Differential Evolution and Dynamic Differential Evolution Variants for High Dimensional Function Optimization : An Empirical Scalability Study

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Abstract— **This paper empirically compares the performance and scalability of Differential Evolution (DE) and Dynamic Differential Evolution (DDE) variants for solving high dimensional unconstrained global optimization functions. Four functions with different modality and decomposability viz unimodal separable, unimodal nonseparable, multimodal separable and multimodal nonseparable were chosen. Fourteen variants of DE and DDE were implemented and tested on these four benchmark functions, for the dimensions of 30,100, 500 and 1000. The performance of the variants are well compared by their mean objective function value (*MOV*), probability of convergence (P_c) and the success performance (*SP*). The analysis, done based on the results obtained for 100 runs for each variant-function-dimension combination, identifies the competitiveness and the scalability of the variants.**

Keywords-Differntiatl Evloution; Dynamic Differential Evolution; Population Convergene; Probability of Convergence; Success Performance;

I. INTRODUCTION

Evolutionary algorithms (EA) have been widely used to solve optimization problems. Differential Evolution [1] is an EA proposed to solve optimization problems, mainly to continuous search spaces. The DE algorithm, a stochastic population-based search method, has been successfully applied to many global optimization problems, benchmark functions and real world applications [2, 3, 4]. DE shares similarities with traditional EAs. As in other EAs, two main processes that derive the evolution are the perturbation process (crossover and mutation) which ensures the exploration of the search space and the selection process which ensures the exploitation properties of the algorithm. In the case of DE, the perturbation of a population element is done by probabilistically replacing it with an offspring obtained by adding to a randomly selected element a perturbation proportional with the difference between other two randomly selected elements. The selection is done by one to one competition between the parent and its offspring.

There are three strategy parameters in DE, the population size *NP*, the crossover rate *CR* and the scaling factor *F*. Many works have been done to study the suitable setting of these control parameters [5, 6]. The conceptual and algorithmic simplicity, high convergence characteristics and robustness of DE has made it one of the popular techniques for real-valued parameter optimization. Nevertheless, from the point of view of population updating, DE is static. The whole population in DE remains unchanged until it is replaced by a new population. Inevitably, it results in slower convergence. To alleviate this problem, a dynamic version of DE called Dynamic Differential Evolution (DDE) has been proposed in [7]. DDE updates the population dynamically and responds to any improvement immediately.

Interestingly, little research effort has been devoted to understand and compare the efficacy of DDE variants Moreover, most of the studies on DE is done on low dimensional benchmark functions (up to only 100). In this paper, an empirical analysis of the performance and scalability of fourteen DE and DDE variants on four benchmark functions has been carried out, for higher dimension (up to 1000). The remainder of the paper is organized as follows. Section 2 describes DE and DDE algorithms. After a brief review of the related work in Section 3, Section 4 details the design of experiments. Section 5 discusses the simulation results and finally Section 6 concludes the paper.

II. DE AND DDE ALGORITHMS

DE algorithm aims at exploring the search space by sampling at multiple, randomly chosen NP *D*-dimensional parameter vectors (population of initial points). The initial population should sufficiently cover the search space as much as possible, by uniformly randomizing individuals. After population initialization an iterative process is started and at each iteration (generation) a new population is produced until a stopping criterion is satisfied.

At each generation, DE employs the differential mutation operations to produce a mutant vector. A number of differential mutation strategies have been proposed in the literature. Along with the strategies came a notation scheme to classify the various DE-variants. The notation is defined by *DE/a/b/c* where *a* denotes the base vector or the vector to be perturbed; *b* denotes the number of vector differences used for perturbation; and *c* denotes the crossover scheme used between the mutant vector and the target vector to create a trial vector. The seven commonly used mutation strategies are *rand/1, best/1, rand/2, best/2, current-to-rand/1, current-to-best/1* and *rand-to-best/1*.

The two crossover schemes are *binomial* and *exponential*. With seven commonly used mutation strategies and two crossover schemes, the fourteen possible variants of DE and DDE are **/rand/1/bin, */rand/1/exp, */best/1/bin, */best/1/exp, */rand/2/bin, */rand/2/exp, */best/2/bin, */best/2/exp, */current-to-rand/1/bin, */current-to-rand/1/exp, */current-to-best/1/bin, */current-to-best/1/exp, */rand-to-best/1/bin* and **/rand-to-best/1/exp*. After the mutation and crossover operations, a one-to-one knockout competition between the target vector and its corresponding trial vector decides the survivor for the next generation. Figure 1 depicts the algorithmic description of general DE.

Population Initialization
$X(0) \leftarrow \{ x_1(0),\ldots,x_{NP}(0) \}$
$g \leftarrow 0$
Compute $\{ f(x_1(g)),\ldots,f(x_{NP}(g)) \}$
While the stopping condition is false do
    for i = 1 to NP do
        $y_i \leftarrow$ generatemutant(X(g))
        $z_i \leftarrow$ crossover($x_i(g)$, y_i)
        if $f(z_i) < f(x_i(g))$ then
            $x_i(g+1) \leftarrow z_i$
        else
            $x_i(g+1) \leftarrow x_i(g)$
        end if
    end for
$g \leftarrow g+1$
Compute $\{f(x_1(g)),\ldots,f(x_{NP}(g))\}$
End while

Figure 1. The algorithmic description of DE

As can be seen from the DE algorithm, in Figure 1, the repeated cycles of differential mutation and crossover do not make use of any progress taking place in the current generation, even if better fit individuals are generated in the current generation they are reserved for use in the next generation. The dynamic differential evolution, shown in Figure 2, employs a dynamic evolution mechanism. The dynamic evolution mechanism in DDE updates both the current optimal individual with the new competitive individual (if better than the current optimal) and the non optimal individuals dynamically. Consequently, the trial vectors are always generated using the newly updated population and thus DDE always responds to any progress immediately.

III. RELATED WORKS

The conceptual simplicity of DE has attracted many researchers, who are working on its improvement, resulting in many variants of the algorithm [8, 9, 10, 11]. However most of the experimental results on DE are obtained using low-dimensional problems, the reported studies on the scalability of DE derivative algorithms are scarce. In contrast, other evolutionary algorithms such as evolutionary programming (EP) have been tested up to 1000 dimension [12].

Population Initialization
$X(0) \leftarrow \{ x_1(0),\ldots,x_{NP}(0) \}$ $g \leftarrow 0$
Compute $\{ f(x_1(g)),\ldots,f(x_{NP}(g)) \}$
While the stopping condition is false do
    for i = 1 to NP do
        $y_i \leftarrow$ generatemutant(X(g))
        $z_i \leftarrow$ crossover($x_i(g)$, y_i)
        if $f(z_i) < f(x_i(g))$ then
            $x_i(g) \leftarrow z_i$
        endif
        if $f(z_i) < f(x_{best}(g))$ then
            best ← i
        endif
    end for
$g \leftarrow g+1$
Compute $\{f(x_1(g)),\ldots,f(x_{NP}(g))\}$
End while

Figure 2. The algorithmic description of DDE

Cooperative co-evolution architecture was firstly proposed by Potter for genetic algorithm, called CCGA [13], and had been successfully applied to other evolutionary algorithms [12, 14 and 15]. In the context of DE, the cooperative co-evolution has also been introduced, and it was proposed as CCDE [16]. However, CCDE only extended the problem domain up to 100 dimensions, which are relatively small. Zhenyu Yang, Ke Tang and Xin Yao, proposed two new DE variants named DECC-I and DECC-II for high dimensional optimization, up to 1000 dimension [8]. These two algorithms are based on a cooperative-coevolution.

Menzura-Montes et. al. [17] empirically compared the performance of eight DE variants on unconstrained optimization problems. The study concluded *rand/1/bin, best/1/bin, current-to-rand/1/bin* and *rand/2/dir* as the most competitive variants. However, the potential variants like *best/2/**, *rand-to-best/1/** and *rand/2/** were not considered in their study.

Babu and Munawar [18] compared the performance of ten variants of DE (excluding the *current-to-rand/1/** and *current-to-best/1/** variants of our variants suite). Qin, Huang and Suganthan [19], recently, proposed a self adaptive DE (SaDE). Four variants viz. *rand/1/bin, rand-to-best/2/bin, rand/2/bin* and *current-to-rand/1/bin* were considered for study.

Qing proposed the dynamic differential evolution in [7] and analyzed the performance of *DDE/best/1* variant on a function minimization problem with 8, 16, 24, 50 and 100 optimization parameters and on a benchmark electromagnetic inverse scattering problem. The study concluded that DDE significantly outperforms the classical DE.

A recent study by Qing [20] compared *DDE/rand/1/bin* and *DDE/best/1/bin* against their classical counterparts. The test bed involved around 37 test functions with dimension less than 10 and three application problems with 6, 8, 9, 16 and 24 dimensions. The work concluded *DDE/best/1/bin* as the most competitive variant among the scrutinized four strategies.

IV. DESIGN OF EXPERIMENTS

In this paper, we compared the performance efficacy of DDE variants against DE counterpart variants, for the dimension *D*=30. The scalability of the variants are compared for the higher dimensions viz. 100, 500 and 1000, on a set of benchmark functions with different features. We have chosen four test functions [17, 22] with different features f_1-Schwefel's problem 2.21 (unimodal separable), f_2-Schwefel's Problem 1.2 (unimodal nonseparable), f_3-Generalized Rastrigin's Function (multimodal separable) and f_4-Ackely's Function (multimodal nonseparable). The details of the functions are presented in Table I. All the functions have its optimum value at zero.

TABLE I. THE DETAILS OF THE BENCHMARK FUNCTIONS

f_1 - Schwefel's Problem 2.21

$$f_{sch3}(x) = \max_i \left\{ |x_i|, 1 \le i \le 30 \right\}$$

$$-100 \le x_i \le 100;\ x^* = (0,0,...,0);\ f_{sch3}(x^*) = 0$$

f_2 – Schwefel's Problem 1.2

$$f_{schDS}(x) = \sum_{i=1}^{30} \left(\sum_{j=1}^{i} x_j \right)^2$$

$$-100 \le x_i \le 100;\ x^* = (0,0,...,0);\ f_{schDS}(x^*) = 0$$

f_3 – Generalized Restrigin's Function

$$f_{Grf}(x) = \sum_{i=1}^{30} \left[x_i^2 - 10\cos(2\pi x_i) + 10 \right]$$

$$-5.12 \le x_i \le 5.12;\ x^* = (0,0,...,0);\ f_{Grf}(x^*) = 0$$

f_4 – Ackley's Function

$$f_{ack}(x) = 20 + e - 20\exp\left(-0.2\sqrt{\frac{1}{30}\sum_{i=1}^{30} x_i^2} \right) - \exp\left(\frac{1}{30}\sum_{i=1}^{30}\cos\left(2\pi x_i\right) \right)$$

$$-30 \le x_i \le 30;\ x^* = (0,0,...,0);\ f_{ack}(x^*) = 0$$

The three crucial control parameters of the DE and DDE algorithms are population size (*NP*), scaling factor (*F*) and crossover rate (*CR*). We fixed the population size (*NP*) as 100, a large population affects the ability of the approach to find the correct search direction, so we fixed a moderate population size in all the experiments, for both DE and DDE variants. We fixed the maximum number of function evaluations as proportional to the dimension, which is equal to *D* * 5000. We also fixed the stopping criteria as an error value of 1 x 10^{-12}. Based on [17, 23], we decided a range for the parameter $F \in [0.3, 0.9]$, this value is generated anew at each generation. The same F value is assigned to *K*, which is used for mutation.

The *CR* parameter is tuned for each Variant-Function combination. 11 different values for the "CR" parameter were tested {0.0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 1.0} for each Variant-Function combination, with dimension D=30. We conducted 50 independent runs for each combination of Variant-Function-CR value. A bootstrap test was conducted for each combination, the "*CR*" value corresponding to the best confidence interval (95%) was chosen. The same *CR* values are adopted for D=100, 500 and 1000 also. The *CR* values of the variants are shown in Table II.

TABLE II. THE FOURTEEN VARIANTS OF DE AND DDE ALONG WITH THE CR VALUES FOR EACH TEST FUNCTION, FOR D = 30

No	Variant	f_1		f_2		f_3		f_4	
		DE	DDE	DE	DDE	DE	DDE	DE	DDE
V1	rand/1/bin	0.9	0.3	0.9	0.9	0.1	0.1	0.9	0.9
V2	rand/1/exp	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
V3	best/1/bin	0.8	0.9	0.5	0.7	0.1	0.1	0.1	0.1
V4	best/1/exp	0	0.1	0.1	0.1	0.1	0.1	0.1	0.2
V5	rand/2/bin	0.9	0.2	0.9	0.9	0.1	0.1	0.9	0.9
V6	rand/2/exp	0	0	0.1	0.1	0.1	0.1	0.1	0.1
V7	best/2/bin	0.1	0.9	0.7	0.8	0.1	0.1	0.5	0.9
V8	best/2/exp	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1
V9	current-to-rand/1/bin	0.3	0.2	0.9	0.9	0.1	0.1	0.1	0.1
V10	current-to-rand/1/exp	0	0	0	0	0.1	0.1	0.1	0.1
V11	current-to-best/1/bin	0.2	0.2	0.9	0.9	0.1	0.1	0.1	0.1
V12	current-to-best/1/exp	0	0	0	0	0.1	0.1	0.1	0.1
V13	rand-to-best/1/bin	0.9	0.3	0.9	0.9	0.1	0.1	0.9	0.9
V14	rand-to-best/1/exp	0.1	0.1	0.1	0.1	0.1	0.1	0.1	0.1

We initialized the population with uniform random initialization with in the search space and 100 independent runs were performed for each Variant-Function-Dimension combination. We recorded, mean objective function value (*MOV*) for 100 runs, probability of convergence (P_c) [21] and measured the success performance [*SP*] [19, 24].

V. RESULTS AND DISCUSSION

In Table III, we present the MOV obtained for f_1, f_2, f_3 and f_4, for D=30. The values with bold and underline are the best MOV for that Function-Variant combination.

Results in Table III shows that, for f_1, none of the variants could solve this function. Most of the DDE variants are outperforming their classical DE counterparts, except *DDE/rand/1/bin, DDE/rand/2/bin, DDE/rand/2/exp, DDE/current-to-rand/1/exp* and *DDE/rand-to-best/1/bin*. The best performing DDE variant is *DDE/best/1/bin*, but its counterpart variant has given contrary result. Except this, it is interesting to note that the best and worst performance for f_1 was provided by similar set of DE and DDE variants. For f_1, the top four best performing variants are **/rand-to-best/1/bin, */rand/1/bin, */best/2/bin, DE/rand/2/bin* and *DDE/best/1/bin*. The worst performing variants are *DE/best/1/exp, DE/best/1/bin, DDE/rand-to-best/1/exp, DDE/rand/1/exp, */current-to-rand/1/exp* and **/current-to-best/1/exp.*

For the function f_2, Table III, most of the DDE variants outperforming their classical DE counterparts, except *DDE/current-to-rand/1/** and *DDE/current-to-best/1/exp*. The superiority of DDE variant is more evident in the case of **/best/1/exp* variants. The top two best and worst performance were provided by the similar set of DE and DDE variants. The *DDE/best/1exp* is one of the best performing variants but its classical counterpart variant shown only moderate performance. The best performing variants were */best/2/bin, */best/2/exp, */rand-to-best/1/bin, */rand/1/bin, */best/1/bin* and *DDE/best/1/exp*. The worst performing variants were */current-to-best/1/** and */current-to-rand/1/**.

TABLE III. MEAN OBJECTIVE FUNCTION VALUES (MOV) FOR DE AND DDE VARIANTS FOR D = 30

Variant	f_1		f_2		f_3		f_4	
	DE	DDE	DE	DDE	DE	DDE	DE	DDE
rand/1/bin	0.91	1.00	0.55	0.30	**0.00**	**0.00**	0.09	**0.00**
rand/1/exp	17.82	17.44	5.71	5.011	97.63	98.32	**0.00**	**0.00**
best/1/bin	28.72	**0.69**	0.79	0.22	3.40	3.138	3.52	3.04
best/1/exp	25.96	2.83	364.74	**0.00**	47.41	45.74	7.39	7.59
rand/2/bin	3.34	3.81	1073.51	891.05	16.40	401.20	0.09	**0.00**
rand/2/exp	7.13	7.87	1049.47	959.51	161.05	159.72	4.04	3.82
best/2/bin	2.60	0.94	**0.00**	**0.00**	0.63	0.67	0.09	2.59
best/2/exp	9.09	2.13	**0.00**	**0.00**	120.97	120.92	0.43	0.52
current-to-rand/1/bin	18.83	17.36	13937.70	14105.89	63.66	64.55	1.58	1.59
current-to-rand/1/exp	55.74	54.88	1262.38	1265.43	249.82	245.68	15.65	15.45
current-to-best/1/bin	17.05	17.03	13749.31	14354.49	64.90	64.06	1.69	1.63
current-to-best/1/exp	55.75	55.26	1259.26	1221.48	245.38	248.02	15.44	15.67
rand-to-best/1/bin	**0.78**	1.00	0.54	0.2788	**0.00**	**0.00**	0.09	**0.00**
rand-to-best/1/exp	17.98	17.4	5.54	4.8769	100.97	98.99	**0.00**	**0.00**

For the function f_3, Table III, we noticed that eight out of fourteen variants of DDE could outperform their classical DE counterparts. The top four best performance were provided by the similar set of DE and DDE variants viz. */rand/1/bin, */rand-to-best/1/bin, */best/2/bin* and */best/1/bin*. The worst performance was provided by */current-to-rand/1/exp, */current-to-best/1/exp* and *DDE/rand/2/bin.* Results in Table III, for the function f_4, shows that the function was solved by most of the variants. For f_4, seven DDE variants have outperformed their classical counterpart DE variants and two DDE variants have shown similar performance as its DE counterpart variants. The best performance were provide by the similar set of DE and DDE variants viz. */rand-to-best/1/exp, */rand/1/exp, DDE/rand/1/bin, DDE/rand/2/bin* and *DDE/rand-to-best/1/bin*. The worst performing variants viz. */rand/2/exp, */best/1/exp, */current-to-best/1/exp* and */current-to-rand/1/exp*.

Next, in our experiment, the probability of convergence (P_c), the percentage of successful runs to total runs, is calculated for each variant-function combination. This measure identifies variants having higher convergence capability to global optimum. It is calculated as the mean percentage of number of successful runs out of total number of runs i.e. $P_c = (nc\ /\ nt)\%$ where *nc* is total number of successful runs made by each variant for all the functions and *nt* is total number of runs, in our experiment *nt* = 4 * 100 =400.

The convergence probability results are shown in Table IV, for dimension D=30. The table presents the number of successful runs made by each variant for each function, the total number of successful runs and P_c. The results show that the variants with higher P_c are */rand/1/bin, */best/2/bin, */best/2/exp, DE/rand-to-best/1/bin, DDE/best/1/bin, DDE/best/1exp* and *DDE/rand/2/bin.* The worst performing variants viz. */current-to-best/1/*, */current-to-rand/1/** and */rand/2/exp* were found to have the least P_c value. It is worth noting that DDE has given overall more number of successful runs than that of DE. Consequently, DDE variants have higher probability of convergence compared to classical DE variants, irrespective of their recombination type.

TABLE IV. P_C VALUES FOR DE AND DDE VARIANTS FOR D=30

Variant	f_1	f_2	f_3	f_4	*nc*	P_c
DE						
DE/rand/1/bin	37	0	100	0	137	34.25
DE/rand/1/exp	0	0	0	10	10	2.5
DE/best/1/bin	0	14	6	0	20	5
DE/best/1/exp	0	0	0	0	0	0
DE/rand/2/bin	0	0	0	0	0	0
DE/rand/2/exp	0	0	0	0	0	0
DE/best/2/bin	0	100	38	0	138	34.5
DE/best/2/exp	0	100	0	0	100	25
DE/current-to-rand/1/bin	0	0	0	0	0	0
DE/current-to-rand/1/exp	0	0	0	0	0	0
DE/current-to-best/1/bin	0	0	0	0	0	0
DE/current-to-best/1/exp	0	0	0	0	0	0
DE/rand-to-best/1/bin	40	0	100	0	140	35
DE/rand-to-best/1/exp	0	0	0	9	9	2.25
DDE						
Variant	f_1	f_2	f_3	f_4	*nc*	P_c
DDE/rand/1/bin	0	0	100	100	200	50
DDE/rand/1/exp	0	0	0	26	26	6.5
DDE/best/1/bin	44	98	9	8	159	39.75
DDE/best/1/exp	0	100	0	0	100	25
DDE/rand/2/bin	0	0	0	100	100	25
DDE/rand/2/exp	0	0	0	0	0	0
DDE/best/2/bin	8	100	37	2	147	36.75
DDE/best/2/exp	0	100	0	55	155	38.75
DDE/current-to-rand/1/bin	0	0	0	0	0	0
DDE/current-to-rand/1/exp	0	0	0	0	0	0
DDE/current-to-best/1/bin	0	0	0	0	0	0
DDE/current-to-best/1/exp	0	0	0	0	0	0
DDE/rand-to-best/1/bin	0	1	0	0	1	0.25
DDE/rand-to-best/1/exp	0	0	0	30	30	7.5

Figure 3 compares the performances of all DE and DDE variants, for dimension 30, by plotting empirical distribution of normalized success performance. The success performance (*SP*) has been calculated as follows.

$$SP = \frac{\text{mean}\left(\text{function evalutions for successful runs}\right) * \left(\#\,\text{total runs}\right)}{\#\,\text{successful runs}} \quad (1)$$

A run is considered *successful* if the global optimum is reached with the given precision, before the maximum number of functions evaluations. The success performances of all 14 variants on each benchmark function are calculated and are normalized by dividing them by the best *SP* on the respective function. As can be seen from equation (1) small values of *SP* and therefore large values in the empirical distribution are preferable.

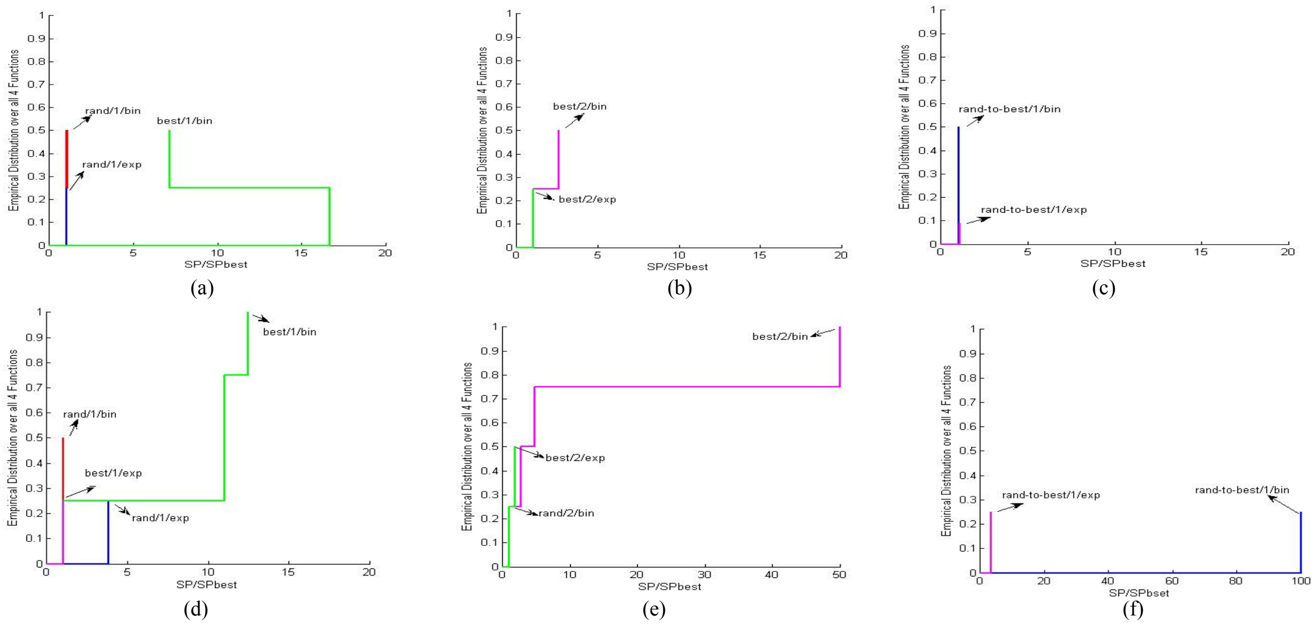

Figure 3. Success Performance of DE variants (a-c) and DDE variants (d-f), for the dimension 30.

The first variant that reaches (earlier) the top of the graph will be regarded as the best variant. For the sake of display, the variants have been plotted in three groups. As can be seen from the first row (a-c) of Figure 3, *DE/rand/1/bin, DE/best/2/bin* and *DE/rand-to-best/1/bin* have displayed overall superior performance. It is worth noting that none of the DE variants reached top of the graph because none of them solved all the benchmark functions even once. However, as is shown in the second row (d-f) of Figure 3, *DDE/best/1/bin* and *DDE/best/2/bin* reached top as they solved all the benchmark problems at least once. *DDE/best/2exp* is performing relatively better than *DE/best/2/exp*. It is worth noting that Figure 3 (a-c) do not display the variants *DE/best/1/exp, DE/rand/2/bin, DE/rand/2/exp, DE/current-to-rand/1/bin, DE/current-to-rand/1/exp, DE/current-to-best/1/bin* and *DE/current-to-best/1/exp* due to their poor overall performance. However Figure 3 (d-f) could display *DDE/best/1/exp* and *DDE/rand/2/bin* due to their better performance over their DE counterparts. Thus the superior performance of DDE over DE is much more evident in case of worst performing variants. Interestingly, as can be seen from the graphs, the binomial variants have shown a relatively better performance against their exponential counterparts.

From the overall analysis, for dimension 30, the best performing variants are */*rand-to-best/1/bin, */rand/1/bin, */best/2/bin* and */*best/1/bin* (closely followed by */*rand/2/bin)*. The identified worst performing variants are */*current-to-rand/1/exp, */current-to-best/1/exp and */best/1/exp*. The variants with binomial recombination have shown superior convergence nature, compared to the variants with exponential crossover. The DDE variants are outperforming their counterpart DE variant for all the functions, in most of the cases.

On identifying the best and worst performing variants for dimension 30, next we focused on analyzing the scalability nature of DE variants and comparing it with the corresponding DDE counterpart variants. The simulations results for the functions f_1, f_2, f_3 and f_4 for the dimensions D = 30, 100, 500 and 1000 are presented in Table V and VI for the DE and DDE variants, respectively.

A. f_1 : Schwefel's problem 2.23 (Unimodal Separable)

In dimension 100, 500 and 1000, the results in Table V and VI, show that f_1 was not solved by any of the variants. In D=100, in the case of DE, comparatively the best results were provided by *DE*/*best/2/bin* and *DE*/*rand/1/bin*, with the MOV of 30.11 and 30.12, respectively. The poorest performances were provided by *DE/current-to-rand/1/exp* and *DE/rand/2/exp* with the MOV of 94.50 and 94.49, respectively. For DDE, the best performance was shown by *DDE/rand/1/bin* and *DDE/rand-to-best/1/bin* with the MOV of 14.12 and 14.21, respectively. For dimension 100, *DDE/rand/1/bin* is competitive than *DE/rand/1/bin*. The worst performance was provided by *DDE/rand/2/exp* and *DDE/current-to-rand/1/exp*, and they outperform their counterpart classical DE variants.

For DE, in D=500, comparatively the best results were provided by *DE/best/1/exp* and *DE/best/1/bin* with the MOV of 41.84 and 64.15, respectively. The poorest performance was provided by *DE/*rand/2/*bin* with MOV of 98.63. For DDE, *DDE/best/1/bin* was the best performing variant and it could outperform *DE/best/1/bin*. The poor performance was provided by *DDE/best/2/bin*.

TABLE V. MOV MEASURED FOR D = 30, 100, 500 AND 1000 FOR DE VARIANTS

f_1					f_2				
Variant	**30**	**100**	**500**	**1000**	**Variant**	**30**	**100**	**500**	**1000**
DE/rand/1/bin	0.91	30.12	98.53	98.78	DE/rand/1/bin	0.55	7061.26	**378461.24**	1302355.44
DE/rand/1/exp	17.82	90.95	98.33	98.72	DE/rand/1/exp	5.71	239074.44	15216875.17	14342315.34
DE/best/1/bin	28.72	50.09	64.15	64.62	DE/best/1/bin	0.79	10464.86	854930.98	3191306.42
DE/best/1/exp	25.96	34.71	**41.84**	**42.79**	DE/best/1/exp	364.74	144233.48	14870737.35	79492389.71
DE/rand/2/bin	3.34	94.47	98.63	98.80	DE/rand/2/bin	1073.51	218155.96	6632782.43	19785758.60
DE/rand/2/exp	7.13	94.49	98.52	98.78	DE/rand/2/exp	1049.47	311891.24	15328136.09	68431518.00
DE/best/2/bin	2.60	**30.11**	88.26	89.65	DE/best/2/bin	**0.00**	**2248.43**	391863.94	1558474.60
DE/best/2/exp	9.09	89.61	98.30	98.67	DE/best/2/exp	**0.00**	209909.29	15183598.61	71901345.60
DE/current-to-rand/1/bin	18.83	81.30	98.55	98.77	DE/current-to-rand/1/bin	13937.70	667057.83	16347074.17	63430502.40
DE/current-to-rand/1/exp	55.74	94.50	98.50	98.82	DE/current-to-rand/1/exp	1262.38	440334.78	8048956.17	28044382.29
DE/current-to-best/1/bin	17.05	72.85	98.45	98.82	DE/current-to-best/1/bin	13749.31	657134.55	15664122.22	62995994.80
DE/current-to-best/1/exp	55.75	30.17	98.57	98.84	DE/current-to-best/1/exp	1259.26	442178.83	7985850.68	28555998.20
DE/rand-to-best/1/bin	**0.78**	91.35	98.46	98.85	DE/rand-to-best/1/bin	0.54	6853.77	388102.10	**1167663.24**
DE/rand-to-best/1/exp	17.98	91.35	98.34	98.62	DE/rand-to-best/1/exp	5.54	246681.38	15443684.83	71024066.00

f_3					f_4				
Variant	**30**	**100**	**500**	**1000**	**Variant**	**30**	**100**	**500**	**1000**
DE/rand/1/bin	**0.00**	1579.21	3782.37	17700.42	DE/rand/1/bin	0.09	1.93	9.05	12.74
DE/rand/1/exp	97.63	599.03	7836.49	9146.20	DE/rand/1/exp	**0.00**	19.56	20.90	21.03
DE/best/1/bin	3.40	1575.65	**607.87**	17733.15	DE/best/1/bin	3.52	3.47	2.06	2.66
DE/best/1/exp	47.41	834.01	7721.86	10808.09	DE/best/1/exp	7.39	20.20	20.98	21.07
DE/rand/2/bin	16.40	1580.49	4448.48	17729.09	DE/rand/2/bin	0.09	**0.02**	16.88	20.89
DE/rand/2/exp	161.05	769.45	7947.06	17741.80	DE/rand/2/exp	4.04	20.03	20.94	21.04
DE/best/2/bin	0.63	1587.85	3920.91	17702.71	DE/best/2/bin	0.09	0.03	**1.75**	**1.52**
DE/best/2/exp	120.97	677.79	7943.69	9785.23	DE/best/2/exp	0.43	19.33	20.93	21.05
DE/current-to-rand/1/bin	63.66	1586.25	7096.76	17679.43	DE/current-to-rand/1/bin	1.58	4.20	20.72	21.00
DE/current-to-rand/1/exp	249.82	1504.33	8343.67	17707.06	DE/current-to-rand/1/exp	15.65	20.42	21.01	21.08
DE/current-to-best/1/bin	64.90	1586.12	7065.75	17714.14	DE/current-to-best/1/bin	1.69	4.25	20.72	20.99
DE/current-to-best/1/exp	245.38	1503.51	8338.84	17734.21	DE/current-to-best/1/exp	15.44	20.42	21.01	21.08
DE/rand-to-best/1/bin	**0.00**	1583.53	3785.98	17681.53	DE/rand-to-best/1/bin	0.09	0.48	4.54	10.23
DE/rand-to-best/1/exp	100.97	**595.62**	7843.83	**9107.64**	DE/rand-to-best/1/exp	**0.00**	19.55	20.90	21.03

In D=1000, in the case of DE, comparatively the best results were provided *DE/best/1/exp* with the MOV of 42.79. The poorest performance was provided by *DE/rand-to-best/1/bin* with the MOV of 98.85. In the case of DDE, the best and worst performance was given by *DDE/best/1/bin* and *DDE/current-to-rand/1/bin*, respectively.

The best performing DE variants for the dimension of 30, 100, 500 and 1000 are *DE/rand-to-best/1/bin* (0.78), *DE/best/2/bin* (30.11), *DE/best/1/exp* (41.84) and *DE/best/1/exp* (42.79), respectively. The best performing DDE variants are *DDE/best/1/bin* (0.69), *DDE/rand/1/bin* (14.12), *DDE/best/1/bin* (54.47) and *DDE/best/1/bin* (55.52), respectively. For the dimension 30 and 100, DDE has given least MOV compared to DE, but for dimension 500 and 1000 DE has given least MOV than DDE.

On the other hand, the worst performing DE variants for the dimension of 30, 100, 500 and 1000 are *DE/current-to-best/1/exp* (55.75), *DE/current-to-rand/1/exp* (94.50), *DE/rand/2/bin* (98.63) and *DE/rand-to-best/1bin* (98.85), respectively. The worst performing DDE variants are *DDE/current-to-best/1/exp* (55.26), *DDE/rand/2/exp* (94.35), *DDE/best/2/bin* (98.62) and *DDE/current-to-rand/1/bin* (98.92), respectively. In the worst performing variants cases, for all the dimension DDE has given, comparatively, least MOV than DE, except for the dimension 1000.

It is observed from the overall comparative performance analysis of the variants, for f_1, that most of the DDE variants are outperforming their classical DE counterparts, irrespective of the dimension, except of few cases. For $f_{1,}$ for the dimensions 30, 100 and 500 nine out of fourteen DDE variants and for the dimension 100 eight out fourteen DDE variants have outperformed their classical DE variant. This shows the superiority of DDE in solving f_1 in all the dimensions.

To say about the scalability of the variants to the higher dimensions, the DE variants are not scaling with the dimension. It is interesting to note that the *DDE/best/1/bin* demonstrates better scalability, it continued to be the best performing DDE variant in all the dimension, except for D=100. *DE/best/1/exp* has shown competitive result for the dimensions of 500 and 1000. *DE/best/2/bin* and *DDE/rand/1/bin* are good in D=100. This shows that the */best/1/* variants are good for high dimensional function optimization.

B. f_2: Schwefel's Problem 1.2 (Unimodal NonSeparable)

The result, Table V and VI, shows for the dimensions of 100, 500 and 1000, f_2 was not solved by any of the variants. In dimension 100, comparatively the best results were provided by *DE/best/2/bin* with the MOV of 2248.43. The poorest performance was provided by *DE/current-to-*

TABLE VI. MOV MEASURED FOR D = 30, 100, 500 AND 1000 FOR DDE VARIANTS

f_1					f_2				
Variant	**30**	**100**	**500**	**1000**	**Variant**	**30**	**100**	**500**	**1000**
DDE/rand/1/bin	1.00	**14.12**	98.47	98.80	DDE/rand/1/bin	0.30	6637.46	383888.18	1299255.38
DDE/rand/1/exp	17.44	90.61	98.27	98.80	DDE/rand/1/exp	5.01	240363.05	14939295.29	75385246.55
DDE/best/1/bin	**0.69**	31.84	**54.47**	**55.52**	DDE/best/1/bin	0.22	**1259.21**	**352902.38**	1510373.51
DDE/best/1/exp	2.83	90.14	98.25	98.60	DDE/best/1/exp	**0.00**	141526.49	15243616.03	72440117.57
DDE/rand/2/bin	3.81	43.54	98.39	98.64	DDE/rand/2/bin	891.05	214154.89	7166948.68	21860325.56
DDE/rand/2/exp	7.87	94.35	98.51	98.84	DDE/rand/2/exp	959.51	314246.18	15066990.09	63196774.55
DDE/best/2/bin	0.94	31.05	98.62	98.76	DDE/best/2/bin	**0.00**	1583.60	370816.19	1457755.15
DDE/best/2/exp	2.13	89.65	98.39	98.68	DDE/best/2/exp	**0.00**	204320.84	15302298.13	74245393.22
DDE/current-to-rand/1/bin	17.36	72.72	98.43	98.92	DDE/current-to-rand/1/bin	14105.89	654438.79	16056102.62	64136432.16
DDE/current-to-rand/1/exp	54.88	94.17	98.54	98.72	DDE/current-to-rand/1/exp	1265.43	438408.69	8174238.38	29423339.00
DDE/current-to-best/1/bin	17.03	73.08	98.43	98.80	DDE/current-to-best/1/bin	14354.49	656880.63	16111978.77	61850086.72
DDE/current-to-best/1/exp	55.26	94.23	98.44	98.72	DDE/current-to-best/1/exp	1221.48	443737.50	8178306.49	28770555.30
DDE/rand-to-best/1/bin	1.00	14.21	98.50	98.80	DDE/rand-to-best/1/bin	0.28	6646.89	380753.96	**1240678.54**
DDE/rand-to-best/1/exp	17.40	91.32	98.30	98.52	DDE/rand-to-best/1/exp	4.88	240388.47	15062434.97	61102382.96
f_3					f_4				
Variant	**30**	**100**	**500**	**1000**	**Variant**	**30**	**100**	**500**	**1000**
DDE/rand/1/bin	**0.00**	320.14	3783.48	9083.81	DDE/rand/1/bin	**0.00**	1.96	9.74	13.32
DDE/rand/1/exp	98.32	1030.92	7834.76	16730.77	DDE/rand/1/exp	**0.00**	19.49	20.90	21.03
DDE/best/1/bin	3.14	**151.30**	**578.48**	**1317.76**	DDE/best/1/bin	3.04	3.43	**2.13**	**2.85**
DDE/best/1/exp	45.74	833.35	7736.94	16670.45	DDE/best/1/exp	7.58	20.20	20.98	21.07
DDE/rand/2/bin	401.20	401.20	4449.98	11182.90	DDE/rand/2/bin	**0.00**	**1.24**	5.16	17.90
DDE/rand/2/exp	159.72	1146.47	7958.94	16868.71	DDE/rand/2/exp	3.82	20.04	20.94	21.05
DDE/best/2/bin	0.67	342.58	3923.32	8985.19	DDE/best/2/bin	2.59	9.46	16.14	16.65
DDE/best/2/exp	120.92	1082.43	7959.34	16861.72	DDE/best/2/exp	0.52	19.25	20.93	21.04
DDE/current-to-rand/1/bin	64.55	580.61	7056.70	16548.26	DDE/current-to-rand/1/bin	1.59	4.26	20.71	21.00
DDE/current-to-rand/1/exp	245.68	1360.12	8336.10	17302.01	DDE/current-to-rand/1/exp	15.45	20.41	21.01	21.09
DDE/current-to-best/1/bin	64.06	574.96	7061.56	16549.52	DDE/current-to-best/1/bin	1.63	4.25	20.72	21.00
DDE/current-to-best/1/exp	248.01	1357.18	8321.75	17298.95	DDE/current-to-best/1/exp	15.67	20.40	21.01	21.09
DDE/rand-to-best/1/bin	**0.00**	323.22	3787.94	9081.63	DDE/rand-to-best/1/bin	**0.00**	2.21	9.77	13.33
DDE/rand-to-best/1/exp	98.99	1032.48	7844.41	16727.88	DDE/rand-to-best/1/exp	**0.00**	19.56	20.90	21.03

best/1/bin and *DE/current-to-rand/1/bin* with the MOV of 657134.55 and 667057.83, respectively. The best performing DDE variant was *DDE/best/1/bin* with MOV of 1259.21, and it has also outperformed *DE/best/1/bin*. The worst results were shown by *DDE/current-to-rand/1/bin* and *DDE/current-to-best/1/bin* with the MOV of 654438.79 and 656880.63, respectively.

In D=500, for DE, comparatively the best results were provided by *DE/rand/1/bin* with the MOV of 378461.23, the poorest performance were provided by *DE/current-to-best/1/bin* and *DE/current-to-rand/1/bin* with the MOV of 15664122.22 and 16347074.17, respectively. For DDE, the best performance was provided by *DDE/best/1/bin* with MOV of 352902.38 and the worst performance was provided by the variants *DDE/current-to-rand/1/bin* and *DDE/current-to-best/1/bin* with MOV of 16056102.62 and 16111978.77, respectively.

In D=1000, for DE, comparatively the best result was provided by the variant *DE/rand-to-best/1/bin* with the MOV of 1167663.24. The poorest performance was provided by the variant *DE/best/1/exp* with the MOV of 79492389.71. For DDE, the best result was provided by *DDE/*rand-to-best/1/bin with the MOV of 1240678.54, and it is failed to outperform its DE counterpart. The worst result was provided by the variant *DDE/rand/1/exp* with the MOV of 75385246.55.

The best performing DE variants for the dimension of 30, 100, 500 and 1000 are *DE/best/2/** (0), *DE/best/2/bin* (2248.43), *DE/rand/1/bin* (378461.24) and *DE/rand-to-best/1/bin* (1167663.24), respectively. The best performing DDE variants are (*DDE/best/2/*, DDE/best/1/exp)* (0), *DDE/best/1/bin* (1259.21), *DDE/best/1/bin* (352902.38) and *DDE/rand-to-best/1/bin* (1240678.54), respectively. For D = 100 and 500 the DDE variants have achieved least MOV.

On the other hand, the worst performing DE variants for the dimension of 30, 100, 500 and 1000 are *DE/current-to-rand/1/bin* (13937.70), *DE/current-to-rand/1/bin* (667057.83), *DE/current-to-rand/1/bin* (16347074.17) and *DE/best/1/exp* (79492389.71), respectively. The worst performing DDE variants are *DDE/current-to-best/1/bin* (14354.49), *DDE/current-to-best/1/bin* (656880.63), *DDE/current-to-best/1//bin* (16111978.77) and *DDE/rand/1/exp* (75385246.55), respectively. For the dimension 100, 500 and 1000, among the worst performing variants itself, the DDE variants have outperformed their counterpart DE variant, by providing least MOV.

For f_2, most of the DDE variants are outperforming their classical DE counterparts, irrespective of the dimension, except of few cases. For the dimensions 30 and 100 the superiority of DDE variants were more evident, in these cases 11 out of 14 DDE variants have outperformed their DE counterparts. For dimension 500 seven, and for dimension 1000 eight, out of fourteen DDE variants have shown their competitiveness compared to their corresponding DE variants. The results show the superiority of DDE variants in solving f_2 in all the dimensions.

The *DDE/best/1/bin* variant is good at dimension 100 and 500, **/rand-to-best/1/bin* variants are good at the dimension 1000. *DE/rand/1/bin* and *DE/best/2/bin* variants have shown competitive results for the dimension 500 and 100, respectively.

C. f_3: Generalized Rastrigin's Function (Multimodal Separable)

The results, Table V and VI, shows the f_3 was not solved by any of the variants, for D = 100, 500 and 1000. In D=100, for DE variants, comparatively the best results were provided by *DE/rand-to-best/1/exp* with the MOV of 595.62, and the poorest performance was given by *DE/best/2/bin* with the MOV of 1587.85, but *DDE/best/2/bin* outperforms *DE/best/2/bin*. For DDE variants, the best performance was shown by *DDE/best/1/bin* with the MOV of 151.30 and the worst performance was given by *DDE/current-to-rand/1/exp* with 1360.12.

In D=500, for the DE variants, comparatively the best results were provided by *DE/best/1/bin* with the MOV of 607.87. The poorest performances were provided by *DE/current-to-best/1/exp* and *DE/current-to-rand/1/exp* with the MOV of 8338.84 and 8343.67, respectively. In DDE also the best and worst performance were provided by the same set of DE variants. The best performance was provided by *DDE/best/1/bin* with MOV of 578.48, ie., it has outperformed *DE/best/1/bin*. The worst performance were provided by *DDE/current-to-best/1/exp* and *DDE/current-to-rand/1/exp* with the MOV of 8321.75 and 8336.10. The superiority of DDE is evident in the case of worst performing variants also.

In D=1000, for DE, comparatively the best results were provided by the variant *DE/rand-to-best/1/exp* with the MOV of 9107.64. The poorest performance was given by the variant *DE/rand/2/exp* with the MOV of 17741.80. For DDE, the best result was provided by *DDE/best/1/bin* with the MOV of 1317.76 and the worst result was provided by *DDE/current-to-rand/1/exp* with the MOV of 17302.01.

The best performing DE variants for the dimension of 30, 100, 500 and 1000 are (*DE/rand/1/bin, DE/rand-to-best/1/bin)* (0), *DE/rand-to-best/1/exp* (595.62), *DE/best/1/bin* (607.87) and *DE/rand-to-best/1/exp* (9197.64), respectively. The best performing DDE variants are (*DDE/rand/1/bin, DDE/rand-to-best/1/bin)* (0), *DDE/best/1/bin* (151.30), *DDE/best/1/bin* (578.48) and *DDE/best/1/bin* (1317.86), respectively. At each dimension, DDE has given least MOV.

On the other hand, the worst performing DE variants for the dimension of 30, 100, 500 and 1000 are *DE/current-to-rand/1/exp* (249.82), *DE/best/2/bin* (1587.85), *DE/current-to-rand/1/exp* (8343.67) and *DE/rand/2/exp* (17741.80), respectively. The worst performing DDE variants are *DDE/rand/2/bin* (401.20), *DDE/current-to-rand/1/exp* (1360.12), *DDE/current-to-rand/1/exp* (8336.10) and *DDE/current-to-rand/1/exp* (17302.01), respectively. For all the dimensions DDE has given comparatively least MOV than DE, except for D==30.

In case of f_3, for the dimension 30, nine out of fourteen DDE variants are outperforming their DE counterparts. For the dimensions 100 seven, and for D=500 and 1000 only two, out of fourteen DDE variants have outperformed their classical DE variant.

On analyzing scalability of the variants, we observed that, the *DDE/best/1/bin* demonstrates better scalability, it continued to be the best performing DDE variant for the dimensions of 100, 500 and 1000. *DE/best/1/bin* also showed its better performance for the dimension 500. Interestingly, the variant *DE/rand-to-best/1/exp* was the best performing DE variant for the dimension 100 and 1000.

D. f_4 : Multimodal Nonseparable(Ackely's Function)

In dimension 100, 500 and 1000, the result, Table V and VI, shows that f_4 was not solved by any of the variants. In D=100, for DE, comparatively the best results were provided by *DE/best/2/bin* and *DE/rand/2/bin* with the MOV of 0.03 and 0.02, respectively. The poorest performance was provided by *DE/current-to-rand/1/exp* and *DE/current-to-best/1/exp* with the MOV of 20.42. For DDE, the best performance was provided by the *DDE/rand/2/bin* with the MOV of 1.24. The worst performance was provided by the variant *DDE/current-to-rand/1/exp*.

In D=500, for DE, comparatively the best result was provided by *DE/best/2/bin* with the MOV of 1.75. The poorest performances were provided by *DE/current-to-best/1/exp* and *DE/current-to-rand/1/exp* with the MOV of 21.01. For DDE, the best performance was provided by *DDE/best/1/bin* with the MOV of 2.13. The worst performance was provided by *DDE/current-to-best/exp* and *DDE/current-to-rand/1/exp* with the MOV of 21.01.

In dimension 1000, for DE, comparatively the best result was provided by *DE/best/2/bin* with the MOV of 1.52. The poorest performance was provided by *DE/current-to-rand/1/exp* and *DE/current-to-best/1/exp* with the MOV of 21.08. For DDE, the best performance was provided by *DDE/best/1/bin* with the MOV of 2.85. The worst performance was provided by *DDE/current-to-rand/1/exp* and *DDE/current-to-best/1/exp* with the MOV of 21.09.

The best performing DE variants for the dimension of 30,100,500 and 1000 are (*DE/rand-to-best/1/exp, DE/rand/1/ep)* (0), *DE/rand/2/bin* (0.02), *DE/best/2/bin* (1.75) and *DE/best/2/bin* (1.52), respectively. The best performing DDE variants are (*DDE/rand-to-best/1/*, DE/rand/1/*, DDE/rand/2/bin) (0) DDE/rand/2/bin* (1.24), *DDE/best/1/bin* (2.13) and *DDE/best/1/bin* (2.85), respectively. For f_4, the DE

TABLE VII. P_c COMPARISON FOR DE AND DDE VARIANTS

Variant	P_c				Variant	P_c			
	D=30	D=100	D=500	D=1000		D=30	D=100	D=500	D=1000
DE/rand/1/bin	34.25	0	0	0	DDE/rand/1/bin	50	0	0	0
DE/rand/1/exp	4	0	0	0	DDE/rand/1/exp	6.5	0	0	0
DE/best/1/bin	3.5	0	0	0	DDE/best/1/bin	39.75	0	0	0
DE/best/1/exp	0	0	0	0	DDE/best/1/exp	25	0	0	0
DE/rand/2/bin	0	1	0	0	DDE/rand/2/bin	25	0	0	0
DE/rand/2/exp	0	0	0	0	DDE/rand/2/exp	0	0	0	0
DE/best/2/bin	34.5	0	0	0	DDE/best/2/bin	36.75	0	0	0
DE/best/2/exp	25	0	0	0	DDE/best/2/exp	38.75	0	0	0
DE/current-to-rand/1/bin	0	0	0	0	DDE/current-to-rand/1/bin	0	0	0	0
DE/current-to-rand/1/exp	0	0	0	0	DDE/current-to-rand/1/exp	0	0	0	0
DE/current-to-best/1/bin	0	0	0	0	DDE/current-to-best/1/bin	0	0	0	0
DE/current-to-best/1/exp	0	0	0	0	DDE/current-to-best/1/exp	0	0	0	0
DE/rand-to-best/1/bin	35	0	0	0	DDE/rand-to-best/1/bin	0.25	0	0	0
DE/rand-to-best/1/exp	2.25	0	0	0	DDE/rand-to-best/1/exp	7.5	0	0	0

TABLE VIII. THE BEST AND WORST PERFORMING DE AND DDE VARIANTS FOR EACH FUNCTION-DIMENSION COMBINATIONS

D=30				
Function	Best Variant		Worst Variant	
	DE	DDE	DE	DDE
f_1	V13	V3	V12	V10
f_2	V7,V8	V4,V7,V8	V9	V11
f_3	V1,V13	V1,V13	V10	V7
f_4	V14,V2	V!,V2,V7,V13,V14	V10	V12
D=100				
Function	Best Variant		Worst Variant	
	DE	DDE	DE	DDE
f_1	V7	V1	V10	V6
f_2	V7	V3	V9	V11
f_3	V14	V3	V7	V7
f_4	V5	V5	V12	V10
D=500				
Function	Best Variant		Worst Variant	
	DE	DDE	DE	DDE
f_1	V4	V3	V5	V7
f_2	V1	V3	V9	V11
f_3	V3	V3	V10	V10
f_4	V7	V3	V10,V12	V10,V12
D=1000				
Function	Best Variant		Worst Variant	
	DE	DDE	DE	DDE
f_1	V4	V3	V13	V9
f_2	V13	V13	V4	V2
f_3	V14	V3	V6	V10
f_4	V7	V3	V10,V12	V10,V12

variants have achieved least MOV than the DDE variants.

On the other hand, the worst performing DE variants for the dimension of 30, 100, 500 and 1000 are *DE/current-to-rand/1/exp* (15.65), *DE/current-to-best/1/exp* (20.42), *DE/current-to-best/1/exp* (20.01) and *DE/current-to-best/1/exp* (21.08), respectively. The worst performing DDE variants are *DDE/current-to-best/1/exp* (15.67), *DDE/current-to-rand/1/exp* (20.41), *DDE/current-to-rand/1/exp* (20.01) and *DDE/current-to-best/1/exp* (21.09), respectively. In the case of worst performing variants, both DE and DDE have shown almost similar performance.

For f_4, most of the DDE variants outperformed their DE counterpart only in the dimension 30. For the higher dimensions, the superiority of the DDE variants is not much evident. For the dimensions 500 and 1000 two, for the dimension 100 four, out of fourteen DDE variants have outperformed their classical DE variant. On analyzing scalability of the variants, once again *DDE/best/1/bin* demonstrates its scalability in the dimensions of 500 and 1000. Similarly, *DE/best/2/bin* performs well in the dimension 500 and 1000. */rand/2/bin* could show its competitiveness in the dimension of 100.

The probability of convergence measured for all the variant-function-dimension combination is presented in Table VII, the results show than none of combination could provide any successful run for the dimension 100, 500 and 1000 (except four successful runs provided by the combination *DE/rand/2bin-f_4*-100). It shows, the *P*c value of the variants is decreasing with the dimension of the chosen problem. The best and worst performing DE and DDE variants for each of the function-dimension combinations are presented in Table VIII. It is observed from the results that even though *DE/best/1/bin* has not shown any superiority in high dimensional cases, the *DDE/best/1/bin* has given best performance in most of the combinations, in all the dimensions. But *DE/best/2/bin* has given competitive results in f_4 for D=500 and 1000, in f_1 and f_2 for D=100.

On the overall analysis for the functions f_1, f_2, f_3 *and* f_4 for the dimensions of 30, 100, 500 and 1000, we can highlight the following points: (1) The DDE variants are, comparatively, showing superior performance in reaching the global optimum with higher probability of convergence, for all the function-dimension combination, except few cases. (2) The variants with binomial recombination type and "*best*" selection scheme are most competitive in solving the function at hand. (3) Most of the best performing variants are "*bin*" variants, and the worst performing variants are the "*exp*" variants, on each function-dimension combinations. (4) The */best/2/bin* and

DDE/best/1/bin variants are found highly suitable for higher dimensional function optimization.

VI. CONCLUSION

An empirical comparative performance analysis of DE and DDE variants for various dimension are attempted. The results indeed identified that most of the DDE variants are outperforming their classical counterpart variants, irrespective of the dimension. The results obtained show that the variants with binomial recombination are relatively performing better than the variants with exponential crossover. With the scalability study of the variants, we found that *DDE/best/1/bin* and *DE/best/2/bin* are able to demonstrate their competitiveness, even in the higher dimensions, up to 1000. The other variants did not scale up to higher dimensions, this is due to the reason that the parameter setup (*CR, NP* and *F)* which is been used for 30 dimension is used for the other dimensions also. Our future work includes analyzing scalability of the variants with proper tuning of the control parameters, for each dimension, with more number of benchmark functions.

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