Performance Evaluation of the Layered Space-Time Receiver Using the QR Detection Method

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***Abstract* -- The conventional V-BLAST detection is a single flow algorithm that consists of the two major operations: 1) the finding of the ordering index and 2) the nulling and symbol cancellation process. This paper focuses on the second operation and provides an alternative way for the nulling and symbol cancellation process using the QR decomposition to modify the V-BLAST algorithm for another way of effective detection. The decomposition of the channel matrix into the orthonormal matrix Q and the triangular matrix R have created a way of achieving symbol cancellation by means of backward substitution. Results are presented for various system configurations that demonstrate the performance of rearranging the channel matrix using the proposed method as compared to random ordering.**

Keywords* -- *MIMO, V-BLAST, Space-Time system, Symbol cancellation, MMSE.

1. INTRODUCTION

Space-time techniques potentially provide significant increases in capacity compared with traditional wireless communication systems for wireless channels that suffer from severe multipath propagation. This is achieved by properly exploiting the diversity that exists within this type of rich scattering channel environment between the multiple antennas at both the transmitter and receiver [1]. MIMO systems, such as V-BLAST [2], use multiple antennas at both the transmitter and receiver to permit very high transmission rate. A number of V-BLAST methods have been proposed recently. In [3], an efficient square-root algorithm for the nulling and cancellation step has been proposed to reduce the computational complexity. In [4], a new decoding algorithm based on the joint maximum likelihood and decision feedback equalization method for V-BLAST has been proposed for performing symbol detection. In [5], a low complexity V-BLAST architecture is proposed where Gram-Schmitt Orthogonalization (GSO) is applied to the channel matrix in order to find the weights.

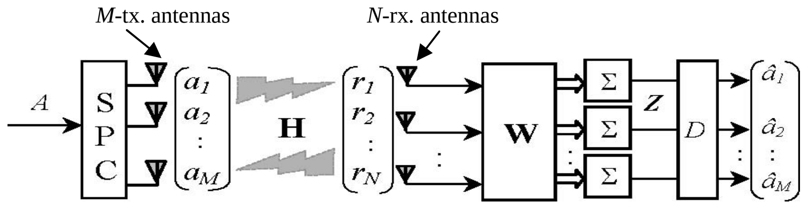
In this paper, we present a simpler concept for the non-linear space-time symbol detection using orthogonal triangularization method to reduce the complexity of the detection process. First, the optimum symbol detection order is obtained based on the post-detection SNR, using criteria such as zero-forcing or minimum mean square error, as in V-BLAST. This order is then used to rearrange the columns of the channel matrix. The new algorithm then applies QR decomposition to this rearranged channel matrix. The resulting triangular properties allow the sequence of symbols to be recovered by simple backward substitution with symbol cancellation.

The paper is organized as follows. Section 2 briefly describes the space-time model and detection algorithm. The proposed nonlinear QR detection algorithm is introduced in Section 3. Section 4 presents the simulation results. Conclusions are given in Section 5.

2. SPACE-TIME MODEL AND DETECTION ALGORITHM

Fig. 1 shows the architecture for the space-time communication system used in this paper. The system consists of M parallel transmit antennas and N parallel receive antennas operated at co-channel mode. The single transmit vector, $\boldsymbol{a}$ (which contains individual transmit symbols a_i from each i^{th} transmit antenna) can be denoted as:

$$\boldsymbol{a} = [a_1, a_2, \text{-----}, a_M]^T \qquad (1)$$



SPC = Serial to Parallel Conversion

Fig.1: Space-time system model with symbol detection.

Each transmit symbol 'a' is a complex component mapped from the QPSK modulation scheme. Each substream is pre-multiplied with a factor of $\sqrt{\rho / M}$, where ρ is the expected signal to noise ratio at the receiver and the total power radiated by each transmit antenna is proportional to $1/M$ in order to provide a constant power to the receiver regardless of the changes in number of transmit antenna. The symbols are sent over a Rayleigh flat-fading MIMO channel, **H** with the matrix dimension of $(N\times M)$, where signals are distorted and superimposed at each receive antenna. The corresponding received signal vector, $\boldsymbol{r}$, (contains N received signal r_j at each j^{th} receive antenna) can be expressed as:

$$\boldsymbol{r} = \mathbf{H}\boldsymbol{a} + \boldsymbol{n} \qquad (2)$$

where $\boldsymbol{r} = [r_1, r_2, \text{-------}, r_N]^T$ and $\boldsymbol{n}$ is the independent identically distributed, (i.i.d.) additive white Gaussian noise (AWGN) vector.

The received signal vector is processed by weighting each element of $\boldsymbol{r}$ with the elements of the weighting matrix, **W**. The weights, ω_{ij} in **W** are obtained according to one of two possible criteria: Zero-Forcing (ZF) or Minimum Mean Square Error (MMSE) [6]. The weighted signals are combined to form $\boldsymbol{Z} = [z_1, z_2, \text{----}, z_M]^T$. Each element in vector $\boldsymbol{Z}$ can be expressed as:

$$z_i = \sum_{j=1}^{N} \omega_{ij} r_j \qquad (3)$$

Using the zero forcing criterion as an example, estimates of the symbol vector, $\hat{\boldsymbol{a}} = [\hat{a}_1, \text{---}, \hat{a}_M]^T$ are recovered by applying the inverse of **H** to, $\boldsymbol{r}$, assuming that the fading coefficients in **H** are perfectly known by the receiver and each element in the received vector $\boldsymbol{r}$ is totally uncorrelated. The process is represented as follows:

$$\mathbf{W} = \mathbf{H}^{+} \qquad (4a)$$
$$\boldsymbol{Z} = \mathbf{W}\boldsymbol{r} \qquad (4b)$$
$$\hat{\boldsymbol{a}} = D(\boldsymbol{Z}) \qquad (4c)$$

Where D is the 'slicing' operator that decides upon the symbol estimate, according to the decision threshold for the modulation scheme employed. The $^{+}$ sign denotes the pseudo-inverse operation and is applied for the case when $N \geq M$. In practice, the channel coefficients must be estimated by some method [7].

2.1 V-BLAST Nonlinear Detection and Symbol Cancellation

The symbol detection algorithm of (4) is a linear process where all symbols in the transmitted vector $\boldsymbol{a}$ can be resolved simultaneously, assuming perfect symbol synchronization. However, superior performance can be obtained if nonlinear detection methods, such as symbol cancellation, are used. The nonlinear detection method resolves each symbol by treating the already-detected symbol components of vector $\boldsymbol{a}$ as symbol interference and they are subsequently eliminated from the received signal vector $\boldsymbol{r}$ one at a time [2]. The V-BLAST process can be separated into two parts; namely a) the optimum ordering process and b) the symbol detection process, as follows:

a) The optimum ordering process:

Initialization

$$i \leftarrow 1 \quad (5a)$$

$$\mathbf{G}_1 = \mathbf{H}^+ \quad (5b)$$

$$v_1 = \underset{j}{\arg\min} \left\| (\mathbf{G}_1)_j \right\|^2 \quad (5c)$$

Recursion

$$\mathbf{G}_{i+1} = \left\{ Z(\mathbf{H})_{vi} \right\}^+ \quad (5d)$$

$$v_{i+1} = \underset{j \notin \{v_1 \cdots v_i\}}{\arg\min} \left\| (\mathbf{G}_{i+1})_j \right\|^2 \quad (5e)$$

$$i \leftarrow i+1 \quad (5f)$$

b) The symbol detection process:

Initialization

$$i \leftarrow 1 \quad (6a)$$

Recursion

$$\mathbf{w}_{vi} = (\mathbf{G}_{\mathrm{i}})_{vi} \quad (6b)$$

$$\hat{\boldsymbol{a}}_{vi} = D(\mathbf{w}_{vi}^{\mathrm{T}} \boldsymbol{r}_i) \quad (6c)$$

$$\boldsymbol{r}_{i+1} = \boldsymbol{r}_i - \hat{\boldsymbol{a}}_{vi} (\mathbf{H})_{vi} \quad (6d)$$

$$i \leftarrow i+1 \quad (6e)$$

where v_i is the order in which the subsequent symbol detection will be carried out and is determined by the post-detection SNR, as described in [2]. The *'Z'* operator in (5*d*) sets the respective column of **H** to zero according to the value of 'v_i'. The same *'D'* operator from (4*c*) is used in (6*c*). The values of the matrix **G** are stored following each iteration of the ordering process and may subsequently be used in the symbol detection process. On completion of the optimum detection ordering process (5*a*)-(5*f*), the detection ordering set is obtained and written as:

$$\Re_{VBLAST} = \{\mathrm{v}_1, \mathrm{v}_2, \text{-----}, \mathrm{v}_M\} \quad (7)$$

The order in (7) may be obtained by other methods that avoid the pseudo-inverse process [3]. In this paper, we assume that we have been able to obtain the optimum ordering and the paper concentrates on the symbol detection process described in the next section.

3. NONLINEAR QR DETECTION ALGORITHM

The method makes use of the orthogonal triangularization process [8,9] to resolve each symbol iteratively by symbol cancellation. First, the order obtained in (7) is used to rearrange the columns of the channel matrix. The algorithm then applies QR decomposition to this rearranged channel matrix, $\mathbf{H}_{new}$. The resulting triangular properties allow the sequence of transmitted symbols to be recovered by simple backward substitution with cancellation. It is important, therefore, that the best estimates of the symbols are used first. The process is described in the following section.

3.1 Channel Matrix Rearrangement

The ordering set obtained by the ordering process can be expressed as:

$$\Re = \{s_1, s_2, \cdots, s_M\} \quad (8)$$

This is simply a restatement of (7) for the general case. Once the ordering is obtained, the rearrangement from **H** to $\mathbf{H}_{new}$ can be accomplished by rearranging columns using the following procedure:

Initialization

$$i \leftarrow 1 \qquad (9a)$$

$$b \leftarrow M \qquad (9b)$$

Recursion

$$(\mathbf{H}_{new})_b = (\mathbf{H})_{s_i} \qquad (9c)$$

$$i \leftarrow i+1 \qquad (9d)$$

$$b \leftarrow b-1 \qquad (9e)$$

The nomenclature of equation (9*c*) implies that the s_i^{th} column of **H** is selected and replaces the b^{th} column of $\mathbf{H}_{new}$ for all *M* columns.

3.2 Orthogonal Triangularization by QR decomposition

The reordered channel matrix, $\mathbf{H}_{new}$, can be decomposed into the orthonormal matrix, $\mathbf{Q}_{new}$, and upper triangular matrix, $\mathbf{R}_{new}$, as follows:

$$\mathbf{H}_{new} = \mathbf{Q}_{new} \begin{bmatrix} \mathbf{R}_{new} \\ \boldsymbol{O} \end{bmatrix} \qquad (10)$$

where $\boldsymbol{O}$ is the zero matrix. To illustrate the method adopted we assume the noise free condition. Hence, from (2), the received vector can be written as:

$$\boldsymbol{r} = \mathbf{Q}_{new} \begin{bmatrix} \mathbf{R}_{new} \\ \boldsymbol{O} \end{bmatrix} \boldsymbol{a}_{new} \qquad (11)$$

A property of an orthogonal matrix is that $\mathbf{Q}^{\mathrm{T}} = \mathbf{Q}^{-1}$ and $\mathbf{Q}^{\mathrm{T}}\mathbf{Q} = 1$, therefore, (11) can be re-written as follows:

$$\mathbf{Q}^{\mathrm{T}}_{new}\,\boldsymbol{r} = \boldsymbol{x} = \begin{bmatrix} \mathbf{R}_{new} \\ \boldsymbol{O} \end{bmatrix} \boldsymbol{a}_{new} \qquad (12)$$

where $\boldsymbol{x}$ denotes the 'orthogonalized' receive vector and its elements q_i can be expressed as: $\boldsymbol{x} = [q_1, q_2, \text{---}, q_N]^{\mathrm{T}}$. Estimates of the symbol elements of vector $\boldsymbol{a}$ are then recovered by a non-linear process that includes backward substitution into the upper triangular matrix $\mathbf{R}_{new}$ in (12) with symbol cancellation.

3.3 Nonlinear QR Symbol Detection Process

Having rearranged **H** to give $\mathbf{H}_{new}$, we now apply QR decomposition to obtain $\mathbf{Q}_{new}$ and $\mathbf{R}_{new}$. At the receiver, the symbol elements in the original transmitted vector $\boldsymbol{a}$ are 'rearranged' so that the received vector $\boldsymbol{r}$ remains the same. The $\mathbf{R}_{new}$ obtained from $\mathbf{H}_{new}$ can be expressed as:

$$\mathbf{R}_{new} = [R_1 \quad R_2 \quad \cdots \quad R_M] = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \cdots & \gamma_{1M} \\ 0 & \gamma_{22} & \cdots & \gamma_{2M} \\ \vdots & 0 & \ddots & \vdots \\ 0 & \cdots & 0 & \gamma_{MM} \end{bmatrix} \qquad (13)$$

where R_i is the i^{th} column vector in $\mathbf{R}_{new}$ and γ_{nm} are the elements in $\mathbf{R}_{new}$. The rearrangement of the columns of **H** is determined by the detection ordering set, $\Re$, such that the first element of $\Re$ corresponds to the last column of $\mathbf{H}_{new}$, etc.

For example, in a (3×3) system, if $\Re$ = {2,1,3}, then **H** is rearranged such that $\mathbf{H}_{new}$ = [H_3 , H_1 , H_2] where H_i denotes the columns of **H**. Once the 'orthogonalized' receive vector, ***x***, in (12) is obtained, the symbol estimates of the transmit vector, $\hat{\boldsymbol{a}}_{new}$ = [$\hat{a}_3$, $\hat{a}_1$, $\hat{a}_2$]^T can be detected by either backward substitution, which is equivalent to linear detection, or the following non-linear approach with symbol cancellation. It is worth noting that QR decomposition applied to $\mathbf{H}_{new}$ is only carried out once and the columns of $\mathbf{R}_{new}$ are used in the following algorithm:

Initialization

$$j \leftarrow M \qquad (14a)$$

$$k = 1 \qquad (14b)$$

Recursion

$$\hat{a}_j = D\left(\frac{q_j^k}{\gamma_{jj}}\right) \qquad (14c)$$

$$x^{k+1} = x^k - \hat{a}_j R_j \qquad (14d)$$

$$j \leftarrow j - 1 \qquad (14e)$$

$$k = k + 1 \qquad (14f)$$

where '*D*' is the same decision operator mentioned in (4c). '*k*' is the recursion index. It can be noticed that (14*c*) resolves each symbol estimate sequentially in a backward manner and (14*d*) performs the symbol cancellation of the already detected component a_j from the orthogonalized received vector ***x***.

3.4 Methods of performing QR decomposition

The QR decomposition can be performed by several methods as mentioned in [10]. For the results presented in this paper, the QR decomposition used Householder's method since it is numerically more stable than the Gram-Schmidt Orthogonalization (GSO) process [8]. The number of computational 'flops' required is also less for the Householder QR factorization as compared to the classical or modified GSO method [8].

4. RESULTS AND DISCUSSION

The performance results of the space-time system were obtained by computer simulation using the proposed QR detection algorithm for both linear and nonlinear detection algorithms. It is assumed that the channel is time varying. However, the fading coefficients are assumed to be relatively constant over a 'block' of symbols and change prior to the next 'block'. For the results described in this paper, a 'block' comprises 100 vectors, ***a***, where each vector comprises of M QPSK symbols. Perfect channel estimation is also assumed.

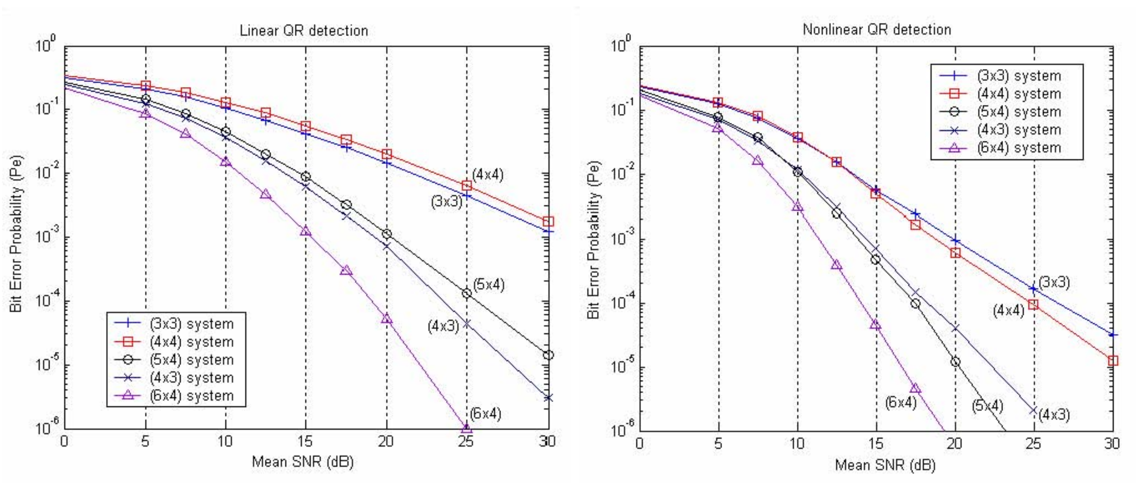


Fig. 2: Bit Error Performance using linear QR & nonlinear QR method.

Fig. 2 shows the bit error probability performance of the space-time system for both linear and nonlinear QR detection algorithms for various systems with configuration of *(N×M)*. The results obtained using the linear QR detection shown in figure 2a is equivalent to the results obtained for the linear ZF algorithm shown in Appendix. This confirms the idea of using QR decomposition by means of the orthogonal triangularisation. The results using the nonlinear QR method is shown in figure 2b, which is found to outperform the linear QR method by a great margin.

It is shown in Fig. 2 that the (6×4) system has better performance than the (3×3) system in all cases. This is due to the fact that higher multipath diversity is achieved by the multi-antenna arrangement. The result of non-linear detection is equivalent to the performance of V-BLAST.

It can also be observed that the nonlinear detection process performs better than the linear detection process for both systems due to the optimum ordering process and symbol cancellation technique used in the detection algorithm. The results are also compared in the following figure:

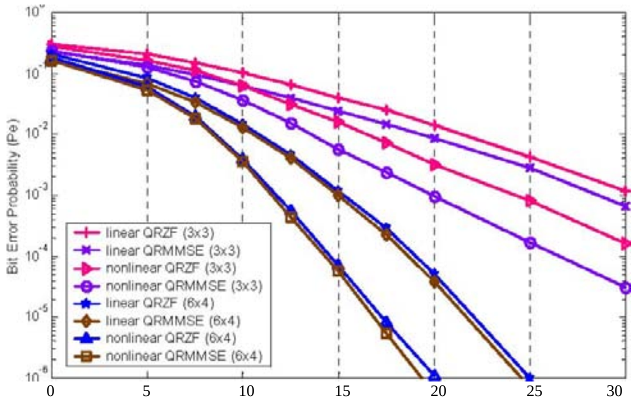


Fig. 3: System performance for the (6×4) & (3×3) system configuration for both linear and nonlinear QR detection using either ZF or MMSE criteria.

Next, in order to assess the performance due to the effect of correct detection ordering, we deliberately conceal the process of channel matrix re-arrangement in the nonlinear QR detection. This is equivalent by either freezing this feature (no re-arrangement made prior to the QR decomposition of **H** – QR decomposition is performed on the original **H**) or by allowing random re-arrangement (meaning that re-arrangement not according to the optimum detection ordering set). The reason to perform this channel matrix re-arrangement process is to effectively incorporate the detection ordering required by the symbol cancellation feature in determining which symbols were to be detected first. We denote this setting as without '$\mathbf{H}_{new}$' and comparison is shown for the results using the original nonlinear QR detection with '$\mathbf{H}_{new}$'.

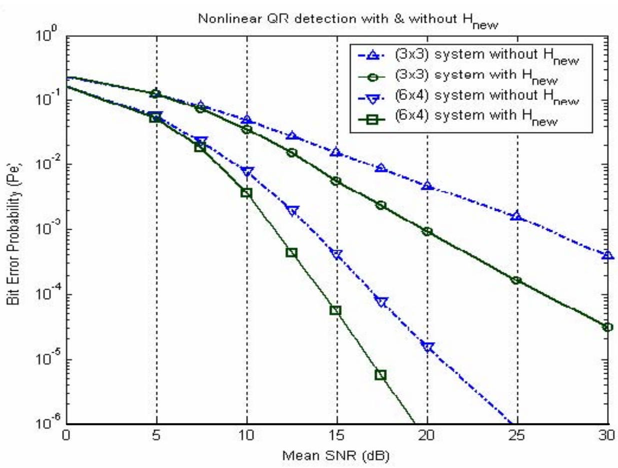


Fig. 4: System performance using nonlinear QR detection a) with $\mathbf{H}_{new}$ & b) without $\mathbf{H}_{new}$

It can be seen clearly in Fig. 4 that the system performance of the case with the proper channel matrix re-arrangement (according to the optimum detection ordering set obtained *a priori*) – with $\mathbf{H}_{new}$, is better than the case without $\mathbf{H}_{new}$. This demonstrates the importance of the including the correction detection ordering in the feature that rearranges the original $\mathbf{H}$ to $\mathbf{H}_{new}$ in the nonlinear QR algorithm. The bit error probability is considerably reduced when the correct order is incorporated during the symbol detection process by the method of backward substitution in the QR algorithm.

5. CONCLUSIONS

In this paper, the nonlinear QR symbol detection algorithm has been successfully implemented for the space-time system in the flat-fading Rayleigh environment. It is found that the proposed method equals the performance of V-BLAST. The paper has also shown the importance of obtaining the correct symbol detection order.

6. APPENDIX

The following results show the performance of V-BLAST system achieved by different detection algorithms in the various (N×M) MIMO systems operated in flat fading MIMO channel.

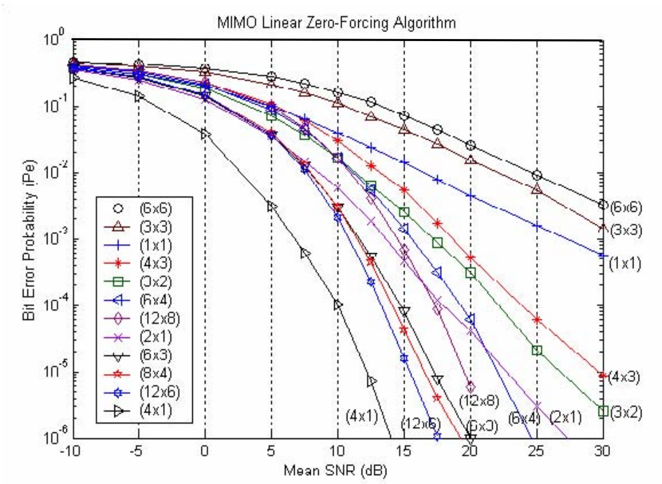


Fig. 5: Performance of layered space-time receiver using linear ZF algorithm.

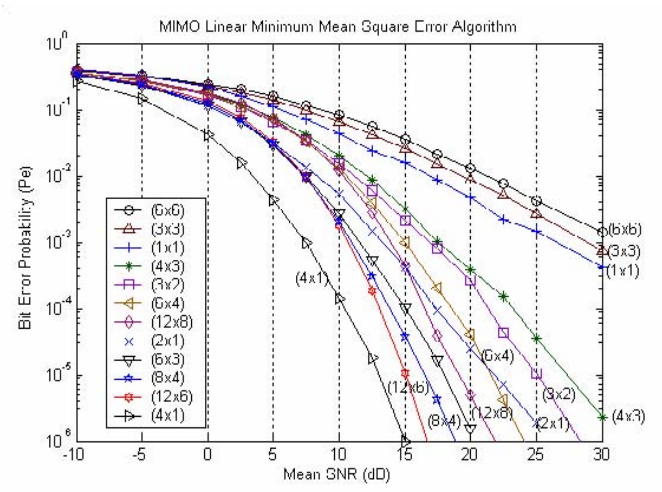


Fig. 6: Performance of layered space-time receiver using linear MMSE algorithm.

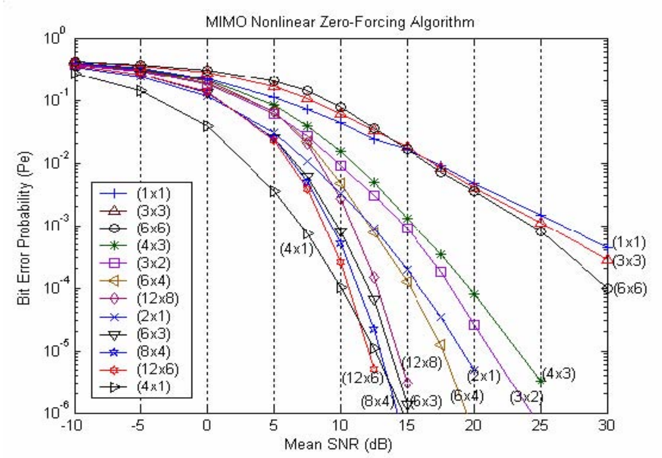


Fig. 7: Performance of layered space-time receiver using nonlinear ZF algorithm.

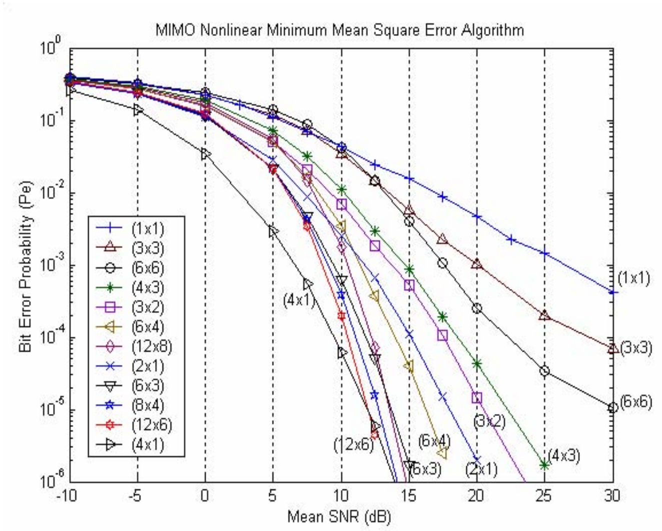


Fig. 8: Performance of layered space-time receiver using nonlinear MMSE algorithm.

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