International Conference on Image and Graphics, 2011, AnHui, China.

[2] M. Sezgin and B. Sankur, “Survey over image thresholding techniques and quantitative performance evaluation,” Journal of Electronic Imaging, Vol. 13 (1), 2004, pp. 146-156.

[3] T. Pun, “Entropic thresholding, a new method,” Computer Graphics Image Process, 1981, pp. 210-239.

[4] J.N. Kapur, P.K. Sahoo, and A.K.C. Wong, “A new method for grey-level picture thresholding using the entropy of the histogram,” Computer Vision Graphics and Image Processing, 1985, pp. 273-285.

[5] M.P. de Albuquerque, I.A. Esquef, A.R.G. Mello, “Image thresholding using Tsallis entropy,” Pattern Recognition Letters, 2004, pp. 1059-1065.

[6] J. C. Yen, F. J. Chang, and S. Chang, ‘‘A new criterion for automatic multilevel thresholding,’’ IEEE Trans. Image Process, IP-4, pp. 370-378, 1995.

[7] P.K Sahoo, C. Willkins, and J. Yeager, “Threshold selection using Renyi’s entropy,” Pattern Recognition, 1997, pp. 71-84.

[8] A.S. Abutaleb, “Automatic thresholding of gray-level picture using 2-D entropies,” Pattern Recognition, 1989, pp. 22-32.

[9] A.D. Brink, “Thresholding of digital images using 2-D entropies,” Pattern Recognition, 1992, pp. 803-808.

[10] P.K.Sahoo, and G. Arora., “A thresholding method based on 2-D Renyi’s entropy,” Pattern Recognition, 2004, pp. 1149-1161

[11] P.K. Sahoo and G. Arora, “Image thresholding using 2-D Tsallis-Havrda-Charvát entropy,” Pattern Recognition Letters, 2006, pp. 520-528.

[12] X. J. Wu, Y. J. Zhang, L. Z. Xia, “A fast recurring two-dimensional entropic thresholding algorithm,” Pattern Recognition, 1994, 32: 2055–2061.

[13] J. Gong, L. Y. Li and W. N. Chen, “Fast recursive algorithms for two-dimensional thresholding,” Pattern Recognition, 1998, 31: 295–300.

[14] Y. G. Tang, Q. Y. Di and X. P. Guan, “Fast recursive algorithm for two-dimensional Tsallis entropy thresholding method,” Journal of Systems Engineering and Electronics, Vol.20, No.3, 2009, pp. 619-624.

[15] F. Shafait, D. Keysers and T. M. Breuel, “Efficient implementation of local adaptive thresholding techniques using integral images,” Proceedings of 15th International Conference on Document Recognition and Retrieval, Vol. 6815, pp. 81510, 2008, San Jose, CA, USA.

[16] P. Viola and M. J. Jones, “Robust real-time face detection,” Int. Journal of Computer Vision, Vol.57, No.2, pp. 137–154, 2004.

[17] http://sipi.usc.edu/database/database.php