A SURVEY OF CALL MARKET (DISCRETE) AGENT BASED ARTIFICIAL STOCK MARKETS

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Abstract – **Artificial stock markets are models of financial markets used to study and understand market dynamics. Agent Based Artificial Stock Markets can be seen as any market model in which prices are formed endogenously as a result of participants' interaction and in which the representation of participants varies from simple equations of forecast functions to intricate software components endowed with human-like artificial-intelligence based adaptive behavior. There are various artificial stock markets in existence that are created using different strategies and customized for specific requirements. Trading sessions may be call market sessions or continuous sessions. Call market(Discrete) sessions occur at predefined intervals of time whereas trading happens continuously in continuous sessions. In this paper, we make a study of five such artificial stock market models namely Santa Fe Artificial Stock Market (SF-ASM), Genoa Artificial Stock Market(GASM), Agent Based Model for Investment (ABMI), Business School (BS) and Baron's Model (BM), all being call market or discrete time sessions. We analyze their features, design and their pros and cons based on a few important parameters.**

Key words— **Agents, artificial stock markets, call auctions, genetic algorithm, classifier systems, market makers, liquidity, efficient market hypothesis (EMH), rational expectations hypothesis(REH), Constant Absolute Risk Aversion (CARA), Constant Relative Risk Aversion (CRRA), forecasting and prediction.**

I. INTRODUCTION

The goal of agent-based modeling of stock markets is to enrich our understanding of fundamental processes that appear in a market. The emergent properties of an agent-based model are the result of "bottom-up" processes, rather than a "top-down" direction. The agent-based approach considers a population of intelligent adaptive agents and lets them interact in order to maximize their financial performance[1,2,3,4,5]. In Artificial Stock Markets (ASM), prices should emerge internally as a result of trading interactions of the market participants represented. ASMs are composed of many heterogeneous, interacting, adaptive agents and enable us study the stock market as a complex adaptive system rich in dynamics, and emergent properties. The ASMs considered in this paper for survey are discrete in nature: The discrete/continuous distinction is applied to the state of the environment, to the way time is handled, and to the percepts and actions of the agent [6]. The criteria for selecting the ASMs under study here is, that they should incorporate an endogenous price formation mechanism, and represent the behavior of market participants. Also the market models should have at least a well-defined price formation mechanism for at least one asset such that prices emerge internally through the interaction of represented market participants. Five ASMs have been shortlisted for the purpose of the study – namely Santa Fe (SF-ASM), Genoa Artificial Stock Market(GASM), Agent Based Model for Investment (ABMI), Business School (BS) and Baron's Model(BM), all being call market or discrete time sessions. We admit that the ASM chosen is selective and incomplete, but it covers the most widely discussed approaches. We first study the Santa Fe artificial stock market, which is considered as a very revolutionary model and has been accepted as a pioneering effort implemented over a decade ago. Then, we explore other models which had come up subsequently and then make a fair comparison of the five models based on a few selected parameters.

II. SANTA FE ARTIFICIAL STOCK MARKET

A. Introduction

The Santa Fe Artificial Stock Market (SF-ASM) [7,8] consists of a central computational market and a number of artificially-intelligent agents. The agents choose between investing in a stock and leaving their money in the bank, which pays a fixed interest rate. Agents make their investment decisions by attempting to forecast the future return on the stock using genetic algorithm to generate test and evolve predictive rules. The artificial market shows two distinct regimes of behavior namely, the rational expectations behavior and the complex realistic market behavior. The parameter settings and the initial conditions control the strategy. One of the parameters that can be used is the exploration rate, which governs how rapidly the agents explore new hypothesis with their genetic algorithms. At low exploration rates, the market settles into rational expectations equilibrium and at high exploration rates it falls into the realistic regime.

In the rational expectations equilibrium theory the agents select their optimum behavior by assuming that the agents have complete information, are perfectly rational, have common expectations and they know that everyone else have the same properties. Because of these assumptions there is neither any dynamics, nor learning nor evolution and everything is decided ab-initio.

B. The Agents.

The agents classify the available information; notice patterns in the information and generalize internal models from the noticed patterns and act on the basis of these models. However, the agents have to evaluate and adapt after seeing how well they work. In actuality, the agents have a number of different ways of predicting the future and they continually compare and evaluate them. The ones which work well gain more weight and are used more often. The market and the agent are co-evolving in the environment each action affecting the each other.

C. Market Structure.

The basic structure of the market is N agents, ranging from 50 to 100, interacting with the central market. The interaction between the agents is not direct but only via the market. A single stock exists with price p(t) per share at time t. The stock pays a dividend of d(t+1) per share at the end of time period t. The dividend time series d(t) is a stochastic process independent of the market and the agents' actions. The dividend d(t) is given by the simple random process

$$d(t+1) = pd(t) + \alpha\, n(t) \qquad (1)$$

where p and α are parameters and *n*(t) is a Gaussian random variable, chosen independently at each time t from a normal distribution with mean 0 and variance σ.

There is also a fixed-rate asset, the bank, which pays a constant rate or return r per period. The agents have to decide how much money they want to put into the stock and how much money they want to leave in the bank. At any time t, each agent *i* holds some number of shares, of stock $h_i(t)$ and has some amount of cash $M_i(t)$ in the bank. Its total wealth is then given by

$$w_i(t) = M_i(t) + h_i(t)p(t) \qquad (2)$$

At the end of the period, one time step later, this portfolio becomes worth

$$w_i'(t+1) = (1+r)M_i(t) + h_i(t)p(t+1) + h_i(t)d(t+1)$$

where the three terms are the money in the bank with interest, the new value of the stock, and the dividend pay-out.

The trading process is managed by a *specialist* inside the market. The specialist also has the job of setting the *p(t+1).* If there are more bids than offers, then the price is raised, so the bids drop and the offers increase, until they match closely.

D. World Bits

The information that is available to the agents at any given time in the market consists of the price, dividend, total number of bids, and total number of offers at each past time step and also includes a predictor of the future dividend and a random "sunspot" variable around which the agents might coordinate their actions. However, this information, known as the *world*, is condensed into a string of 80 bits and some recent price and dividend information, called as the *world bits,* each of which is either *true* or *false*.

E. Structure of Agents

The agents decide whether to invest in the stock or the bank. The *forecasting agents* are considered that use a number of predictors each of which attempts to predict the future return (price plus dividend). By seeing how well their predictors work, the agents can estimate their accuracy (prediction variance) and update or replace poor ones.

F. Constant Absolute Risk Aversion (CARA)

Because they know the variance of their overall predictions, the agents can also perform a risk aversion analysis- Constant Absolute Risk Aversion (CARA). When the mean and variance of the expected return is known for each asset, an optimal division of funds between two possible assets is made based on an exponential utility function. If agent *i*'s estimate of the mean return is $E_i[p(t+1)+d(t+1)]$ with variance *v,* then under CARA, the optimum number of shares to hold is given by

$$h_i^{desired}(t) = \frac{E_i[p(t+1)+d(t+1)]}{\lambda v} \qquad (3)$$

where λ is the degree of relative risk aversion.

The agents' predictors actually consist of two parts, a *condition part* and the *forecast part*. The *condition part* determines when each particular predictor is *activated,* as explained below. Only activated predictors produce forecasts, using their forecast part which is a linear rule

$$E_{ij}[p(t+1)+d(t+1)] = a_{ij}(p(t) + d(t)) + b_{ij} \qquad (4)$$

where $E_{ij}[]$ means the expected (predicted) value for i's j^{th} predictor and a_{ij} and b_{ij} are the coefficients that constitute the forecast part of this predictor.

G. Classifier Systems

The condition part of the predictor is implemented with a *classifier system*, in which the condition part is represented by a ternary string of the symbols {0,1,#}, one for each of the world bits that the agent can observe. 0 means *false,* 1 means *true* and # means either *true* or *false*.

H. Genetic Algorithm

Some of the agents' predictors may give good predictions when they are activated, while others may not. A genetic algorithm is used to adjust and evolve a better set of predictors. The genetic algorithm eliminates some of the worst predictors, those that have the highest variance, and generates some new ones to replace them. To generate new predictors, cloning, crossover and mutation are carried out.

I. Reported Results

The agents are given the initial beliefs in the rational expectations result by setting the initial conditions for a_{ij} and b_{ij} to calculated rational expectations values which is a local attractor, resulting in a very stable market with very little trading, and homogeneous agent behavior. Two regimes of behavior are seen viz, the rational expectations regime and the complex regime. Varying the parameter K indicated how often the genetic algorithm is run which controlled how often the new predictors were evolved. K=250 gave fast exploration (complex regime) and K=1000 slow exploration (rational expectations regime).

J. Discussion

SF-ASM was a trendsetter, one of the most complex artificial markets of the time. This market allows agents to explore a fairly wide range of possible forecasting rules. They have flexibility in using and ignoring different pieces of information. The interactions that cause trend following rules to persist are endogenous, they are not forced to be in the market. On the other hand, the market is relatively difficult to track in terms of a computer study. This makes it harder to make strong theoretical conclusions about the reflections of this market on real markets. Further, a few factors that are not considered or explored in SF-ASM include:

- Multiple stocks
- Impact of wealth
- Improved prediction
- Transition details
- Information control
- Strategic behavior

Hence we shift our focus towards another ASM model, the Genoa Artificial Stock Market(GASM), where we see that a few of the above factors are considered.

III. GENOA ARTIFICIAL STOCK MARKET(GASM)

A. Introduction

Genoa Artificial Stock Market (GASM) is, characterized with heterogeneous agents, which exhibit random buy or sell patterns and interact with each other[9]. The orders thus placed are processed by a module called the Market Maker, which decides the price of the asset. Orders which find their limit price compatible with the fixed asset price are satisfied.

B. Micro-structure of the GASM:

1) Traders

Each trader is modeled as an autonomous agent with certain amount of cash and stocks at the beginning. It is up to the trader to decide whether to sell his stock or use his cash to buy more stock. Their decisions depend on their current state ie, the cash he possesses at hand and the stocks he owes. The system has mainly three state variables(degrees of freedom):-

- The amount of cash in the system
- The number of stock in the system
- The price of each stock

2) Market Maker

The purpose of the market maker is to fix the price of the stock. It does so from the demand and supply curves. Demand curve gives the price per stock against the demand for the stock (ordered quantity). Similarly supply curve gives the price per stock against the ordered quantity. The price formation process is given by the intersection point of these two curves.

Only orders compatible with prices can be executed. The market maker can also add cash into the system or add assets to the system. The size of the buy order or sell order may vary. If the size of sell orders is larger than the size of the buy orders then the market maker adds cash to the system and subtracts assets from it and vice versa for the reverse. The market maker may thus be thought of as having unlimited cash and assets capable of satisfying any order. All orders that do not satisfy the clearing price are discarded.

3) Functioning of the Market

At the beginning of the simulation, the current price $p(0)$ is set and each trader is given an amount of cash and an amount of stocks. The trader issues buy and sell orders with the same probability and are totally independent from each other. In this model, each trader is marked with a tendency to be optimist or pessimist. At each time step, random links are added among traders sharing the same tendency, with a probability P_a , hence clusters of traders sharing the same opinion gradually take shape. A link between two traders belonging to different clusters, results in merging of clusters into a bigger one.

At each simulation step clusters of both optimist and pessimist traders are randomly chosen with probability P_c. All traders belonging to a chosen cluster receive a message to buy (if they are optimist) or to sell (if they are pessimist) as far as they can. After the aggregate orders are placed, all links of traders belonging to the chosen clusters are broken, and these traders change their tendency. As optimists have bought almost all the stocks they could, their tendency switches to pessimist as they don't have any more cash for buying, but only stocks to sell (vice-versa for pessimists).

C. Discussion

The market's main features include the following:

- It has been developed using efficient programming techniques which makes it easily upgradeable and modifiable.
- The portfolio and cash of every single trader, order and transaction are tracked.
- It follows the realistic price formation mechanism.
- With a mechanism called aggregation of traders, the GASM is able to reproduce the phenomenon of *"fat-tails"* seen in real markets.

This aggregate behavior reflects in a simplified way *mechanisms of opinion formation* actually in place in real markets. However, in this model, there is no learning for the agents and hence there is no evolution in their behavior. Since evolution is a very important aspect of artificial stock markets, we move on to the next model, Agent Based Model for Investment which includes models for price formation and agents' behavior.

IV. AGENT BASED MODEL FOR INVESTMENT(ABMI)

A. Introduction

This ASM[10] illustrates how simple agent-based systems can be used for modeling and studying stock markets. There are a few types of investors and a market maker, all represented as agents. The role of the market maker is to adjust prices as a function of the order imbalance. The study shows in what sense the market mechanism matters. Risk-averse behavior of the market maker, for example, introduces trends in prices. This is caused by the fact that if the market maker acquires a position he wants to get rid of it. Structure in price series creates opportunity for technical traders. In the model there is a point at which the market is efficient (i.e. everyone breaks even). The authors analyze under which conditions the market will converge to this point.

B. Market-Making Model.

The ABMI model is based on an order-driven market. Traders place orders—in this case, market orders—and the market maker provides liquidity by buying and selling. The agents do not know at what price the orders will be filled, so the transactions automatically take place out of equilibrium. A single asset and a single representative market maker is assumed. Market orders only are allowed and actions are synchronized, so that trades take place at t, t + 1, and so on, using the following price-formation rule:

$$p(t+1) - p(t) = \frac{1}{\lambda}\left[\sum_{i=1}^{N} \omega^i(t) - \beta X(t)\right]. \quad (5)$$

where
ω^i = market order of agent i
p = log price
ω^$i(t + 1) = x^i(t + 1) - x^i(t)$
λ = liquidity
X = market-maker position
β = market-maker risk aversion
x^i = position of agent i
The change in the logarithm of the price is proportional to the sum of the net order imbalance. The first term is the sum of the orders that are placed by the agents at time t, and the second term is the order placed by the market maker, which is always a fraction β of the market maker's current position. The variable X, the market maker's total position, is the total amount that supply and demand are out of balance in the market. The constant of proportionality $1/\lambda$, can be thought of as liquidity, and it determines the amount that an order of a given size will move the price. Any particular family of agent behaviors can be studied by specifying a set of functions $x^i(t)$ and iterating the resulting equations.

C. Behavioral Models

The behavioral model involves four classes of investors: Market makers as seen above, fundamental (or value) investors, technical traders (or chartists), and liquidity demanders (or noise traders).Value investors take a position based on the perceived fundamental value of an asset. The more underpriced the asset, the larger the position that value investors take in the asset. The asset's perceived fundamental value will change over time according to a logarithmic random walk. Thus, it is assumed that there is some positive number that the value investor perceives as being associated with the asset, which changes randomly with time.

D. Discussion

The agent based model replicates some of the properties of real prices. This model is simple, but only certain basic points are illustrated. The market mechanism operation is illustrated: Using an order-based market with market makers as liquidity providers can result in patterns in prices that sustain trend followers. When a large number of different strategies are introduced and the market let to select them, the dynamics of the strategies interacting with each other become prominent.

V. BUSINESS SCHOOL(BS)

A. Introduction

The Business School[11,12,13]is an agent-based model of a so-called "school" (actually strategies) which is used to forecast future values and then evolves over time as a function of their performance. Investors update from time to time the forecast function selected from the school if it does not predict satisfactorily.

B. Traders

All traders share the same constant absolute risk aversion (CARA) utility function. Traders can accumulate their wealth by making investments. There are two assets available for traders to invest in. One is the riskless interest-bearing asset (money), and the other is the risky asset(stock). Stocks pay dividends following a stochastic process not known to traders. The goal of each trader is to myopically maximize the one-period expected utility function The key point in relation to this ASM is the formation of expectation, which is modeled by genetic programming. The call market sessions are implemented, traders simultaneously submit orders that are centrally matched at a price at equilibrium.

C. Evolution

The population of forecast functions in the school is evaluated and evolves over time using a GA-based technique. In an evolution phase badly performing strategies are eliminated and give place to new strategies. At BS at every trading period in the experiments there is a probability for each trader to go back to learn. This probability depends on the relative net change in wealth (compared to all traders) and on the growth-rate of wealth. Learning means choosing a forecast function (randomly) from the set of functions that would have performed better for the latest given number of periods.

D. Discussion

In the BS, the fundamental value depends on the current price and the dividend paid which result in a random independent and identically distributed (IID) return series in a world with technical traders. The basic framework is the standard asset pricing model. The dynamics of the market are determined by the interactions of many heterogeneous agents. Each of them, based on his forecast of the future, maximizes his expected utility. This models an individual as a collection of decision rules. These decision rules are continuously under review and revision; new decision rules are tried and tested against experience, and rules that produce desirable outcomes supplant those that do not. This ASM has successfully demonstrated the emergence of macro-phenomena of financial markets, endogenously generated from interactions among evolving decentralized system of autonomous adaptive agents without exogenously imposing any conditions.

VI BARON'S MODEL (BM)

A. Introduction

The design proposed by LeBaron[14,15,16] is reviewed in the following paragraphs. The investment decisions of agents are based upon an information set. A Walrasian auction is adopted to determine the price. The model contains two assets for investment – cash and equity. Cash pays a constant guaranteed rate of return r_f (risk free). The equity pays a dividend at each time step. This is random and the log-dividend follows a random walk:

$$\log(d_{t+1}) = \log(d_t) + \varepsilon_t \qquad (6)$$

where d_t is the dividend and ε_t is a Gaussian random variable $N(\mu,\sigma)$. The equity is available in a fixed supply of one share for the population. If s_i is the share holding of agent i, the summation of holding of all agents will be always maintained at unity.The equity price arises through the interactions of the agents.

B. Agents

The model contains a number of agents with a certain wealth and at each time step it decides how much of its wealth should be allocated to equity and how much to cash. The agents are of Constant Relative Risk Aversion (CRRA) of logarithmic form and at time t makes these decisions in an attempt to maximise its lifetime utility. The optimal amount of wealth to consume at a single time step is taken as a constant proportion of wealth. The agent restricts itself over the next single time step. In order to maximise the utility, the expected log-return is maximised:

$$E_t \log[1 + \alpha_t r_{t+1} + (1 - \alpha_t) r_f] \qquad (7)$$

where α_t is the proportion of wealth allocated to equity, r_{t+1} is the return achieved from equity in the period (t, t+1) and r_f the constant cash return. Since the equity returns distribution is not known in advance (these arise from the interaction of the agents), the agents maximise a sample expectation taken from historic returns. Because the distribution of returns may change over time, agents look at the last Ti periods, to maximise

$$1/Ti \sum_{K=1}^{Ti} \log[1 + (\alpha_{t-k}\ r_{t+1-k}) + (1 - \alpha_{t-k})\, r_f] \qquad (8)$$

where Ti is a constant for agent i. The choice of Ti will affect an agent's performance. Allocation to equities is done using one of a pool of rules. A rule recommends the proportion of savings an agent should allocate to equities, taking information about the current state of the market. The rules are implemented as simple feed forward neural network(FFNN) with a single hidden unit giving an output.

C. The Information Set

The information set consists of six items reflecting various fundamental and technical trading strategies. The first three technical trading inputs are the returns on equity in the previous three time-steps. The fourth is a measure of how the current price differs from the rational-expectations price. The last two inputs measure the ratio between the current price and exponentially weighted moving averages of the price.

D. Trading and price-setting

For a given share price p, each agent can determine how much of its wealth is to be invested in shares and arrives at a demand function for shares

$$d_{i,t}(p_t) = [\alpha_i(p_t, I_t)\ \beta\ W_{i,t}] / p_t \qquad (9)$$

where i denotes the agent, t refers to time, and I_t represents the information set. A Walrasian auction is then used to find the price p_t.

$$\sum_{I=1}^{N_{agents}} d_{i,t}(p_t) = N_{shares} \qquad (10)$$

where N_{agents} is the number of agents and N_{shares} the number of shares. This non-linear equation is solved using complex recursive function which searches for a value of p_t that satisfies these equations starting from the price at the previous time-step.

E. Adaptation and evolution

The model contains three forms of adaptation and evolution :-

- At each time step a proportion of the agents adapt by selecting a randomly chosen rule after comparing with the current rule.
- Agents evolve at each time step, wherein agents with the least wealth is removed and replaced by a new agent.
- The rules are also evolved by being replaced if it has not been used for 10 time steps.

F. Discussion

This ASM demonstrates some of the empirical features generated in an agent-based computational stock market with market participants adapting and evolving over time.

Investors view differing lengths of past information as being relevant to their investment decision making process. The interaction of these memory lengths in determining market prices creates a kind of market ecology in which it is difficult for the more stable longer horizon agents to take over the market. The market generates some features that are similar to those from actual data, viz, magnifying the volatility from the dividend process, inducing persistence in volatility and volume, and generating fat-tailed return distributions.

One of the goals of this market has been to streamline some of the complexities of the Santa Fe Artificial Market.

VII COMPARATIVE STUDY OF ASMs

A. Basis of Comparison

The design and mechanisms of these ASMs is analysed based on the following aspects:-

- Structure.
- Formation of price.
- Traders' behavior.

B. Structure. The various architectural elements of the markets they model is given below.

- *Assets traded*

  In general to reduce computational complexity, ASMs trade only few assets, usually two types of assets viz one risk-free and one risky stock. Exception being GASM where two risky assets can be traded. Risk free assets might represent PPF or Govt bonds paying a constant interest rate. Dividends paid by risky assets are represented in s SF-ASM, BS and BM, wherein additional dynamics are observed. Dividends are generally modeled as stochastic processes. In the BS the fundamental value depends on the current price and the dividend paid. In the SF-ASM and BM, the dividend paid by the risky stock is compared to the interest rate of the risk-free stock to get its real value.

- *Orders Generated*

  Trading is either by market or by limit orders. In GASM and SF-ASM it is limit orders.

- *Market Participants*

  Individual investors are simulated in all ASMs. Brokers are not modeled. The behavior of market makers is modeled in SF-ASM, BM and ABMI.

- *Execution System*

  In all models traders simultaneously submit orders that are centrally matched at a price at equilibrium - the execution system being "single-price auction".

C. Formation of Price

1) Placement of Order

a) Investor Objectives

The main investment objectives are to maximize profits by fundamental or technical strategy as follows:-

- ABMI - Arbitrage opportunities.
- SF-ASM,BS and BM - Utility function.
- GASM - Portfolio optimization.

b) Time Horizon

The majority of the objectives is long-term as they remain fixed during the full length of the experiments. In BM however, Long and Short horizons are considered.

c) Attitude to Risk

Investors' attitude to risk is modeled by some ASMs by introducing risk averse traders:-

- SF-ASM and BS - CARA.
- BM- CRRA of logarithmic form.

d) Investment strategy

Investors are classified as informed or fundamentalist traders:-

- Fundamentalists or informed traders are in ABMI and GASM. At SF-ASM and BM, investors compare the dividend paid to the interest rate of the risk free asset.
- Technical trading strategies are considered in all ASMs. While in ABMI it is trend followers, In GASM the technical trading strategies include mean-variance trading.
- Both fundamental and technical strategy is used by traders at SF-ASM and BM for forecasting future values, applying moving average functions.
- In BS forecast functions generated are combinations based on past prices and dividends.

e) Learning

- In SF-ASM, BS and BM, traders can switch strategies if they are not successful enough.
- At SF-ASM and BM, each trader has its own set of strategies, from which they choose the most suitable one every trading round.
- Neural networks and evolutionary algorithms are two commonly used techniques to implement learning. SF-ASM , BS and BM apply this technique.
- Selection, mutation and crossover are applied to adapt the set of strategies to the changing conditions.
- At SF-ASM and BM agents who learn are selected centrally with some probability every given trading period.
- In the BS model, investors try to find the best trading strategies by genetic programming.
- In BS at every trading period in the experiments there is a probability for each trader to learn.
- Fitness function in GA is maximum return, wealth, utility, or the minimum forecast error .
- At SF-ASM, the distance of the forecast value from the real outcome indicates the fitness.

f) Various Timing Issues

- In all ASMs except BM, the time-horizon of the investment objectives hold during the whole experiment.
- In BM, long and short memory traders are modeled.

- Forecast horizon of the investment strategies is one period ahead.
- All traders simultaneously make a trading decision whereby it usually results in placing a new order.
- Asynchronous behavior is modeled by selecting only a fraction of traders at any time.

2) Execution of Order. The brokers are not represented in any of these ASMs.

a) Order execution by market makers.

- At ABMI the market maker trades based on his position. All orders are market orders, and price is centrally set according to an automated mechanism. The market maker is represented by a simple equation.
- At SF-ASM and BM, the market price is defined by an auctioneer.

b) Equilibrium price.

- At SF-ASM, equilibrium is determined at the price at which trading volume is maximized.
- In GASM a new market price is often at the intersection of demand and supply curves.
- At BS price is based on the excess demand/ supply discounted with some adjustment value.
- At BM, Walrasian auction is done.

VIII. FINDINGS OF ASMs

The large scale of design and implementation approaches applied in the ASMs studied here demonstrates that there are many methods to represent traders, to determine a forecasting strategy, to implement learning, to construct a portfolio and develop other decision strategies that lead the investors to place certain orders. In addition a variety of order execution and price setting mechanisms are represented. ASMs mainly focus on the analysis of price or return dynamics.

A. Support for Classical Theory of EMH and REH

In the studies on ASMs, depending on the methodology adopted, support for both classical theory and empirical “stylized facts” has been observed. All ASMs show that it will not be possible to earn above-average profits, thereby implying that the given market is efficient.

- Evidence for the Efficient Market Hypothesis (EMH) and the Rational Expectations Hypothesis(REH) is found within the BS model.
- In ABMI, where interacting investors have credit limits, the market tends to a point where everything is in equilibrium, i.e the wealth of traders breaks even.
- The case with slowly learning traders at SF-ASM or the Long Horizon agents in BM approximates the equilibrium point predicted by the REH.

B. PatternsObserved

Many ASMs find evidence for stylized facts.

- Fat tails and volatility clusters are observed within SF-ASM, BM and GASM.
- In SF-ASM and BM, a correlation between the trading volume and volatility is found.
- A reversion to mean is reported in the study on the GASM market.
- Trend followers in the ABMI, mean-reversion traders at the GASM market, and fast learning technical traders at SF-ASM and BM can systematically dominate the market, meaning that the market is not efficient in these cases.

C. Summary of important parameters

Table1 gives a comparative study of important parameters having a predominant impact on the market dynamics. These parameters serve as factors that help in distinctly drawing out the lines of difference in the ASMs.

The investment objective parameter analyses the goal of the ASM. In Santa Fe, the agents form their demands for stock based on a utility factor, CARA, and in BM it is CRRA. In ABMI, agents have the simple objective of maximizing the profit. GASM aims at maximizing liquidity.

The execution systems applied is the Single price auction system. Also based on market participants, ABMI and GASM models can have any number of individual investors, Santa Fe accommodates 50-100 individual investors.

VI CONCLUSION

The thrust of this paper is a survey on agent-based artificial stock markets based on call based trading session (Discrete time simulation). We looked at several ASMs and analyzed how they cover the important organizational and behavioral aspects of stock markets. A vast number of trading strategies in a broad range of market organizations is illustrated by the ASMs presented. Based on the comparison we deduce the following main conclusions[17]:

- Autonomous asynchronous behavior of traders is rarely represented.
- There are many ways to represent the aspects of price formation and trading behavior.
- Most ASMs focus on representing only the investor.
- Representations of traders in ASMs are not autonomous.
- In all above ASMs, traders place orders at discrete points in time. Orders are then aggregated and the new price is determined at some equilibrium point(call auctions). Therefore, call market(discrete) sessions ASMs lack in the following representations: *continuous trading, asynchronous behavior, autonomous behavior, and the representation of brokers.*

REFERENCES

[1] L. Tesfatsion. Agent-based computational economics: Growing economies from the bottom up. Artificial Life, 8:55–82, 2002.

[2] Tesfatsion, L. (2006), Agent-based computational economics: A constructive approach to economic theory, in L. Tesfatsion and K. L.

Judd, eds, 'Handbook of Computational Economics: Agent-Based Computational Economics', Elsevier North-Holland.

[3] LeBaron, B. (2000), 'Agent-based computational finance: Suggested readings and early research',Journal of Economic Dynamics and Control 24, 679–702.

[4] LeBaron, B. (2001), 'A builder's guide to agent based financial markets', Quantitative Finance 1(2), 254–261.

[5] LeBaron, B., Arthur, W. B. and Palmer, R. (1999), 'Time series properties of an artificial stock market', Journal of Economic Dynamics and Control 23, 1487–1516.

[6] Russell, S. and Norvig, P. (2003), Artificial Intelligence. A Modern Approach, Third edition, Prentice Hall.

[7] Blake LeBaron, Building the Santa Fe Artificial Stock Market, Brandeis University, June 2002.

[8] Tesfatsion, L. (2004), 'Notes on the Santa Fe artificial stock market model' http://www.econ.iastate.edu/classes/ econ308 /tesfatsion/sfistock.htm.

[9] Michele Marchesi, Silvano Cincotti, Sergio Focardi, Marco Raberto, Development and testing of an artificial stock market, Genoa Artificial Stock Market, September, 2005.

[10] Farmer, J. (2001), Toward agent-based models for investment, benchmarks and attribution analysis, Working paper, Association for Investment Management and Research.

[11] S.-H. Chen, C.-H. Yeh, Evolving traders and the business school with genetic programming: a new architecture of the agent-based artificial stock market, Journal of Economic Dynamics and Control 25 (2001) 363–393.

[12] S.-H. Chen, C.-H. Yeh, On the emergent properties of artificial stock markets, Journal of Economic Behavior and Organization 49 (2002) 217–239.

[13] S.-H. Chen, C.-H. Yeh, C.-C. Liao, On AIE-ASM: a software to simulate artificial stock markets with genetic programming, in: S.-H. Chen (Ed.), Evolutionary Computation in Economics and Finance, Physica-Verlag, Heidelberg, 2002, pp. 107–122.

[14] B. LeBaron. Empirical regularities from interacting long- and short-memory investors in an agent based stock market.IEEE Transactions on Evolutionary Computation,5(5):442–455,Oct 2001.

[15] B. LeBaron. Evolution and time horizons in an agent based stock market. Macroeconomic Dynamics, 5:225–254, 2001.

[16] B. LeBaron. A builder's guide to agent based financial markets. Quantitative Finance, 1(2):254–261, 2001

[17] Katalin Boer-Sorban,(2008), Agent-Based Simulation of Financial Markets- A Modular, Continuous-Time Approach, PhD Thesis, RSM Erasmus University / Erasmus School of Economics

Table1: A Comparative Study of Important Parameters in Selected Artificial Stock Markets (Call Market)

ASM / *PARAMETERS*	*Santa Fe* (*SF-ASM*)	*Agent Based Model for Investment* (*ABMI*)	*Genoa Artificial Stock Market* (*GASM*)	*Business School* (*BS*)	*Baron's Model* (*BM*)
Traded Assets	Experiments conducted with both risky and risk free assets.	Experiments conducted with only risky assets.	Experiments conducted with both risky and risk free assets.	Experiments conducted with both risky and risk free assets.	Experiments conducted with both risky and risk free assets.
Fundamental Value	Price, Dividend & Risk free Interest rate	Log random walk	-	Price, Dividend	Price, Dividend & Risk free Interest rate
Dividend	Auto Regressive	-	-	IID(stochastic)	Auto Regressive
Types of orders	Limit Orders	Market Orders	Limit Orders	Market Orders	Limit Orders
Trading Sessions	Discrete Time (Call Based Trading Session)	Discrete Time (Call Based Trading Session)	Discrete Time (Call Based Trading Session)	Discrete Time (Call Based Trading Session)	Discrete Time (Call Based Trading Session)
Execution System	Single Price Auctions	Single Price Auctions	Single Price Auctions	Single Price Auctions	Single Price Auctions
Investment Objective	Maximize CARA utility	Arbitrage/ Maximize profit	Optimize by maximizing utility/ liquidity	Maximise CARA	CRRA of logarithmic form
Market Participants	50 – 100 individual investors 1 Market Maker	N individual investors 1 Market Maker	N individual investors No Market maker	500	N individual investors 1 Market Maker
Evolution	Evolving(NN & GA)	No evolution	No evolution	Evolving(NN & GA)	Evolving(NN & GA)