Anti-Synchronization of the Hyperchaotic Liu and Hyperchaotic Qi Systems by Active Control

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***Abstract*—This paper investigates the problem of anti-synchronization of identical hyperchaotic Liu systems (2008), hyperchaotic Qi systems (2008) and non-identical hyperchaotic Liu and hyperchaotic Qi systems using active nonlinear control. Sufficient conditions for achieving anti-synchronization of the identical and different hyperchaotic Liu and hyperchaotic Qi systems using active nonlinear control are derived based on Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active control method is very effective and convenient to achieve anti-synchronization of identical and different hyperchaotic Liu and hyperchaotic Qi systems. Numerical simulations are shown to demonstrate the effectiveness of the anti-synchronization schemes derived in this paper.**

Keywords-chaos; anti-synchronization; active control; hyperchaotic Liu system; hyperchaoticQi system.

I. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. The sensitive nature of chaotic systems is commonly called as the *butterfly effect* [1].

Since the pioneering work of Pecora and Carroll [2], chaos synchronization has been studied extensively and intensively in the last two decades [2-17]. Chaos theory has been explored in a variety of fields including physical systems [3], chemical systems [4] and ecological systems [5], secure communications [6-8] etc.

In the recent years, various schemes such as PC method [2], OGY method [9], active control [10-12], adaptive control [13-14], time-delay feedback approach [15], backstepping design method [16], sampled-data feedback synchronization method [17], sliding mode control [18], etc. have been successfully applied for chaos synchronization. Recently, active control has been applied to anti-synchronize identical chaotic systems [19-20] and different hyperchaotic systems [21].

In most of the chaos anti-synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of the anti-synchronization is to use the output of the master system to control the slave system so that the states of the slave system have the same amplitude but opposite signs as the states of the master system asymptotically. In other words, the sum of the states of the master and slave systems are expected to converge to zero asymptotically when anti-synchronization appears.

In this paper, we derive new results for the anti-synchronization of identical Liu systems (2004), identical Chen systems (1999) and non-identical Liu and Chen chaotic systems using the active nonlinear control method. The stability results derived in this paper are established using Lyapunov stability theory.

This paper has been organized as follows. In Section II, we give the problem statement and our methodology. In Section III, we discuss the chaos anti-synchronization of two identical hyperchaotic Liu systems ([22], 2008). In Section IV, we discuss the chaos anti-synchronization of two identical hyperchaotic Qi systems ([23], 2008). In Section V, we discuss the anti-synchronization of hyperchaotic Liu and hyperchaotic Qi systems. In Section VI, we summarize the main results obtained in this paper.

II. Problem Statement and Our Methodology using Active Control

Consider the chaotic system described by

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of the system parameters and $f : R^n \to R^n$ is the nonlinear part of the system. We consider the system (1) as the *master* or *drive* system.

As the *slave* or *response* system, we consider the following chaotic system described by the dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system, B is the $n \times n$ matrix of the system parameters, $g : R^n \to R^n$ is the nonlinear part of the system and $u \in R^n$ is the controller of the slave system.

If $A = B$ and $f = g$, then x and y are the states of two *identical* chaotic systems.

If $A \neq B$ or $f \neq g$, then x and y are the states of two *different* chaotic systems.

In the nonlinear feedback control approach, we design a feedback controller u, which anti-synchronizes the states of the master system (1) and the slave system (2) for all initial conditions $x(0), y(0) \in R^n$.

If we define the *anti-synchronization error* as

$$e = y + x, \tag{3}$$

then the error dynamics is obtained as

$$\dot{e} = By + Ax + g(y) + f(x) + u \tag{4}$$

Thus, the global chaos anti-synchronization problem is essentially to find a feedback controller u so as to stabilize the error dynamics (4) for all initial conditions $e(0) \in R^n$.

Hence, we find a feedback controller u so that

$$\lim_{t \to \infty} \|e(t)\| = 0 \text{ for all } e(0) \in R^n. \tag{5}$$

We take as a candidate Lyapunov function

$$V(e) = e^T Pe, \tag{6}$$

where P is a positive definite matrix.

Note that

$$V : R^n \to R$$

is a positive definite function by construction.

We assume that the parameters of the master and slave system are known and that the states of both systems (1) and (2) are measurable.

If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Qe, \tag{7}$$

where Q is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.

Thus, by Lyapunov stability theory [24], the error dynamics (4) is globally exponentially stable and hence the condition (5) will be satisfied.

Hence, the states of the master system (1) and the slave system (2) will be globally and exponentially anti-synchronized for all initial conditions $x(0), y(0) \in R^n$.

III. ANTI-SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC LIU SYSTEMS

A. *Theoretical Results*

In this section, we apply the active control method for the global chaos anti-synchronization of identical hyperchaotic Liu systems.

The hyperchaotic Liu system is one of the paradigms of the four-dimensional hyperchaotic systems discovered by L Liu, C. Liu and Y. Zhang ([22], 2008).

Thus, the master system is described by the hyperchaotic Liu dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 + x_1x_3 - x_4 \\ \dot{x}_3 &= -x_1x_2 - cx_3 + x_4 \\ \dot{x}_4 &= dx_1 + x_2 \end{aligned} \tag{8}$$

where x_1, x_2, x_3, x_4 are state variables of the system and a, b, c, d are positive, constant parameters of the system.

The four-dimensional system (8) is hyperchaotic when the parameter values are taken as

$$a = 10, \quad b = 35, \quad c = 1.4 \text{ and } d = 5.$$

The state orbits of the hyperchaotic Liu system (8) are illustrated in Fig. 1.

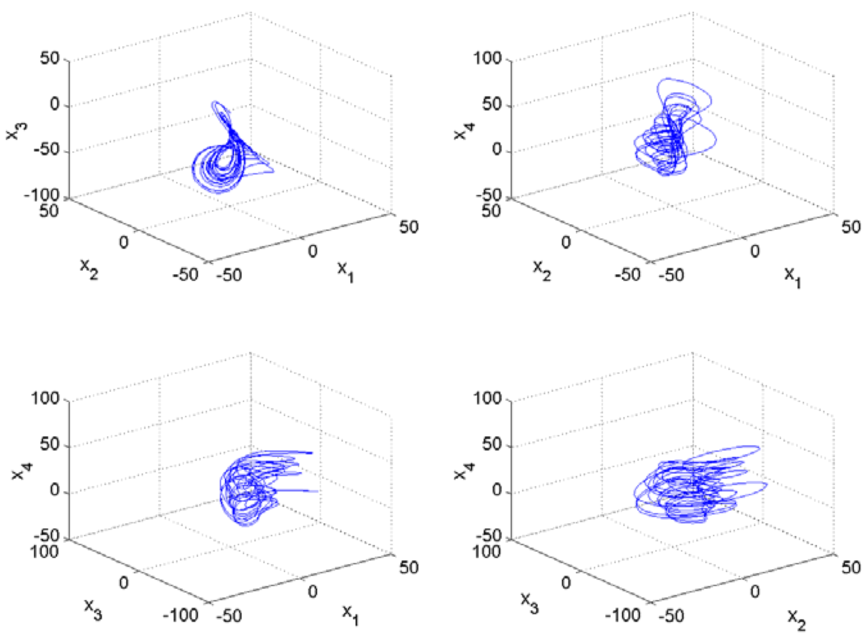

Figure 1. State Portrait of the Hyperchaotic Liu System

The slave system is described by the controlled hyperchaotic Liu dynamics

$$\begin{aligned}
\dot{y}_1 &= a(y_2 - y_1) + u_1 \\
\dot{y}_2 &= by_1 + y_1 y_3 - y_4 + u_2 \\
\dot{y}_3 &= -y_1 y_2 - cy_3 + y_4 + u_3 \\
\dot{y}_4 &= dy_1 + y_2 + u_4
\end{aligned} \tag{9}$$

where y_1, y_2, y_3, y_4 are state variables and u_1, u_2, u_3, u_4 are the controllers to be designed.

The anti-synchronization error is defined by

$$e_i = y_i + x_i, \ (i = 1, 2, 3, 4) \tag{10}$$

The error dynamics is easily obtained as

$$\begin{aligned}
\dot{e}_1 &= a(e_2 - e_1) + u_1 \\
\dot{e}_2 &= be_1 - e_4 + y_1 y_3 + x_1 x_3 + u_2 \\
\dot{e}_3 &= -ce_3 + e_4 - y_1 y_2 - x_1 x_2 + u_3 \\
\dot{e}_4 &= de_1 + e_2 + u_4
\end{aligned} \tag{11}$$

We choose the active nonlinear controller as

$$\begin{aligned}
u_1 &= -ae_2 \\
u_2 &= -be_1 - e_2 + e_4 - y_1 y_3 - x_1 x_3 \\
u_3 &= -e_4 + y_1 y_2 + x_1 x_2 \\
u_4 &= -de_1 - e_2 - e_4
\end{aligned} \tag{12}$$

Substituting (12) into (11), we obtain the linear error system

$$\begin{aligned}
\dot{e}_1 &= -ae_1 \\
\dot{e}_2 &= -e_2 \\
\dot{e}_3 &= -ce_3 \\
\dot{e}_4 &= -e_4
\end{aligned} \tag{13}$$

Next, we consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{14}$$

Differentiating V along the trajectories of (13), we get

$$\dot{V}(e) = -ae_1^2 - e_2^2 - ce_3^2 - e_4^2, \tag{15}$$

which is a negative definite function on R^4.

Thus, the error dynamics (13) is globally exponentially stable for all initial conditions $e(0) \in R^4$.

Hence, we obtain the following result.

Theorem 1. The identical hyperchaotic Liu systems (8) and (9) are globally and exponentially anti-synchronized for all initial conditions by the active nonlinear controller defined by (12). ■

B. Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic Liu systems (8) and (9) with the active controller u given by (12) using MATLAB.

In the hyperchaotic case, the parameter values are

The initial values of the master system (21) are taken as

$$x_1(0) = 36, \quad x_2(0) = 14, \quad x_3(0) = 25, \quad x_4(0) = 18$$

and the initial values of the slave system (22) are taken as

$$y_1(0) = 12, \quad y_2(0) = 28, \quad y_3(0) = 34, \quad y_4(0) = 27$$

Fig. 2 illustrates the complete synchronization of the identical hyperchaotic Liu systems (8) and (9).

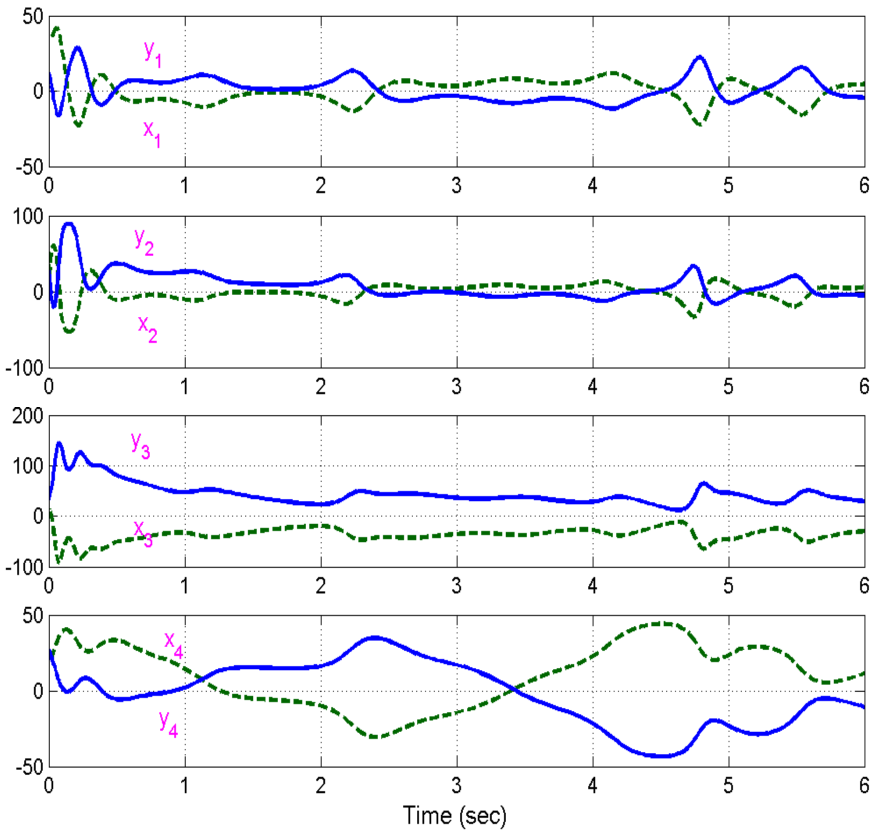

Figure 2. Anti-Synchronization of Identical Hyperchaotic Liu Systems

IV. ANTI-SYNCHRONIZATION OF IDENTICAL HYPERCHAOTIC QI SYSTEMS

A. *Theoretical Results*

In this section, we apply the active control method for the global chaos anti-synchronization of identical hyperchaotic Qi systems.

The hyperchaotic Qi system is one of the paradigms of the four-dimensional hyperchaotic systems discovered by G. Qi, M.A. Wyk, B.J. Wyk and G. Chen ([23], 2008).

Thus, the master system is described by the hyperchaotic Qi dynamics

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) + x_2 x_3 \\ \dot{x}_2 &= \beta(x_1 + x_2) - x_1 x_3 \\ \dot{x}_3 &= -\gamma x_3 - \varepsilon x_4 + x_1 x_2 \\ \dot{x}_4 &= -\delta x_4 + f x_3 + x_1 x_2 \end{aligned} \tag{16}$$

where x_1, x_2, x_3, x_4 are state variables of the system and $\alpha, \beta, \gamma, \delta, \varepsilon, f$ are positive, constant parameters of the system.

The four-dimensional system (16) is hyperchaotic when the parameter values are taken as

$$\alpha = 50,\ \beta = 24,\ \gamma = 13,\ \delta = 8,\ \varepsilon = 33 \text{ and } f = 30.$$

The state orbits of the hyperchaotic Qi system (16) are illustrated in Fig. 3.

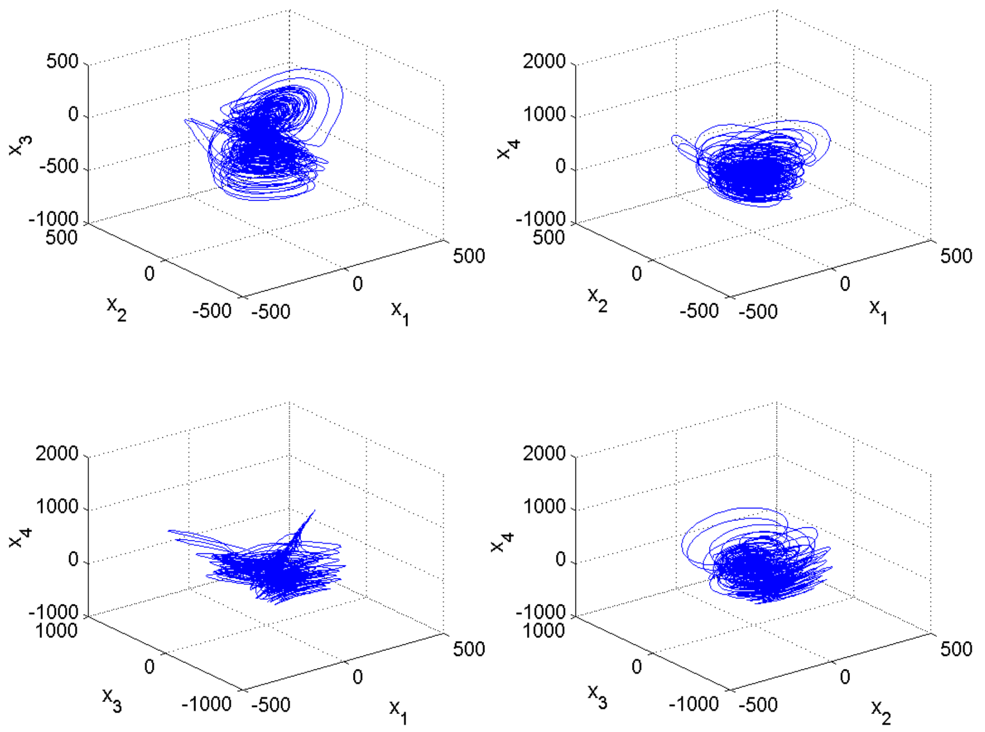

Figure 3. State Orbits of the Hyperchaotic Qi System

The slave system is described by the controlled hyperchaotic Qi dynamics

$$\begin{aligned}
\dot{y}_1 &= \alpha(y_2 - y_1) + y_2 y_3 + u_1 \\
\dot{y}_2 &= \beta(y_1 + y_2) - y_1 y_3 + u_2 \\
\dot{y}_3 &= -\gamma y_3 - \varepsilon y_4 + y_1 y_2 + u_3 \\
\dot{y}_4 &= -\delta y_4 + f y_3 + y_1 y_2 + u_4
\end{aligned} \tag{17}$$

where y_1, y_2, y_3, y_4 are state variables and u_1, u_2, u_3, u_4 are the controllers to be designed.

The anti-synchronization error is defined by

$$e_i = y_i + x_i, \quad (i = 1, 2, 3, 4) \tag{18}$$

The error dynamics is easily obtained as

$$\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + y_2 y_3 + x_2 x_3 + u_1 \\
\dot{e}_2 &= \beta(e_1 + e_2) - y_1 y_3 - x_1 x_3 + u_2 \\
\dot{e}_3 &= -\gamma e_3 - \varepsilon e_4 + y_1 y_2 + x_1 x_2 + u_3 \\
\dot{e}_4 &= -\delta e_4 + f e_3 + y_1 y_2 + x_1 x_2 + u_4
\end{aligned} \tag{19}$$

We choose the active nonlinear controller as

$$\begin{aligned} u_1 &= -\alpha e_2 - (a-\alpha)(x_2 - x_1) - y_2 y_3 \\ u_2 &= -\beta e_1 - (\beta+1)e_2 - (b-\beta)x_1 + \beta x_2 + x_4 + y_1 y_3 - x_1 x_3 \\ u_3 &= \varepsilon e_4 - (\gamma - c)x_3 - (\varepsilon+1)x_4 - y_1 y_2 + x_1 x_2 \\ u_4 &= -fe_3 - dx_1 - x_2 + fx_3 - dx_4 - y_1 y_2 \end{aligned} \tag{20}$$

Substituting (20) into (19), we obtain the linear error system

$$\begin{aligned} \dot{e}_1 &= -\alpha e_1 \\ \dot{e}_2 &= -e_2 \\ \dot{e}_3 &= -\gamma e_3 \\ \dot{e}_4 &= -\delta e_4 \end{aligned} \tag{21}$$

Next, we consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{22}$$

Differentiating V along the trajectories of (21), we get

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \gamma e_3^2 - \delta e_4^2, \tag{23}$$

which is a negative definite function on R^4.

Thus, the error dynamics (21) is globally exponentially stable for all initial conditions $e(0) \in R^4$.

Hence, we obtain the following result.

Theorem 2. The identical hyperchaotic Qi systems (16) and (17) are globally and exponentially anti-synchronized for all initial conditions by the active nonlinear controller defined by (20). ■

B. Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic Qi chaotic systems (16) and (17) with the active controller u given by (20) using MATLAB.

Also, the parameter values are taken as $\alpha = 50,\ \beta = 24,\ \gamma = 13,\ \delta = 8,\ \varepsilon = 33$ and $f = 30.$

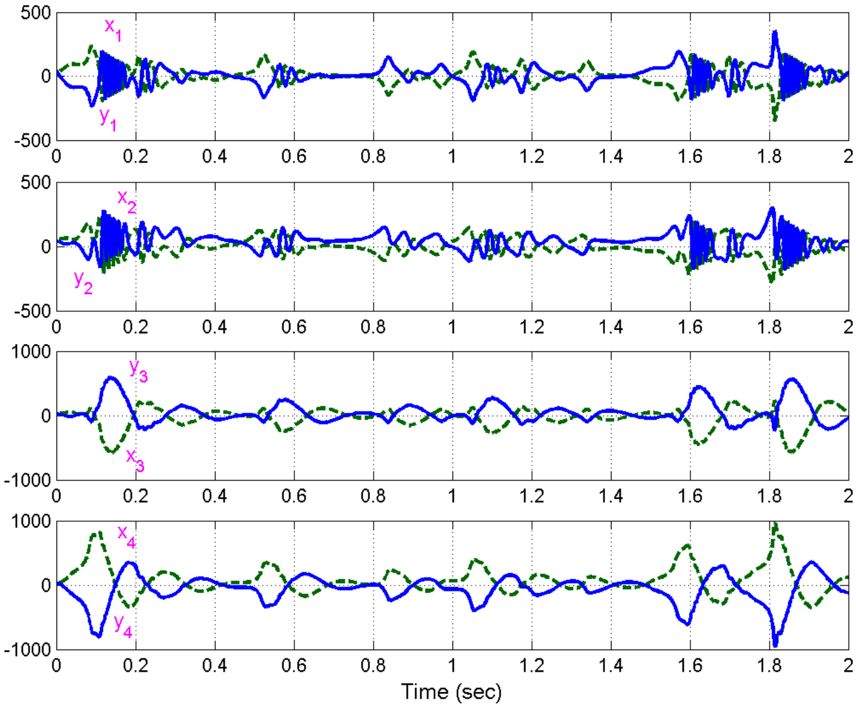

Figure 4. Anti-Synchronization of Identical Hyperchaotic Qi Systems

The initial values of the master system (21) are taken as

$$x_1(0) = 12, \quad x_2(0) = 44, \quad x_3(0) = 32, \quad x_4(0) = 16$$

and the initial values of the slave system (22) are taken as

$$y_1(0) = 42, \quad y_2(0) = 38, \quad y_3(0) = 17, \quad y_4(0) = 25$$

Fig. 4 illustrates the complete synchronization of the identical hyperchaotic Qi systems (21) and (22).

V. Anti-synchronization of Hyperchaotic Liu and Hyperchaotic Qi Systems

A. *Theoretical Results*

In this section, we apply the active control method for the global chaos anti-synchronization of non-identical hyperchaotic Liu and hyperchaotic Qi systems. We take the hyperchaotic Li system ([22], 2008) as the master system and the hyperchaotic Qi system ([23], 2008) s the slave system.

Thus, the master system is described by the hyperchaotic Liu dynamics

$$\begin{aligned} \dot{x}_1 &= a(x_2 - x_1) \\ \dot{x}_2 &= bx_1 + x_1 x_3 - x_4 \\ \dot{x}_3 &= -x_1 x_2 - cx_3 + x_4 \\ \dot{x}_4 &= dx_1 + x_2 \end{aligned} \tag{24}$$

where x_1, x_2, x_3, x_4 are state variables and a, b, c, d are positive, constant parameters of the system.

The slave system is described by the controlled hyperchaotic Qi dynamics

$$\begin{aligned}
\dot{y}_1 &= \alpha(y_2 - y_1) + y_2 y_3 + u_1 \\
\dot{y}_2 &= \beta(y_1 + y_2) - y_1 y_3 + u_2 \\
\dot{y}_3 &= -\gamma y_3 - \varepsilon y_4 + y_1 y_2 + u_3 \\
\dot{y}_4 &= -\delta y_4 + f y_3 + y_1 y_2 + u_4
\end{aligned} \tag{25}$$

where y_1, y_2, y_3, y_4 are state variables, $\alpha, \beta, \gamma, \delta, \varepsilon, f$ are positive, constant parameters of the system and u_1, u_2, u_3, u_4 are the controllers to be designed.

The anti-synchronization error is defined by

$$\begin{aligned}
e_1 &= y_1 + x_1 \\
e_2 &= y_2 + x_2 \\
e_3 &= y_3 + x_3 \\
e_4 &= y_4 + x_4
\end{aligned} \tag{26}$$

The error dynamics is easily obtained as

$$\begin{aligned}
\dot{e}_1 &= \alpha(e_2 - e_1) + (a - \alpha)(x_2 - x_1) + y_2 y_3 + u_1 \\
\dot{e}_2 &= \beta(e_1 + e_2) + (b - \beta)x_1 - \beta x_2 - x_4 - y_1 y_3 + x_1 x_3 + u_2 \\
\dot{e}_3 &= -\gamma e_3 - \varepsilon e_4 + (\gamma - c)x_3 + (\varepsilon + 1)x_4 + y_1 y_2 - x_1 x_2 + u_3 \\
\dot{e}_4 &= -\delta e_4 + f e_3 + d x_1 + x_2 - f x_3 + \delta x_4 + y_1 y_2 + u_4
\end{aligned} \tag{27}$$

We choose the active nonlinear controller as

$$\begin{aligned}
u_1 &= -\alpha e_2 - (a - \alpha)(x_2 - x_1) - y_2 y_3 \\
u_2 &= -\beta e_1 - (\beta + 1)e_2 - (b - \beta)x_1 + \beta x_2 + x_4 + y_1 y_3 - x_1 x_3 \\
u_3 &= \varepsilon e_4 - (\gamma - c)x_3 - (\varepsilon + 1)x_4 - y_1 y_2 + x_1 x_2 \\
u_4 &= -f e_3 - d x_1 - x_2 + f x_3 - \delta x_4 - y_1 y_2
\end{aligned} \tag{28}$$

Substituting (28) into (27), we obtain the linear error system

$$\begin{aligned}
\dot{e}_1 &= -a e_1 \\
\dot{e}_2 &= -e_2 \\
\dot{e}_3 &= -c e_3 \\
\dot{e}_4 &= -e_4
\end{aligned} \tag{29}$$

Next, we consider the quadratic Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_4^2\right), \tag{30}$$

which is a positive definite function on R^4.

Differentiating V along the trajectories of (29), we get

$$\dot{V}(e) = -a e_1^2 - e_2^2 - c e_3^2 - e_4^2, \tag{31}$$

which is a negative definite function on R^4.

Thus, the error dynamics (29) is globally exponentially stable for all initial conditions $e(0) \in R^n$.

Hence, we obtain the following result.

Theorem 3. The non-identical hyperchaotic Liu system (24) and hyperchaotic Qi system (25) are globally and exponentially anti-synchronized for all initial conditions by the active nonlinear controller defined by (28). ■

B. Numerical Results

For the numerical simulations, the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ is used to solve the hyperchaotic systems (24) and (25) with the active controller u given by (28) using MATLAB.

For the hyperchaotic Liu system (24), the parameter values are taken as

$$a = 10, \quad b = 35, \quad c = 1.4 \quad \text{and} \quad d = 5.$$

For the hyperchaotic Qi system (25), the parameter values are taken as

$$\alpha = 50, \ \beta = 24, \ \gamma = 13, \ \delta = 8, \ \varepsilon = 33 \text{ and } f = 30.$$

The initial values of the master system (21) are taken as

$$x_1(0) = 25, \quad x_2(0) = 10, \quad x_3(0) = 38, \quad x_4(0) = 48$$

and the initial values of the slave system (22) are taken as

$$y_1(0) = 14, \quad y_2(0) = 20, \quad y_3(0) = 34, \quad y_4(0) = 12$$

Fig. 5 illustrates the complete synchronization of the non-identical hyperchaotic Liu system (24) and hyperchaotic Qi system (25).

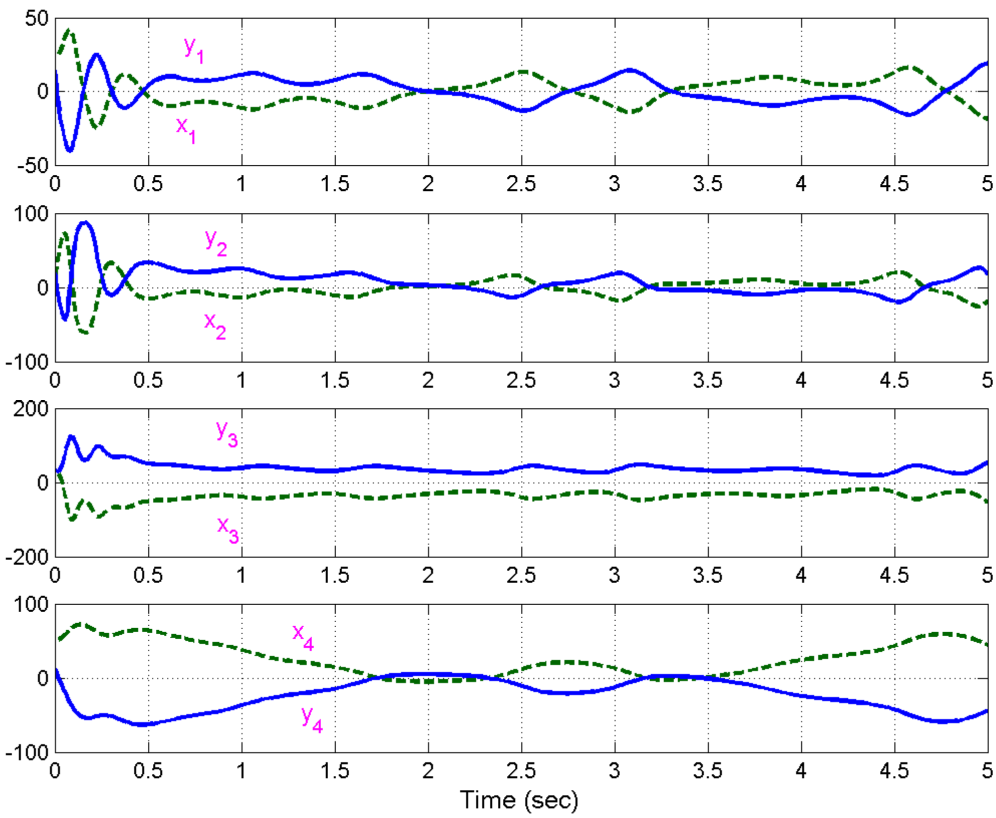

Figure 5. Anti-Synchronization of Non-Identical Hyperchaotic Liu and Hyperchaotic Qi Systems

VI. CONCLUSIONS

In this paper, we have applied active control method for the derivation of state feedback control laws so as to achieve global chaos anti-synchronization of identical hyperchaotic Liu systems (2008), identical hyperchaotic Qi systems (2008) and non-identical hyperchaotic Liu and Qi systems. Our anti-synchronization results have been proved using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the active control method is very effective and convenient to achieve global chaos anti-synchronization for the identical and non-identical hyperchaotic Liu and Qi systems. Numerical simulations have been shown to demonstrate the effectiveness of the anti-synchronization schemes derived in this paper.

REFERENCES

[1] K.T. Alligood, T. Sauer and J.A. Yorke, Chaos: An Introduction to Dynamical Systems, Springer, New York, 1997.
[2] L.M. Pecora and T.L. Carroll, “Synchronization in chaotic systems,” Physical Review Letters, vol. 64, pp. 821-824, 1990.
[3] M. Lakshmanan and K. Murali, Nonlinear Oscillators: Controlling and Synchronization, World Scientific, Singapore, 1996.
[4] S.K. Han, C. Kerrer and Y. Kuramoto, “Dephasing and bursting in coupled neural oscillators,” Physical Review Letters, vol. 75, pp. 3190-3193, 1995.
[5] B. Blasius, A. Huppert and L. Stone, “Complex dynamics and phase synchronization in spatially extended ecological system,” Nature, vol. 399, pp. 354-359, 1999.
[6] K.M. Cuomo and A.V. Oppenheim, “Circuit implementation of synchronized chaos with applications to communications,” Physical Review Letters, vol. 71, pp. 65-68, 1993.
[7] L. Kocarev and U. Parlitz, “General approach for chaotic synchronization with applications to communication,” Physical Review Letters, vol. 74, pp. 5028-5030, 1995.
[8] Y. Tao, “Chaotic secure communication systems – history and new results,” Telecommunication Review, vol. 9, pp. 597-634, 1999.
[9] E. Ott, C. Grebogi and J.A. Yorke, “Controlling chaos,” Physical Review Letters, vol. 64, pp. 1196-1199, 1990.
[10] M.C. Ho and Y.C. Hung, “Synchronization of two different chaotic systems using generalized active control,” Physics Letters A, vol. 301, pp. 424-428, 2002.
[11] L. Huang, R. Feng and M. Wang, “Synchronization of chaotic systems via nonlinaer control,” Physics Letters A, vol. 320, pp. 271-275, 2005.
[12] H.K. Chen, “Global chaos synchronization of new chaotic systems via nonlinear control,” Chaos, Solitons and Fractals, vol. 23, pp. 1245-1251, 2005.
[13] J. Lu, X. Wu, X. Han and J. Lü, “Adaptive feedback synchronization of a unified chaotic system,” Physics Letters A, vol. 329, pp. 327-333, 2004.
[14] S.H. Chen and J. Lü, “Synchronization of an uncertain unified system via adaptive control,” Chaos, Solitons and Fractals, vol. 14, pp. 643-647, 2002.
[15] J.H. Park and O.M. Kwon, “A novel criterion for delayed feedback control of time-delay chaotic systems,” Chaos, Solitons and Fractals, vol. 17, pp. 709-716, 2003.
[16] X. Wu and J. Lü, “Parameter identification and backstepping control of uncertain Lü system,” Chaos, Solitons and Fractals, vol. 18, pp. 721-729, 2003.
[17] J. Zhao and J. Lü, “Using sampled-data feedback control and linear feedback synchronization in a new hyperchaotic system,” Chaos, Solitons and Fractals, vol. 35, pp. 376-382, 2006.
[18] H.T. Yau, “Design of adaptive sliding mode controller for chaos synchronization with uncertainties”, Chaos, Solitons and Fractals, vol. 22, pp. 341-347, 2004.
[19] G.H. Li, “Synchronization and anti-synchronization of Colpitts oscillators using active control”, Chaos, Solitons and Fractals, vol. 26, pp. 87-93, 2005.
[20] J. Hu, S. Chen and L. Chen, “Adaptive control for anti-synchronization of Chua’s chaotic system”, Phys. Lett. A, vol. 339, pp. 455-460, 2005.
[21] X. Zhang and H. Zhu, “Anti-synchronization of two different hyperchaotic systems via active and adaptive control”, Internat. J. Nonlinear Science, vol. 6, pp. 216-223, 2008.
[22] L. Liu, C. Liu and Y. Zhang, “Analysis of a novel four-dimensional hyperchaotic system,” Chinese Journal of Physics, vol. 46, no. 4, pp. 386-393, 2008.
[23] G. Qi, M.A. Wyk, B.J. Wyk and G. Chen, “On a new hyperchaotic system,” Physics Letters A, vol. 372, pp. 124-136, 2008.
[24] W. Hahn, The Stability of Motion, Springer, New York, 1967.

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