Global Chaos Synchronization of Four-Scroll and Four-Wing Attractors by Active Nonlinear Control

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***Abstract*—This paper investigates the global chaos synchronization of identical four-scroll attractors (Liu and Chen, 2004), identical four-wing attractors (Liu, 2009) and non-identical four-scroll and four-wing attractors by active nonlinear control. The stability results derived in this paper for the global chaos synchronization using active nonlinear control are established using Lyapunov stability theory. Since the Lyapunov exponents are not required for these calculations, the proposed active nonlinear control method is very effective and convenient to achieve global chaos synchronization of four-scroll and four-wing chaotic attractors. Numerical simulations are shown to illustrate the effectiveness of the synchronization schemes derived in this paper for the identical and non-identical four-scroll and four-wing chaotic attractors.**

Keywords- chaos synchronization; active nonlinear control; four-scroll attractor; four-wing attractor.

I. INTRODUCTION

Chaotic systems are dynamical systems that are highly sensitive to initial conditions. This sensitivity is popularly known as the butterfly effect [1].

Chaos synchronization problem was first described by Fujisaka and Yemada [2] in 1983. This problem did not receive great attention until Pecora and Carroll [3-4] published their results on chaos synchronization in early 1990s. From then on, chaos synchronization has been studied extensively and intensively in the literature in the last three decades [3-22]. Chaos theory has been explored in a variety of fields including physical systems [5], chemical systems [6], ecological systems [7], secure communications [8-10], etc.

Synchronization of chaotic systems is a phenomenon that may occur when two or more chaotic oscillators are coupled or when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect, which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the chaos synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the idea of the chaos synchronization is to use the output of the master system to control the slave system so that the output of the slave system tracks the output of the master system asymptotically.

Since the pioneering work by Pecora and Carroll [3-4], a variety of impressive techniques have been proposed for the synchronization of chaotic systems such as PC method [3-4], the sampled-data feedback synchronization method [10-11], OGY method [12], time-delay feedback method [13], active control method [14-15], backstepping design method [16], adaptive control method [17-20], sliding mode control method [21], hyperchaos [22], etc.

In this paper, we derive new results for the global chaos synchronization for identical and different four-scroll chaotic attractors ([23], Liu and Chen, 2004) and four-wing chaotic attractors ([24], Liu, 2009) using active nonlinear control. The stability results derived in this paper for the global chaos synchronization have been established using Lyapunov stability theory.

This paper has been organized as follows. In Section II, we detail the problem statement and our methodology. In Section III, we detail the global chaos synchronization of two identical four-scroll chaotic attractors ([23], Liu and Chen, 2004) using active nonlinear control. In Section IV, we detail the global chaos synchronization of two identical four-wing chaotic attractors ([24], Liu, 2009) using active nonlinear control. In Section V, we detail the global chaos synchronization of the non-identical four-scroll and four-wing chaotic attractors using active nonlinear control. In Section VI, we summarize the main results obtained in this paper.

II. PROBLEM STATEMENT AND OUR METHODOLOGY USING ACTIVE NONLINEAR CONTROL

Consider the chaotic system described by the dynamics

$$\dot{x} = Ax + f(x) \tag{1}$$

where $x \in R^n$ is the state of the system, A is the $n \times n$ matrix of system parameters and $f : R^n \to R^n$ is the nonlinear part of the system. We consider the system (1) as the master or drive system.

As the slave or response system, we consider the following chaotic system described by the dynamics

$$\dot{y} = By + g(y) + u \tag{2}$$

where $y \in R^n$ is the state of the system, B is the $n \times n$ matrix of system parameters, $g : R^n \to R^n$ is the nonlinear part of the system and $u \in R^n$ is the controller of the slave system.

If $A = B$ and $f = g$, then x and y are the states of two *identical* chaotic systems. If $A \neq B$ and $f \neq g$, then x and y are the states of two *different* chaotic systems.

In the nonlinear feedback control approach, we design a feedback controller u, which synchronizes the states of the master system (1) and the slave system (2) for all initial conditions $x(0), z(0) \in R^n$.

If we define the synchronization error as

$$e = y - x, \tag{3}$$

then the error dynamics is obtained as

$$\dot{e} = By - Ax + g(y) - f(x) + u, \tag{4}$$

Thus, the global chaos synchronization problem is essentially to find a feedback controller u so as to stabilize the error dynamics (4) for all initial conditions $e(0) \in R^n$.

Hence, we find a feedback controller u so that

$$\lim_{t \to \infty} \|e(t)\| = 0 \quad \text{for all } e(0) \in R^n \tag{5}$$

We take as a candidate Lyapunov function

$$V(e) = e^T P e,$$

where P is a positive definite matrix.

We note that $V : R^n \to R$ is a positive definite function by construction.

We assume that the parameters of the master system (1) and slave system (2) are known.

We also assume that the states of both systems (1) and (2) are measurable.

If we find a feedback controller u so that

$$\dot{V}(e) = -e^T Q e,$$

where Q is a positive definite matrix, then $\dot{V} : R^n \to R$ is a negative definite function.

Thus, by Lyapunov stability theory [25], the error dynamics (4) is globally exponentially stable and hence the condition (5) will be satisfied.

Hence, the states of the master system (1) and slave system (2) will be globally and exponentially synchronized for all initial conditions $x(0), y(0) \in R^n$.

III. GLOBAL CHAOS SYNCHRONIZATION OF IDENTICAL FOUR-SCROLL ATTRACTORS

A. Theoretical Results

In this section, we apply the active nonlinear control method for the global chaos synchronization of two identical four-scroll attractors ([23], Liu and Chen, 2004). The Liu-Chen four-scroll attractor is one of the paradigms of the three-dimensional chaotic systems proposed by W. Liu and G. Chen (2004).

Thus, the master system is described by the Liu-Chen dynamics as

$$\begin{aligned} \dot{x}_1 &= ax_1 - x_2 x_3 \\ \dot{x}_2 &= -bx_2 + x_1 x_3 \\ \dot{x}_3 &= -cx_3 + x_1 x_2 \end{aligned} \tag{6}$$

where x_1, x_2, x_3 are the states of the system and a, b, c are positive, constant parameters of the system such that $b + c > a.$.

The slave system is also described by the Liu-Chen dynamics as

$$\begin{aligned} \dot{y}_1 &= ay_1 - y_2 y_3 + u_1 \\ \dot{y}_2 &= -by_2 + y_1 y_3 + u_2 \\ \dot{y}_3 &= -cy_3 + y_1 y_2 + u_3 \end{aligned} \tag{7}$$

where y_1, y_2, y_3 are the states of the system and u_1, u_2, u_3 are the active nonlinear controls to be designed.

The Liu-Chen system (6) is chaotic when $a = 0.4,\ b = 12$ and $c = 5.$ Fig. 1 illustrates the chaotic portrait of the Liu-Chen four-scroll attractor (6).

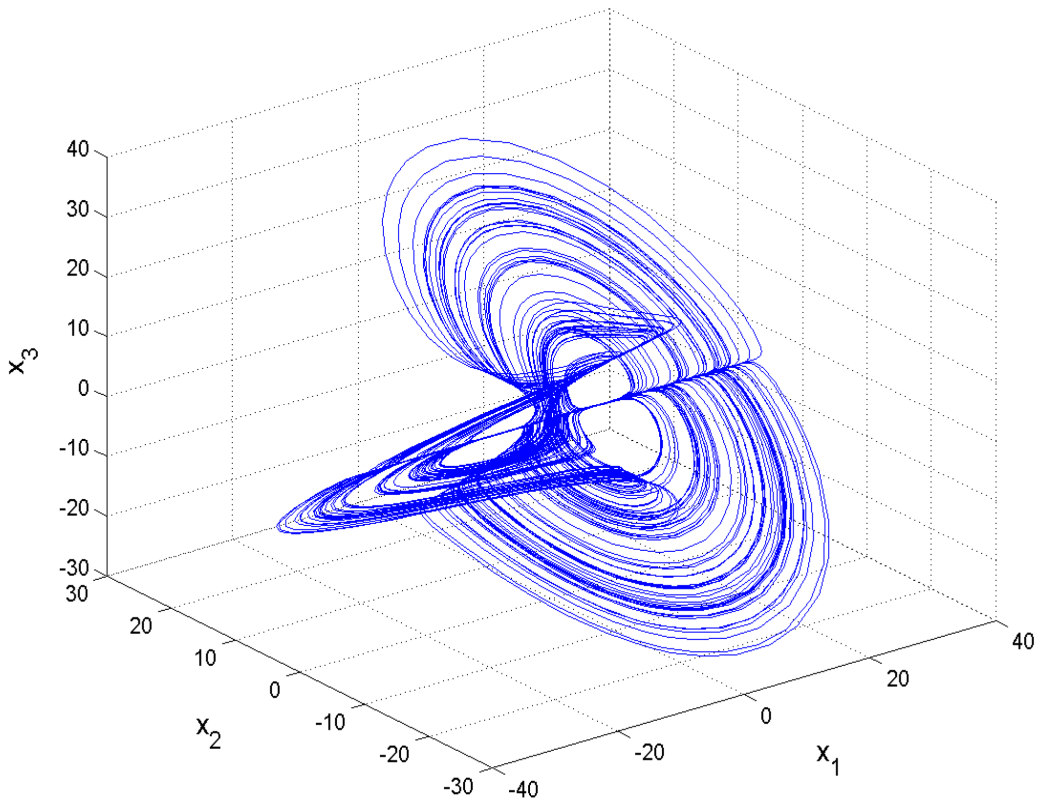

Figure 1. Chaotic Portrait of the Liu-Chen Four-Scroll Attractor

The synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3) \tag{8}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= ae_1 - y_2 y_3 + x_2 x_3 + u_1 \\ \dot{e}_2 &= -be_2 + y_1 y_3 - x_1 x_3 + u_2 \\ \dot{e}_3 &= -ce_3 + y_1 y_2 - x_1 x_2 + u_3 \end{aligned} \tag{9}$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= -(a+1)e_1 + y_2 y_3 - x_2 x_3 \\ u_2 &= -y_1 y_3 + x_1 x_3 \\ u_3 &= -y_1 y_2 + x_1 x_2 \end{aligned} \tag{10}$$

Substituting (10) into (9), we obtain the linear system

$$\begin{aligned} \dot{e}_1 &= -e_1 \\ \dot{e}_2 &= -be_2 \\ \dot{e}_3 &= -ce_3 \end{aligned} \tag{11}$$

We consider the candidate Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{12}$$

which is a positive definite function on R^3.

Differentiating (12) along the trajectories of (11), we get

$$\dot{V}(e) = -e_1^2 - be_2^2 - ce_3^2,$$

which is a negative definite function on R^3.

Thus, the error dynamics (11) is globally exponentially stable and hence we arrive at the following result.

Theorem 1. The identical Liu-Chen four-scroll chaotic attractors (7) and (8) are globally and exponentially synchronized for all initial conditions with the nonlinear controller u defined by (10). ■

B. Numerical Results

For numerical simulations, we use the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ to solve the differential equations (7) and (8) with the active nonlinear controller defined by (10).

The parameters of the Liu-Chen four-scroll chaotic attractor are chosen as

$$a = 0.4, \ b = 12 \ \text{ and } \ c = 5$$

so that the systems (7) and (8) are chaotic.

The initial conditions of the master system (7) are chosen as

$$x_1(0) = 20, \ \ x_2(0) = 12, \ \ x_3(0) = 18.$$

The initial conditions of the slave system (8) are chosen as

$$y_1(0) = 10, \ \ y_2(0) = 21, \ y_3(0) = 26.$$

Fig. 2 shows the complete synchronization of the master system (7) and the slave system (8).

Thus, we have verified that the identical Liu-Chen four-scroll chaotic attractors (7) and (8) are globally and exponentially synchronized by the active nonlinear controller defined by (10).

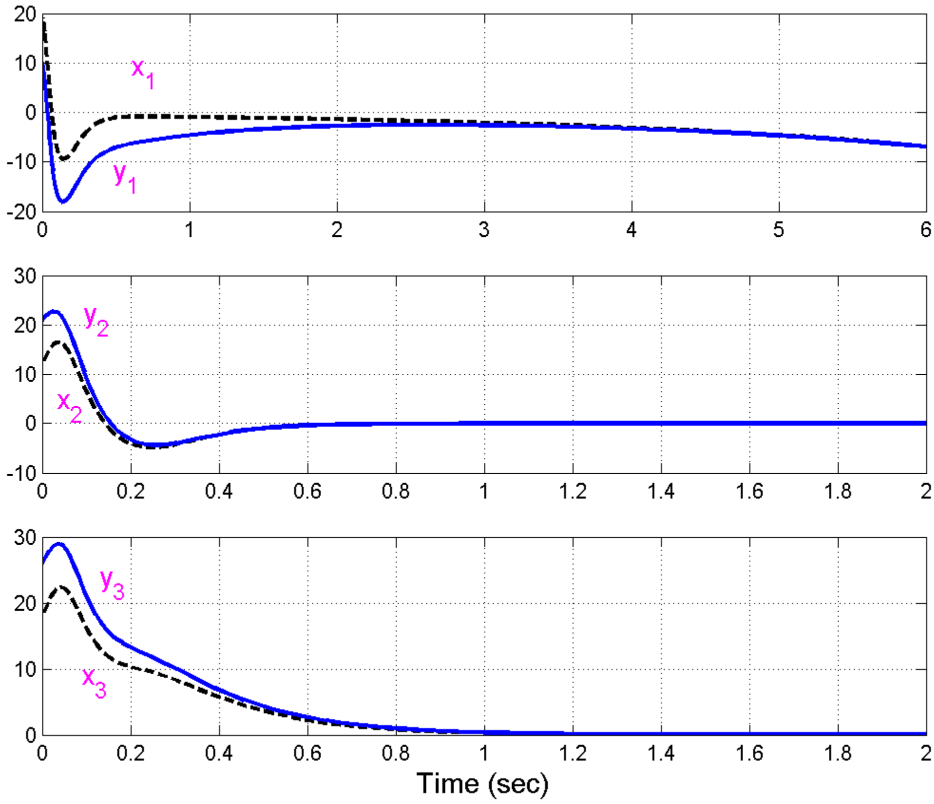

Figure 2. Complete Synchronization of the Identical Liu-Chen Four-Scroll Attractors

IV. Global Chaos Synchronization of Identical Four-Wing Attractors

A. Theoretical Results

In this section, we apply the active nonlinear control method for the global chaos synchronization of two identical four-wing attractors ([24], Liu, 2009). The Liu four-wing attractor is one of the paradigms of the three-dimensional chaotic systems proposed by X. Liu (2009).

Thus, the master system is described by the Liu dynamics as

$$\begin{aligned} \dot{x}_1 &= \alpha(x_2 - x_1) + x_2 x_3^2 \\ \dot{x}_2 &= \beta(x_1 + x_2) - x_1 x_3^2 \\ \dot{x}_3 &= -\gamma x_3 + \varepsilon x_2 + x_1 x_2 x_3 \end{aligned} \tag{13}$$

where x_1, x_2, x_3 are the states of the system and $\alpha, \beta, \gamma, \varepsilon$ are positive, constant parameters of the system.

The slave system is also described by the Liu dynamics as

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + y_2 y_3^2 + u_1 \\ \dot{y}_2 &= \beta(y_1 + y_2) - y_1 y_3^2 + u_2 \\ \dot{y}_3 &= -\gamma y_3 + \varepsilon y_2 + y_1 y_2 y_3 + u_3 \end{aligned} \tag{14}$$

where y_1, y_2, y_3 are the states of the system and u_1, u_2, u_3 are the active nonlinear controls to be designed.

The Liu four-wing system (13) is chaotic when $\alpha = 50,\ \beta = 13,\ \gamma = 13$ and $\varepsilon = 6.$ Fig. 3 illustrates the chaotic portrait of the Liu four-wing attractor (13).

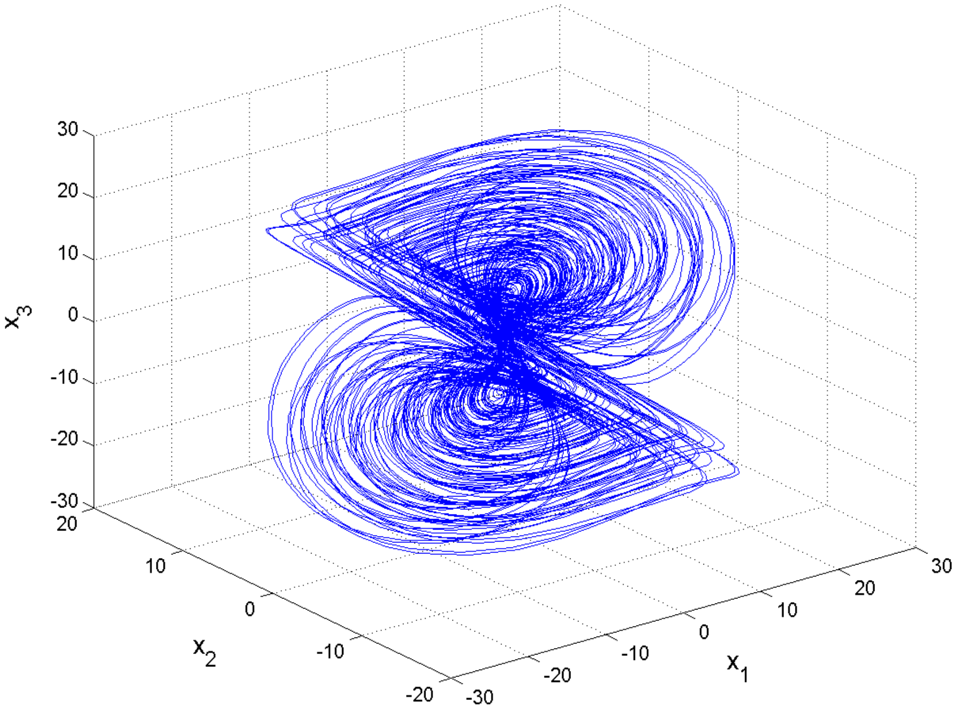

Figure 3. Chaotic Portrait of the Liu Four-Wing Attractor

The synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3) \tag{15}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + y_2 y_3^2 - x_2 x_3^2 + u_1 \\ \dot{e}_2 &= \beta(e_1 + e_2) - y_1 y_3^2 + x_1 x_3^2 + u_2 \\ \dot{e}_3 &= -\gamma e_3 + \varepsilon e_2 + y_1 y_2 y_3 - x_1 x_2 x_3 + u_3 \end{aligned} \tag{16}$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= -\alpha\alpha_2 - y_2 y_3^2 + x_2 x_3^2 \\ u_2 &= -\beta e_1 - (\beta + 1)e_2 + y_1 y_3^2 - x_1 x_3^2 \\ u_3 &= -\varepsilon e_2 - y_1 y_2 y_3 + x_1 x_2 x_3 \end{aligned} \tag{17}$$

Substituting (17) into (16), we obtain the linear system

$$\begin{aligned} \dot{e}_1 &= -\alpha e_1 \\ \dot{e}_2 &= -e_2 \\ \dot{e}_3 &= -\gamma e_3 \end{aligned} \tag{18}$$

We consider the candidate Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{19}$$

which is a positive definite function on R^3.

Differentiating (19) along the trajectories of (18), we get

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \gamma e_3^2,$$

which is a negative definite function on R^3.

Thus, the error dynamics (18) is globally exponentially stable and hence we arrive at the following result.

Theorem 2. The identical Liu four-wing chaotic attractors (13) and (14) are globally and exponentially synchronized for all initial conditions with the nonlinear controller u defined by (17). ■

B. Numerical Results

For numerical simulations, we use the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ to solve the differential equations (13) and (14) with the active nonlinear controller defined by (17).

The parameters of the Liu four-wing chaotic attractor are chosen as

$$\alpha = 50, \ \beta = 13, \ \gamma = 13 \quad \text{and} \quad \varepsilon = 6$$

so that the systems (13) and (14) are chaotic.

The initial conditions of the master system (13) are chosen as

$$x_1(0) = 6, \ \ x_2(0) = 30, \ \ x_3(0) = 12.$$

The initial conditions of the slave system (14) are chosen as

$$y_1(0) = 20, \ \ y_2(0) = 12, \ y_3(0) = 22.$$

Fig. 4 shows the complete synchronization of the master system (13) and the slave system (14).

Thus, we have verified that the identical Liu-Chen four-scroll chaotic attractors (13) and (14) are globally and exponentially synchronized by the active nonlinear controller defined by (17).

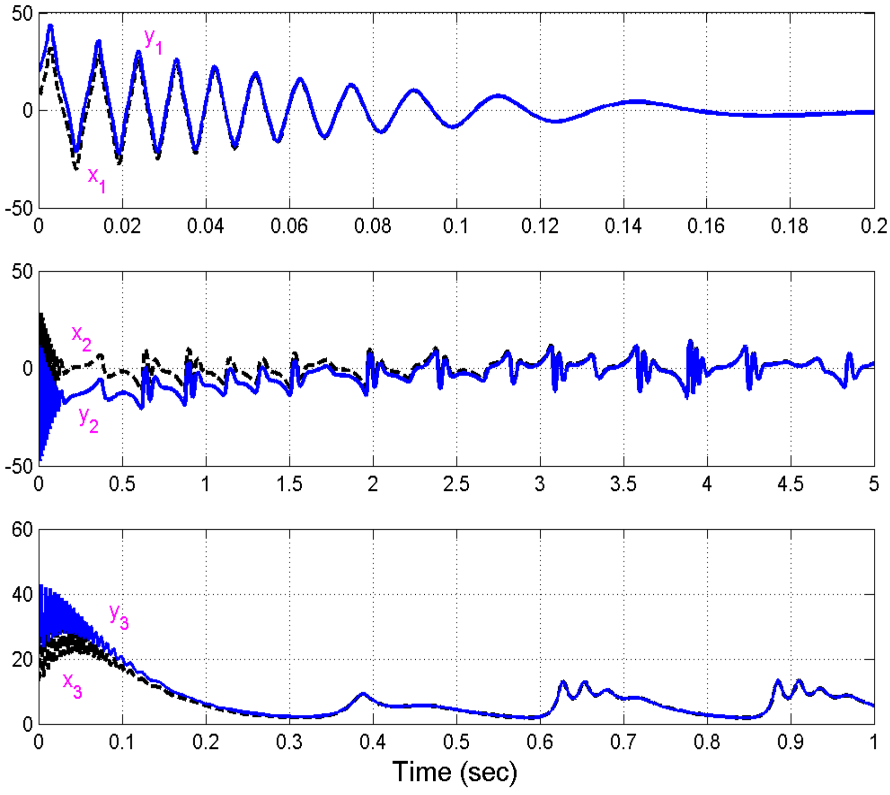

Figure 4. Complete Synchronization of the Identical Liu Four-Wing Attractors

V. Global Chaos Synchronization of Four-Scroll and Four-Wing Attractors

A. *Theoretical Results*

In this section, we apply the active nonlinear control method for the global chaos synchronization of two non-identical chaotic systems, viz. the Liu-Chen four-scroll attractor ([23], Liu and Chen, 2004) as the master system and Liu four-wing attractor ([24], Liu, 2009) as the slave system.

Thus, the master system is described by the Liu-Chen dynamics as

$$\begin{aligned} \dot{x}_1 &= ax_1 - x_2x_3 \\ \dot{x}_2 &= -bx_2 + x_1x_3 \\ \dot{x}_3 &= -cx_3 + x_1x_2 \end{aligned} \tag{20}$$

where x_1, x_2, x_3 are the states of the system and a, b, c are positive, constant parameters of the system.

The slave system is described by the Liu dynamics as

$$\begin{aligned} \dot{y}_1 &= \alpha(y_2 - y_1) + y_2y_3^2 + u_1 \\ \dot{y}_2 &= \beta(y_1 + y_2) - y_1y_3^2 + u_2 \\ \dot{y}_3 &= -\gamma y_3 + \varepsilon y_2 + y_1y_2y_3 + u_3 \end{aligned} \tag{21}$$

where y_1, y_2, y_3 are the states of the system, $\alpha, \beta, \gamma, \varepsilon$ are constant parameters of the system and u_1, u_2, u_3 are the active nonlinear controls to be designed.

The synchronization error is defined by

$$e_i = y_i - x_i, \quad (i = 1, 2, 3) \tag{22}$$

The error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= \alpha(e_2 - e_1) + \alpha x_2 - (\alpha + a)x_1 + y_2y_3^2 + x_2x_3 + u_1 \\ \dot{e}_2 &= \beta(e_1 + e_2) + \beta x_1 + (\beta + b)x_2 - y_1y_3^2 - x_1x_3 + u_2 \\ \dot{e}_3 &= -\gamma e_3 + \varepsilon e_2 + \varepsilon x_2 + (c - \gamma)x_3 + y_1y_2y_3 - x_1x_2 + u_3 \end{aligned} \tag{23}$$

We choose the nonlinear controller as

$$\begin{aligned} u_1 &= -\alpha e_2 - \alpha x_2 + (\alpha + a)x_1 - y_2y_3^2 - x_2x_3 \\ u_2 &= -\beta e_1 - (\beta + 1)e_2 - \beta x_1 - (\beta + b)x_2 + y_1y_3^2 + x_1x_3 \\ u_3 &= -\varepsilon e_2 - \varepsilon x_2 - (c - \gamma)x_3 - y_1y_2y_3 + x_1x_2 \end{aligned} \tag{24}$$

Substituting (17) into (16), we obtain the linear system

$$\begin{aligned} \dot{e}_1 &= -\alpha e_1 \\ \dot{e}_2 &= -e_2 \\ \dot{e}_3 &= -\gamma e_3 \end{aligned} \tag{25}$$

We consider the candidate Lyapunov function defined by

$$V(e) = \frac{1}{2} e^T e = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2\right) \tag{26}$$

which is a positive definite function on R^3.

Differentiating (26) along the trajectories of (25), we get

$$\dot{V}(e) = -\alpha e_1^2 - e_2^2 - \gamma e_3^2,$$

which is a negative definite function on R^3.

Thus, the error dynamics (25) is globally exponentially stable and hence we arrive at the following result.

Theorem 3. The non-identical Liu-Chen four-scroll attractor (20) and Liu four-wing attractor (21) are globally and exponentially synchronized for all initial conditions with the nonlinear controller u defined by (24). ■

B. Numerical Results

For numerical simulations, we use the fourth-order Runge-Kutta method with time-step $h = 10^{-6}$ to solve the differential equations (20) and (21) with the active nonlinear controller defined by (24).

The parameters of the four-scroll attractor (20) and four-wing attractor (21) are chosen as

$$a = 0.4,\ b = 12,\ c = 5, \alpha = 50,\ \beta = 13,\ \gamma = 13 \text{ and } \varepsilon = 6$$

so that the systems are chaotic.

The initial conditions of the master system (20) are chosen as

$$x_1(0) = 24,\ \ x_2(0) = 10,\ \ x_3(0) = 12.$$

The initial conditions of the slave system (21) are chosen as

$$y_1(0) = 8,\ \ y_2(0) = 16,\ y_3(0) = 20.$$

Fig. 5 shows the complete synchronization of the master system (20) and the slave system (21).

Thus, we have verified that the identical Liu-Chen four-scroll chaotic attractors (20) and (21) are globally and exponentially synchronized by the active nonlinear controller defined by (24).

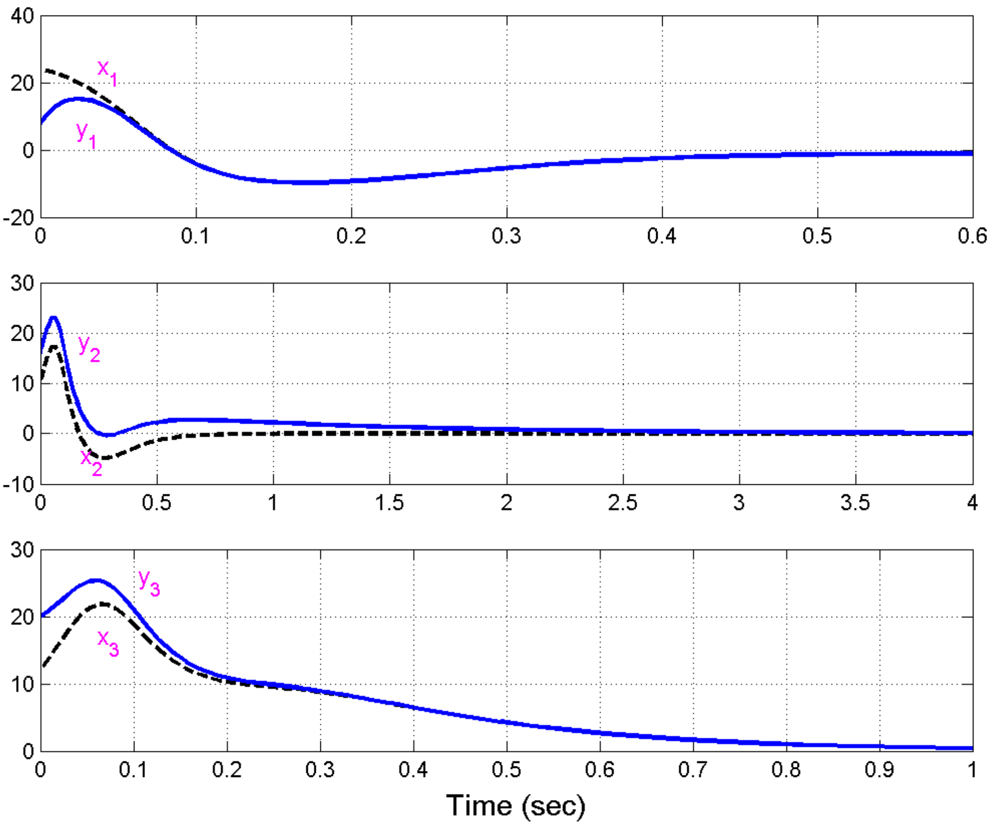

Figure 5. Complete Synchronization of the Four-Scroll and Four-Wing Chaotic Attractors

VI. CONCLUSIONS

In this paper, we have used active nonlinear control method based on Lyapunov stability theory to achieve global chaos synchronization for the following types of chaotic systems:

(A) Identical Liu-Chen four-scroll chaotic attractors (Liu and Chen, 2004)

(B) Identical Liu four-wing chaotic attractors (Liu, 2009)

(C) Non-identical Liu-Chen four-scroll and Liu four-wing chaotic attractors.

Liu-Chen four-scroll chaotic attractors and Liu four-wing chaotic attractors are recently discovered important paradigms of three-dimensional chaotic systems, which have applications in secure communication devices and secure data encryption. Numerical simulations have been given in detail to validate the proposed synchronization approach as well as to illustrate the effectiveness of the synchronization schemes derived in this paper for the three types of chaotic systems described in (A), (B) and (C). Since the Lyapunov exponents are not required for these calculations, the proposed active nonlinear control method is very effective and convenient to completely synchronize the chaotic systems discussed in this paper.

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