Mathematical modeling And Fuzzy logic control of Inspired Isoflurane to obtain minimal flow of Anesthesia

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***Abstract*—In this paper, a fuzzy logic controller is designed and developed for control of Mean Arterial Pressure (MAP) of a patient during anesthesia using the anesthetic drug Isoflurane. Isoflurane is an anesthetic drug administered through a vaporizer based system. The main purpose of this paper is to provide a base for further real application .Here the patient is represented with a linear mathematical model that includes time delay elements. The parameters of the fuzzy controller are tuned in order to obtain a robust control performance .The simulation result showing the robust control performance of the proposed fuzzy controller under the parameter variations is given. Mean Arterial Pressure (MAP) is used as an inferential variable to indicate level of unconsciousness.**

Simulation experiments showed that excellent regulation of blood pressure around set point target.

Keywords- Mean Arterial pressure (MAP), anaesthesia,Isoflurane, fuzzy controller, robust control performance .

I. INTRODUCTION

Anesthesia is the art or science of removing sensation of and reaction to a surgical procedure. Anesthesia aims at loss of sensation with regards to pain, touch, temperature or position [1-3].This complex branch of medical area follows the three triads of muscle relaxation, depth of anesthesia (DOA) and relaxation of pain i.e. analgesia. The anesthetic and analgesic drug in the right combination achieves the perfect amount of depth of anesthesia. The DOA is associated with the application of feedback control. The fact that anesthesia variables interact and produce a complex system, leads to the need of a controller where these variables are integrated. With the extra ordinary advancements made in the field of artificial intelligence and with the respective set of advantages & disadvantages, fuzzy logic based controller is the best choice as it has the ability to perform logical inference under uncertain conditions.

II. MATHEMATICAL MODELLING OF A PATIENT UNDER ANAESTHESIA

In this paper a mathematical model has been formulated wherein there is a relation between the input variable i.e inflow of isoflurane x(t) and output variable namely the Mean Arterial Pressure (MAP) denoted by y(t) [1].This model features a semi closed circuit used to deliver an anesthetic agent to a patient. The unit step response is as follows [4]-

$y(t)=k_1[1-e^{-a}{}_1(t-T_1)]u(t-T_1)+k_2[1-e^{-a}{}_2(t-T_2)]u(t-T_2)$ ……………………(1)

Here $k_1=-3$, $k_2=-7.3$, $T_1=23s$, $T_2=101s$, $a_1=0.01$, and $a_2=0.006$

Using Laplace Transform, we get- $y(s)=k_1\left[\frac{1}{s}-\frac{1}{s+a_1}\right]e^{-T_1 s}+k_2\left[\frac{1}{s}-\frac{1}{s+a_2}\right]e^{-T_2 s}$ ……..….(2), *Since* $u(s)=\frac{1}{s}$

The transfer function between output y(s) and input x(s) becomes

$$G_p(s)=\frac{y(s)}{x(s)}=k_1\left[1-\frac{s}{s+a_1}\right]e^{-T_1s}+k_2\left[1-\frac{s}{s+a_2}\right]e^{-T_2s} \quad \ldots\ldots\ldots\ldots(3)$$

$$Now \quad G_{P1}(s)=\frac{k_1 a_1 e^{-T_1(s)}}{s+a_1} \quad \ldots\ldots\ldots\ldots(4)$$

$$Now \quad G_{P2}(s)=\frac{k_2 a_2 e^{-T_2(s)}}{s+a_2} \quad \ldots\ldots\ldots\ldots(5)$$

To take care of the time delay elements, we use *PADE* approximations [5]. But since the design and simulations will be in discrete time, it is desirable to use modified Z Transform which is generally applied to systems having time delay elements [6].
As such applying modified Z Transform on (4) we get

$$G_P(z)=\frac{z-1}{z}Z\left\{\frac{z^{-n_1}a_1k_1e^{m_1Ts}}{s(s+a_1)}\right\}+\frac{z-1}{z}Z\left\{\frac{z^{-n_2}a_2k_2e^{m_2Ts}}{s(s+a_2)}\right\} \quad \ldots (6)$$

Here,
n: is an integer
m: is a positive real number
T : Sampling Time
T_1 : n_1T-m_1T
T_2 : n_2T-m_2T
Substituting the values of the parameters earlier considered in the Equation (6),we get the mathematical model of the patient under anesthesia
Here , $k_1=-3$, $k_2=-7.3$, $T_1=23s$, $T_2=101s$, $a_1=0.01$, and $a_2=0.006$.

III. THE CONTROLLER FORMULATION

The controller to be used is based on the Generalized Predictive control strategy proposed by Clarke [7].The basic outline of this strategy is briefly explained. Consider a linearised discrete model based on inverse Z Transform or the backward shift operator of z^{-1}

$$A(z^{-1})\Delta x(t) = B(z^{-1})\Delta u(t-1) + C(z^{-1})\xi(t) \quad \ldots\ldots\ldots(7)$$

Where, $A(z^{-1}) =1+a_1z^{-1}+a_2z^{-2}+\ldots\ldots+a_nz^{-n}$
$B(z^{-1}) =b_1+b_2z^{-1}+b_3z^{-2}+\ldots\ldots+b_nz^{-m+1}$
$C(z^{-1}) =c_0+c_1z^{-1}+c_2z^{-2}+\ldots\ldots+c_pz^{-p}$
Here $\xi(t)$ is a random sequence
$\Delta = 1-z^{-1}$
U(t) is control input
X(t) is variable

The controller is used to calculate vector of the control system using an optimization function

$$J\sum_{j=C_1}^{C_2}\left[\left\{p(z^{-1})x(t+j)\hat{-}w(t+j)\right\}^2\right]+\sum_{j=1}^{co}\left[\lambda(j)\{\Delta u(t+j-1)\}^2\right]$$

C_1 → minimum costing output
C_2 → maximum costing output
C_0 → Control Space
W → future set point
$\lambda(j)$ → Control Weighing Sequence
$P(z^{-1})$ → inverse model

Here the control space CO, reflects the degree of freedom for the controller which is greater than 1.

IV. FUZZY PROCESS MODEL

One common characteristic of all Predictive Control Systems is the designing of an accurate model. But in practical situations as the complexity of the system increases, they become less manageable by direct mathematical modeling. As such the modeling problem instead of being confined within a strict analytical framework is based on empirical details.

For the modeling part, the *Takagi And Sugeno* Principle has been adopted where the fuzzy set is only the premise part and the conclusion part is a regression part. Consider a single input, single output system which can be modeled using *Takagi And Sugeno* system[9]. Let the input space be partitioned using p fuzzy partitions, and let the system be represented by fuzzy implications. For example , If $x(t)= A_i$

Then $x_m(t+1) = a_1^i x(t)+$………………… $+a^i jx(t-j+1)+b_1 iu(t-1+1)+k_i$
Further using the above fuzzy implications, the fuzzy linear model has been designed which is as follows [10]
IF x(t) is A^i, THEN $\Delta x_m(t+1) = -a^1{}_i\Delta x(t)-\ldots\ldots-a^i{}_j\Delta x(t-n_a+1) +b^i{}_1\Delta u(t)+..+ \; b^i{}_l\Delta u(t-n_b+1)$ ….(8) in the matrices form this model appears as

$$Z=\begin{bmatrix} a_1^1\cdots\cdots & a_{n_a}^1 & b_1^1\cdots\cdots & b_{n_b}^1 \\ \vdots & & \vdots & \\ \vdots & & \vdots & \\ a_1^p\cdots\cdots & a_{n_a}^p & b_1^p\cdots\cdots & b_{n_b}^p \end{bmatrix}$$

The output fuzzy model which is in the incremental form can be written as

$$x(t)=\begin{bmatrix} -\Delta_x(t) & -\Delta_x(t-1)\cdots & \cdots \\ -\Delta x(t-n_a+1) & \Delta u(t) & \Delta u(t-1)\cdots & \cdots \\ \Delta u(t-n_b+1) & & \end{bmatrix} T^{-1}$$

$$\alpha = [\alpha_1 \quad \alpha_2 \quad \alpha_3\ldots\ldots\ldots\alpha_i\ldots\ldots\ldots\ldots \alpha_p]$$

$$\alpha_i = \frac{A^i[x(t)]}{\sum_{i=1}^{p} A^i[x(t)]}$$

Here $A^i[x(t)]$ is a grade membership of x(t) in A^i and α is vector of weights assigned to each of the p implications

V. SIMULATION RESULT

The simulation study involves continuous non-linear systems, represented in MATLAB.The sampling interval is of 1 minute.

For the parameter estimation, an upper diagonal factorization method is used on the incremental data. At the time t=0 an initial pressure corresponding to MAP_0=90mmHg was assumed. For a total simulation time of 400min, the set point command was 70mmhg and 80mmHg.The partition of the input space was done using Triangular Membership.

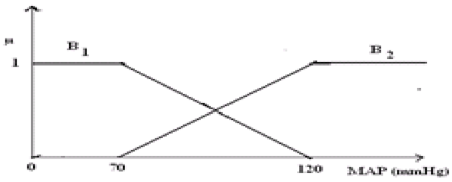

Fig:1- Partition of the Fuzzy input space using triangular membership.

Further the three type of constraints used in the algorithm are as follows-

$-0.2 \le \Delta u(t+j-1) \le 0.2$

$0 \le u(t+j-1) \le 5$

$w-5 \le \Phi\,(t+j) \le w+5$

where j = 1,2,3………………………………..CO.

The observation that came out was that when air flow of 2litres/hr and a Fuzzy partition of two for the input space was used, the output was obtained as follows.

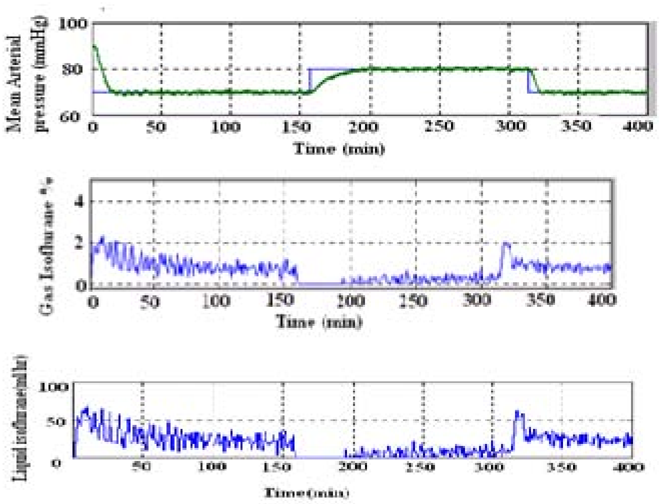

Fig:2 Different outputs

VI. CONCLUSION

A comprehensive study of the above output shows that,when an unconstrained Predictive Control algorithm was applied, The result obtained was-

A. Output tracking was good

B. Control activity was very high

C. Consumption of anaesthesia by the patient was slow, facilitating fast recovery apart from being cost effective.

Thus this simulation study suggests the possibility of the application of overall closed loop control system in the operation theatre for the administration of Isoflurane as an anesthetic agent during surgery.

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