Fuzzy Shortest Path For Steiner Tree Problem

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Abstract— In this paper, a modification of the shortest path approximation based on the fuzzy shortest paths evaluations. The Steiner tree problem on a graph in which a fuzzy number instead of a real number is assigned to each edge. Here, to solve the fuzzy shortest path using a new approach ranking method.

Keywords- fuzzy ranking, single shortest path problem, Steiner tree.

I. INTRODUCTION

The Steiner tree problem in graphs [4], [5] is connected with a subset of vertices at a minimal cost. The given undirected connected graph G = (V, E,w) with a set of vertices V, a set of edges E and the nonnegative weights associated with edges w.The Steiner tree problem is superficially similar to the minimum spanning tree problem, given a set *V* of vertices, interconnect them by a graph of shortest length, where the length is the sum of the lengths of all edges. The difference between the Steiner tree problem and the minimum spanning tree problem is that, in the Steiner tree problem, extra intermediate vertices and edges may be added to the graph in order to reduce the length of the spanning tree. These new vertices introduced to decrease the total length of connection are known as *Steiner points or Steiner vertices.* It has been proved that the resulting connection is a tree, known as the *Steiner tree*. The Steiner tree problem has applications in circuit layout or network design. Most versions of the Steiner tree problem are NP-complete, i.e., thought to be computationally hard.

II. PRELIMINARIES

A fuzzy number a is an upper semi-continuous, normal and convex fuzzy subset of the real line R such that $\mu_{\tilde{a}}$: R→[0; 1] where $\mu_{\tilde{a}}$ (x) is the membership function of $\tilde{a}$, ∃x such that $\mu_{\tilde{a}}(x)=1$, $\mu_{\tilde{a}}(\lambda x1 + (1-\lambda)x2) \geq \mu_{\tilde{a}}(x1) \wedge \mu_{\tilde{a}}(x2)$ f or $\lambda \in [0; 1]$. The α-level set of $\tilde{a}$ is the set $(\tilde{a})_x = \{x \mid \mu_{\tilde{a}}(x) \geq \alpha\}$ where $\alpha \in (0; 1]$. The lower and upper bounds of any α-level set $(\tilde{a})_x$ are equal to $\inf_x \in R\ (\tilde{a})\alpha$ and $\sup_x \in R\ (\tilde{a})\alpha$. An L–R type fuzzy number is denoted as $(m;\alpha;\beta)_{LR}$, where α , β are the left- and right-hand spreads. Let a fuzzy number $\tilde{a}$ be positive if its membership function is such that $\mu_{\tilde{a}}(x)=0$; ∀x<0. As in the special case of L–R fuzzy numbers, triangular fuzzy numbers with linear reference functions are often used. Throughout this paper, triangular fuzzy number $\tilde{a}$ is denoted as $\tilde{a}$ =(a,b,c) or $\tilde{a}$ =(a1,a2,a3). Triangular fuzzy numbers are employed for numerical examples and for the easy computation.

A. Triangular Fuzzy Number

The fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ is a triangular number, denoted by (a_1, a_2, a_3), its membership function $\mu_{\tilde{a}}$ is given by

$$\mu_{\tilde{a}}(x) = \begin{cases} 0 & , if \ 0 \le x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1} & , if \ a_1 \le x \le a_2 \\ 1 & , if \ x = a_2 \\ \dfrac{x - a_3}{a_2 - a_3} & , if \ a_2 \le x \le a_3 \\ 0 & , if \ x \ge a_3 \end{cases}$$

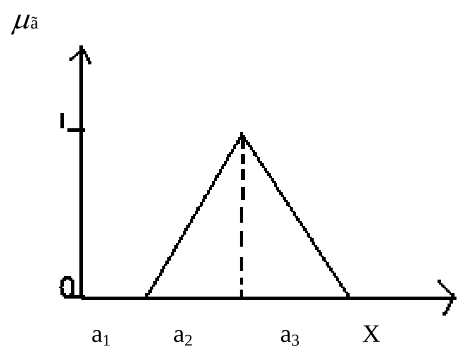


Fig.1. Membership function of a fuzzy number ã

B. Positive Fuzzy Number

A fuzzy number ã is called a positive fuzzy number if its membership function is such that $\mu_{\tilde{a}}(x) = 0 \ \forall \ x < 0$.

C. Addition Of Two Fuzzy Numbers

Let ã and $\tilde{b}$ be two triangular fuzzy numbers. An addition of fuzzy numbers is $\tilde{c} = \tilde{a} \oplus \tilde{b}$ defined by the membership function.

$$\mu_{\tilde{a}}(t) = \underset{t=u+v}{Sup} \min\{\mu_{\tilde{a}}(u), \mu_{\tilde{b}}(v)\}$$

Addition of ã and $\tilde{b}$ is represented as $(a_1, a_2, a_3) \oplus (b1, b_2, b_3)$. Therefore, the function principle is

$\tilde{c} = \tilde{a} \oplus \tilde{b} = (a_1, a_2, a_3) \oplus (b_1, b_2, b_3) = (a_1+b_1, a_2+b_2, a_3+b_3)$.

D. Order Relation

Consider an order relation among fuzzy numbers. A variety of methods for the ordering and ranking of fuzzy numbers has been proposed in the literature. These methods have been reviewed and tested by Bortolan and Degani [13].

*RULE:*Let ã and $\tilde{b}$ be two triangular fuzzy numbers such that ã = (a_1, a_2, a_3) and $\tilde{b}$ =$(b1, b_2, b_3)$.Then ã ≤ $\tilde{b}$ iff the following inequalities are 1. $a_1 \le b_1$ 2. $a_2 \le b_2$ 3. $a_3 \le b_3$

E. Fuzzy Ranking

The ranking or ordering methods of fuzzy quantities has been proposed by many authors. The previous methods are difficulties with comparisons of fuzzy numbers. Most of them were summarized in [12], [10], [11]. Zadeh [9] shows that fuzzy graphs may be viewed as generalizations of the calculi of crisp graphs. Blue *et al*. [3] gives taxonomy of graph fuzziness that distinguishes five basic types combining fuzzy or crisp vertex sets with fuzzy or crisp edge sets and fuzzy weights and fuzzy connectivity. Some of the approaches are rather theoretical and do not address the implementation points of view. These methods are not commonly accepted. Here the fuzzy ranking method described in [1], modified for the case of triangular numbers. The ranking function is defined as the distance between the centroid point $(x_0,y_0)$and the origin,

$$R(A) = \sqrt{(x_0)^2 + (y_0)^2}$$

where,

$$x_o = \frac{\int_{SuppA} x\mu_A(x)dx}{\int_{\sup pA} \mu_A(x)dx}$$

$$= \frac{a_1^{\,3}(a_3-a_2)+a_2^{\,3}(a_3-a_1)+a_3^{\,3}(a_2-a_1)}{3(a_2-a_1)(a_3-a_2)(a_3-a_1)}$$

$$y_o = \frac{a_1+4a_2+a_3}{3(a_1+2a_2+a_3)}$$

Fuzzy numbers A, B are ranked by their ranking function values R(A) and R(B).

III. FUZZY SHORTEST PATH

Finding the shortest path from a specified *source* to a destination is a fundamental problem in transportation, routing, and communications applications. Alternatively, the problem can be formulated as finding the shortest paths from a source to all other locations. The computational problem is called the *single source shortest path problem* (SPP). In this algorithm, priority_queue, Delete and ExtractMin operations can be found in all books dealing with algorithms and data structures (e.g. [2], [6]). Dijkstra's algorithm for the "deterministic" SPP finds the shortest paths from a source *s* to all other vertices. Now, the case of fuzzy edge lengths, Let EL[*v*] be the fuzzy number corresponding to the length of the shortest path from *s* to *v*. Initially, EL[*s*]=(0,0,0) and all other EL[*v*] values are set to (∞, ∞, ∞). The algorithm is based on gradual improvements of the shortest paths distance from *s* to the other vertices. Consider an edge(*u*, *v*) whose weight is *w*(*u*, *v*) and suppose that have already computed current estimates of *EL*[*u*], *EL*[*v*]. If $R(EL[u]\oplus w(u,v))<R(EL[v])$, then $EL[u]\oplus w(u,v)$ becomes a new estimate of EL[*v*]. The process by which an estimate is updated is called *Updation*. The shortest way back to the source is through *u* by updating the predecessor pointer. If the updation is repeated for all edges then EL[*v*] values converge to the shortest paths of vertices *v* to the source. Here is the algorithm.

A. ALGORITHM

EL - Edge Length
pre[] - previous edge
visit[] - known or unknown vertex

Initialization
Step -1

Initialize {
EL[v] = (0, ∞, ∞)
visit[v] = False
pre[v] = null } ∀ V.

Step – 2

Let EL[s] = (0,0,0).

Choosing next vertex
Step-3

Place all the vertices in Q = priority_queue(v).

Step – 4
Choose u = MinLen(Q).

Extract minimum distance edge
Step-5

for all v $\in$ Adj[u]

Step-6

if (R(EL[u]) $\oplus$ w(u, v)) < R(EL[v]) using ranking fuzzy distance

$$R(A) = \sqrt{(x_0)^2 + (y_0)^2}$$

Step-7

If true, Replace EL[v] = EL[u] $\oplus$ w (u, v). Delete (Q, v, EL[v]) and pre[v] = u.

Step-8

Assign visit[u] = True.

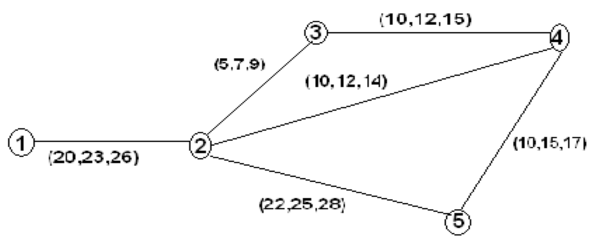
Step-9

Repeat the steps from step-4 to step-8 until the priority queue is empty.

Step-10

Thus obtain the shortest path in the given network.

IV. NUMERICAL EXAMPLE



Initialization

Initialize EL[u] = (0, ∞, ∞), visit[u] = False and pre[u] = null.

Let EL[1] = (0,0,0)

Choosing next vertex

Q = priority_queue (1, 2, 3, 4, 5).
Choose u = 1

Determine minimum distance edge

Here, only one adjacent vertex for vertex 1 in the queue. Now, Add with adjacent edge is EL[1] $\oplus$ w(1,2)

Calculate ranking fuzzy distance of R (1, 2)

R(EL[1] $\oplus$ w(1,2)) = (0,0,0) $\oplus$ (20,23,26)

R(1,2) = (20,23,26)

$$x_0 = \frac{20^3(26-23)+23^3(26-20)+26^3(23-20)}{3(23-20)(26-23)(26-20)}$$

$$x_0 = \frac{8000(3)+12167(6)+17576(3)}{3(3)(3)(6)}$$

$$x_0 = \frac{24000+73002+52728}{162} \qquad x_0 = \frac{149730}{162}$$

$$x_0 = 924.2593$$

$$y_0 = \frac{20+4(23)+26}{3(23+2(23)+26)}$$

$$y_0 = \frac{138}{276} \qquad y_0 = 0.5$$

$$\begin{aligned} R(1,2) &= \sqrt{(924.2593)^2 + (0.5)^2} \\ &= \sqrt{854255.25 + 0.25} \\ &= \sqrt{854255.5} \\ &= 924.25943 \end{aligned}$$

R(EL[2]) = ∞

if(EL[1,2] < EL[2])
(i.e.) if(924.25943 < ∞) -----(true)

If true, Replace d[2] = 924.25943 , delete the vertex 1 from priority queue and pre[2] = 1.
Assign visit[1] = True.

Choosing next vertex
Q = priority_queue (2, 3, 4, 5).
Choose u = 2
From vertex 2, the adjacent vertices are 3, 4 and 5. Perform the same steps once again.

Determine minimum distance edge
Find the next minimum length edge among (2, 3), (2, 4) and (2, 5).

R(2,3) = (1,2) $\oplus$ (2,3)
=(20,23,26) $\oplus$ (5,7,9)=(25,30,35)

$$\begin{aligned} x_0 &= \frac{25^3(35-30)+30^3(35-25)35^3(30-25)}{3(30-25)(25-30)(35-25)} \\ &= \frac{15625(5)+27000(10)+42875(5)}{3(5)(5)(10)} \\ &= \frac{78125+270000+214375}{750} = \frac{562500}{750} \quad = 750 \end{aligned}$$

$$y_0 = \frac{25 + 4\,(30) + 35}{3\,(25 + 2\,(30) + 35)} = \frac{25 + 120 + 35}{3(120)}$$

$$= \frac{180}{360} = 0.5$$

$$R(2,3) = \sqrt{(750)^2 + (0.5)^2} = \sqrt{562500 + 0.25}$$

$$= \sqrt{562500.25} = 750.0002$$

R(2,4) = (1,2) $\oplus$ (2,4)

= (20,23,26) $\oplus$ (10,12,14) = (30,35,40)

$$x_0 = \frac{30^3\,(40 - 35) + 35^3\,(40 - 30)\,40^3\,(35 - 30)}{3\,(40 - 35)\,(40 - 30)\,(35 - 30)}$$

$$= \frac{27000(5) + 42875(10) + 64000(5)}{3(5)(10)(5)}$$

$$= \frac{135000 + 428750 + 320000}{750} = \frac{883750}{750} = 1178.3333$$

$$y_0 = \frac{30 + 4\,(35) + 40}{3\,(30 + 2\,(35) + 40)} = \frac{30 + 140 + 40}{3(140)}$$

$$= \frac{210}{420} = 0.5$$

$$R(2,4) = \sqrt{(1178.3333)^2 + (0.5)^2} = \sqrt{1388469.3659 + 0.25}$$

$$= \sqrt{1388469.6159} = 1178.3334$$

R(2,5) = (1,2) $\oplus$ (2,5)

=(20,23,26) $\oplus$ (22,25,28)=(42,48,54)

$$x_0 = \frac{42^3\,(54 - 48) + 48^3\,(54 - 42)5\,4^3\,(48 - 42)}{3\,(48 - 42)\,(54 - 48)\,(54 - 42)}$$

$$= \frac{74088(6) + 110592(12) + 157464(6)}{3(6)(12)(6)}$$

$$= \frac{444528 + 1327104 + 944784}{1296} = \frac{2716416}{1296} = 2096$$

$$y_0 = \frac{42 + 4\,(48) + 54}{3\,(42 + 2\,(48) + 54)} = \frac{42 + 192 + 54}{3(192)} = \frac{288}{576} = 0.5$$

$$R(2,5) = \sqrt{(2096)^2 + (0.5)^2} = \sqrt{4393216 + 0.25}$$

$$= \sqrt{4393216.25} = 2096$$

Now, The values of R(2, 3), R(2, 4) and R(2, 5) are 750.0, 1178.3334 and 2096 respectively. Compare and Replace d[3] = 750.0 , delete the vertex 2 from Q and pre[3] = 2.Assign visit[2] = True.

Choosing next vertex
Q = priority_queue (3, 4, 5).
Choose u = 3
Determine minimum distance edge
From vertex 3, the adjacent vertex is 4. Perform the same steps once again.
Similarly, it reaches the vertex 5 at the same time queue is also empty. Finally, the path is 1 – 2 – 3 – 4 – 5.

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