Adjoint LMS (ALMS) Algorithm Based Active Noise Control with Feedback Path Modeling

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Abstract: In active noise control (ANC) systems, there exists an inherent feedback from the loudspeaker to the primary microphone. Adjoint least mean square (ALMS) algorithm is known to be an alternative to the widely used filtered x LMS (FxLMS) for reducing the computational complexity and memory requirements, especially in the case of multi-channel systems. Further FxLMS algorithm is based on the assumption that the order of the weighing filter and secondary path can be commuted which is not always true in practice. Though ALMS do not make such an assumption, neither FxLMS nor the ALMS algorithms consider the feedback path effect that is inherent in ANC systems. We propose a feedback ANC system based on ALMS algorithm which is analogous to the system based on FxLMS. Detailed computational complexity analysis for addition and multiplication requirements is presented and are compared with those of its counterpart to establish its usefulness. Simulation results show the convergence characteristics of the ALMS based ANC with feedback path modeling is on par with that based on FxLMS.

Index Terms: Active noise control (ANC), feedback path effect, Filtered–x LMS (FxLMS) algorithm, Adjoint LMS (ALMS) algorithm, computational complexity.

1. Introduction

Active noise control (ANC) has been a field of growing interest over the past few decades [1]. The underlying principle of ANC system is the superposition of the noise signal that is to be suppressed (called the primary noise) and the anti-noise signal. The anti-noise signal must have the same amplitude as that of the primary noise, but with an opposite phase [1,2]. The implementation is generally very complex, especially when the overall system parameters change dynamically.

In ANC applications such as duct of an air-conditioning system, there exist an inherent feedback from the loudspeaker (used for generating anti-noise signal) to the reference microphone. Several algorithms have been proposed in the literature for the active mitigation of the noise, of which some of them are computationally less intense, [3] but does not take into consideration the feedback that exist from the secondary source to the reference microphone. Other algorithms that are recent and quite useful in the nonlinear noise case systems can be seen in [4, 5] but they also does not consider the feedback path effect. Of the several methods proposed in the literature, to compensate the effect of feedback, the very recent one by M. T. Akhtar et. Al., [6, 7] appears to be the best which is based on FxLMS. In the literature, adjoint least mean square (ALMS) algorithm is known to be an alternative to the widely used FxLMS for reducing the computational complexity and memory requirements [8, 9]. We propose a feedback ANC system based on ALMS algorithm which is analogous to a similar system based on FxLMS.

The positive features of ALMS as given in [8, 9] are preserved in the proposed ALMS based ANC with feedback path modeling. In practice the secondary path changes with time, which necessitates the online secondary path modeling. The advantage with the ANC structure given in [6, 7] and also in the proposed algorithm is that it can be easily extended to include the online secondary path modeling.

The organization of this paper is as follows: Section 2 gives a brief review of the existing ANC with online feedback path modeling based on FxLMS. In Section 3, the adjoint LMS algorithm replaces the FxLMS used in Section 2. Computational complexity of ALMS with and without online feedback path modeling is summarized in Section 4 and is compared against those of the ANC with feedback path modeling based on FxLMS. Appropriate MATLAB simulations are shown in Section 5. Section 6 concludes the paper.

Notation convention:

$x(n)$: Input signal which upon filtering with adaptive filter $W(z)$ yields the anti-noise signal, $y(n)$

$r(n)$: Reference noise signal generated by noise source.

$e(n)$: Residual error at the n^{th} time instant.

$y_f(n)$: Feedback component due to the anti-noise signal.

$v_f(n)$: Feedback component due to the modeling signal $v(n)$.

$d(n)$: Signal at the end of primary path near the error microphone, also called the desired response.

L_w, L_f : Lengths of the adaptive filter $W(z)$ and the feedback path modeling filter $\hat{F}(z)$, respectively.

L_h, L_s : Tap-weight length of the adaptive noise cancellation filter $H(z)$ and the secondary path filter $\hat{S}(z)$, respectively.

$S(z)$, $\mathbf{s}(n)$: Matrix forms of secondary path transfer function (in the z-domain) and its impulse response at the n^{th} instant, respectively, that connects the secondary source to the error microphone.

μ_w: Step size for the updation of the adaptive filter $W(z)$.

2. ANC with Online Acoustic Feedback Path Modeling Based on FxLMS

Referring to Fig. 1, which is reproduced from [7], it can be observed that there exists an acoustic feedback path, $F(z)$ from the loudspeaker to the reference microphone. The acoustic feedback path is modeled using a white noise signal $v(n)$ as the input. Let $r(n)$ denote the actual reference noise and $y_f(n)$ be the feedback due to the anti-noise signal $y(n)$. If $v_f(n)$ is the feedback component due to the white noise $v(n)$, then effectively, the signal entering the reference microphone will be

$$s(n) = r(n) + y_f(n) + v_f(n). \qquad (1)$$

Estimating the feedback path by a mathematical model is a system identification problem.
The filter $\hat{F}(z)$ models the secondary path and is referred to as feedback path modeling (FBPM) filter.
The governing equations for the ANC system under consideration are:

$$x(n) = s(n) - \hat{y}_f(n) - \hat{v}_f(n), \qquad (2)$$

where $x(n)$ is the input to the adaptive filter $W(z)$.
The output of the adaptive filter $W(z)$ is given by

$$y(n) = \mathbf{w}^T(n)\mathbf{x}_{L_w}(n), \qquad (3)$$

where $\mathbf{w}^T(n) = \left[w_0(n) \;\; w_1(n) \;\; \ldots \;\; w_{L_w-1}(n)\right]$

and

$$\mathbf{x}_{L_w}(n) = \left[x(n) \;\; x(n-1) \;\; \ldots \;\; x(n-L_w+1)\right]^T$$

.

An estimate of the feedback due to the anti-noise signal
$y(n)$ is given by

$$\hat{y}_f(n) = \hat{\mathbf{f}}^T(n)\mathbf{y}_{L_f}(n), \qquad (4)$$

where $\hat{\mathbf{f}}^T(n) = \left[f_0(n) \;\; f_1(n) \ldots f_{L_f-1}(n)\right]$

and

$$\mathbf{y}_{L_f}(n) = \left[y(n-1) \;\; y(n-2) \;\; \ldots \;\; y(n-L_f)\right]^T.$$

White noise $v(n)$ gets added to the output of the adaptive filter $W(z)$ and comes out through the secondary source. The output of the secondary source passes through the acoustic medium, commonly referred to as the secondary path $S(z)$, before entering into the error microphone.

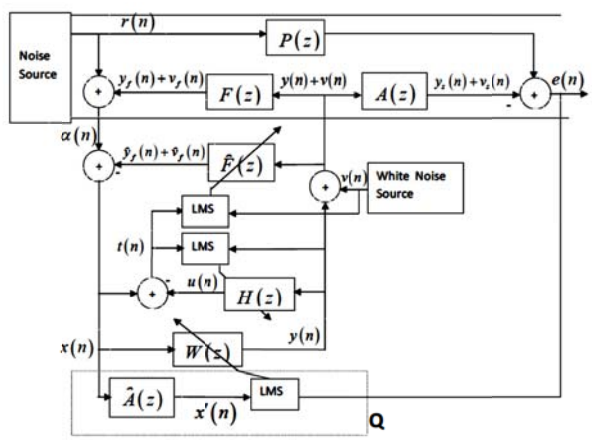

Fig. 1: ANC with online feedback path modeling (based on FxLMS) reproduced from [7]

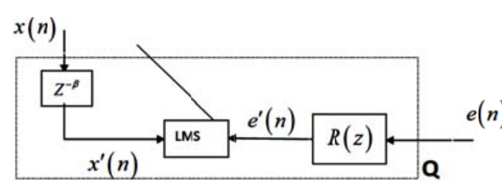

Fig. 2: The modification made to the dashed block Q of Fig. 1 to make the ANC system to be based on adjoint LMS rather than FxLMS

In order to compensate for the secondary path effect, the weight updation of the filter $W(z)$ is done using the secondary path filtered inputs [see Eqns. (7) and

(8)], as in the FXLMS algorithm. An adaptive noise cancellation (ADNC) filter, $H(z)$, is used to remove the disturbance in the desired response of the FBPM filter. The output of the ADNC filter $H(z)$ is given by

$$u(n) = \mathbf{h}^T(n)\mathbf{y}_{L_h}(n), \quad (5)$$

where $\mathbf{h}^T(n) = \left[h_0(n) \;\; h_1(n) \;\; \ldots \;\; h_{L_h-1}(n)\right]$

and

$$\mathbf{y}_{L_h}(n) = \left[y(n-1) \;\; y(n-2) \;\; \ldots \;\; y(n-L_h)\right]^T.$$

An estimate of the feedback component due to white noise $v(n)$ is given by

$$\hat{v}_f(n) = \hat{\mathbf{f}}^T(n)\mathbf{v}(n), \quad (6)$$

where

$$\mathbf{v}(n) = \left[v(n) \;\; v(n-1) \;\; \ldots \;\; v(n-L_f+1)\right]^T.$$

The three adaptive filters are updated as outlined in the following equations:

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_w e(n)\mathbf{x}'(n), \quad (7)$$

where

$$\mathbf{x}'(n) = \left[x'(n) \;\; x'(n-1) \;\; \ldots \;\; x'(n-L_a+1)\right]^T$$

and $$x'(n) = \hat{\mathbf{a}}^T(n)\mathbf{x}_{L_a}(n). \quad (8)$$

$$\hat{\mathbf{f}}(n+1) = \hat{\mathbf{f}}(n) + \mu_f\left[d_f(n) - \hat{v}_f(n)\right]\mathbf{v}(n) \quad (9)$$

$$\mathbf{h}(n+1) = \mathbf{h}(n) + \mu_h d_f(n)\mathbf{y}_{L_h}(n) \quad (10)$$

3. Proposed online feedback path modeling for ANC with ALMS algorithm

For the ALMS based system, the part enclosed within the dashed block, marked 'Q' in Fig. 1 gets modified to that shown in Fig. 2, from which it can be seen that to compensate for the secondary path effect a block of $Z^{-\beta}\hat{A}(z^{-1}) = R(z)$ is placed as in Fig. 2. The delay factor $Z^{-\beta}$ has been introduced to maintain causality, because $\hat{A}(z^{-1})$ may turn out to be non-causal. Then to match this delay in the error path, a delay element $Z^{-\beta}$ is introduced in place of the secondary path estimate, $\hat{A}(z)$ in the process of getting $x'(n)$, as shown in Fig. 2.

With these modifications in effect, the equations (1) – (10) are still valid except (7), (8) and we further have,

$$\mathbf{w}(n+1) = \mathbf{w}(n) + \mu_w e'(n)\mathbf{x}(n-\beta), \quad (11)$$

where,

$$e'(n) = \hat{A}(z^{-1}, n)e(n) = \sum_{n=0}^{M} a_k(n)e(n-\beta) \quad (12)$$

β and M are chosen so as to maintain causality as discussed in [8].

The filtered error $e'(n)$ is obtained by filtering the error $e(n)$ by $R(z) = \hat{A}(z^{-1}, n)$. $\hat{A}(z^{-1}, n)$ is obtained by replacing z with z^{-1} in the causal secondary path estimate $\hat{A}(z)$ and then multiplying the result with $z^{-\delta}$ (δ should be non-negative integer and as minimum as possible) so that it becomes a causal filter.

Table 1: Computational complexity contributed by various equations

Equation	Number of multiplications with feedback path modeling		Number of additions with feedback path modeling	
	FxLMS based	ALMS based	FxLMS based	ALMS based
2	0	0	2	2
3	L_w	L_w	$L_w - 1$	$L_w - 1$
4	L_f	L_f	$L_f - 1$	$L_f - 1$
5	L_h	L_h	$L_h - 1$	$L_h - 1$
6	L_f	L_f	$L_f - 1$	$L_f - 1$
7, 11	L_w	L_w	L_w	L_w
8	L_a		$L_a - 1$	
9	L_f	L_f	$L_f + 1$	$L_f + 1$
10	L_h	L_h	L_h	L_h
Filtering $e(n)$		L_a		$L_a - 1$
Total	$2L_w + L_a + 3L_f + 2L_h$	$2L_w + L_a + 3L_f + 2L_h$	$2L_w + L_a + 3L_f + 2L_h - 2$	$2L_w + L_a + 3L_f + 2L_h - 2$

4 Computational complexity

The computations contributed by various equations are listed in Table 1. This shows the total number of multiplications and additions required per output sample by the Akhtar's method based on FxLMS and the proposed method based on ALMS algorithm. The unfilled cells indicate that the corresponding equations are not relevant for the particular method. By choosing $L_w = L_f = L_h = L_a$ we obtain the number of multiplications and additions for both FxLMS and ALMS methods to be $8L_w$ and $8L_w - 2$ respectively. For a specific case with $L_w = L_f = L_h = L_a = 128$, we need 1024 multiplications and 1022 additions.

5 Simulations

The FxLMS based ANC algorithm with online feedback path modeling is simulated with the following specifications to check its convergence characteristics.
The primary path, secondary path and feedback path transfer functions are taken respectively as:

$P(z) = 0.71z^{-5} - 0.59z^{-6} + 0.9z^{-7}$,

$A(z) = -0.5z^{-6} + z^{-7}$ and

$F(z) = 0.01z^{-5} - 0.9z^{-6} + 0.09z^{-7}$.

The step sizes are chosen to be μ_w = 0.06, μ_h = 0.06 and μ_f = 0.9.

***Experiment 1*: Random input:**

The random input is generated with 1,00,000 samples in the range of [-0.5, 0.5] with zero-mean. The normalized mean-square error in dB (NMSE) plot is drawn by considering an average of 20 independent trials and is shown in Fig. 3. It is seen that the performance regarding convergence is equivalent in both the ALMS and FxLMS based ANC systems when feedback path effect is considered. The weight convergence is shown for the two algorithms in Fig. 4, from which an interesting conclusion can be made. Though the error curves in Fig. 3 almost coincide, the weight curve does not (and need not be). The important thing is that individually both the weight curves converge to some finite value.

***Experiment 2*: Chaotic input**:

Chaotic input is generated by using the recursive relation

$$x(n+1) = \lambda x(n)\big(1 - x(n)\big), \quad (13)$$

with $\lambda = 4$ and $x(0) = 0.9$, which is a non-linear relation. The simulation result for this noise input is given in Fig. 5 which shows the convergence of the NMSE, computed over 20 independent trials.

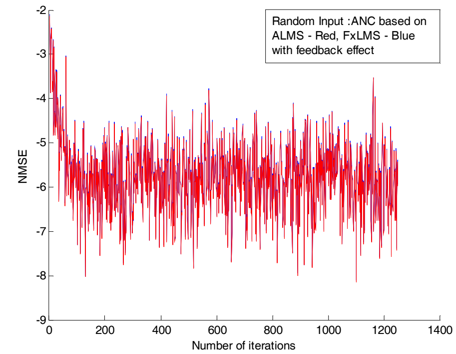

Fig. 3: ANC system with feedback path modeling based on ALMS (Red) and FxLMS (Blue) algorithm for random input.

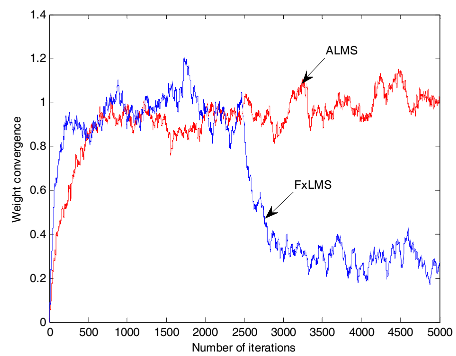

Fig. 4: Weight convergence for the ANC system with feedback path modeling based on ALMS (Red) and FxLMS (Blue) algorithm for random input.

***Experiment 3*: Chaotic input with variable system parameters**:

The primary, secondary and feedback path transfer functions are varied deliberately after some iterations to

$P(z) = -0.1z^{-6} + 0.3z^{-7}$, $A(z) = -0.4z^{-6} + 0.9z^{-7}$

and

$F(z) = 0.005z^{-5} - 0.9z^{-6} + 0.09z^{-7}$.

This is done to test the robustness of the algorithm to sudden changes in the system parameters. The simulation result is given in Fig. 6 which gives the convergence of NMSE, computed over 20 independent trials.

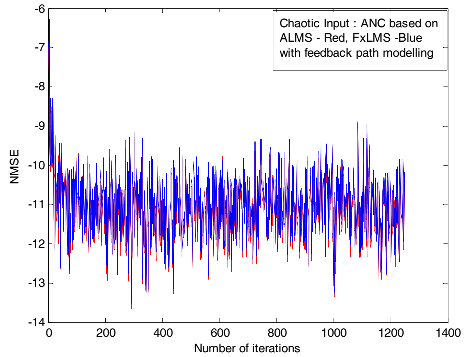

Fig. 5: ANC system with feedback path modeling based on ALMS (Red) and FxLMS (Blue) algorithm for chaotic input

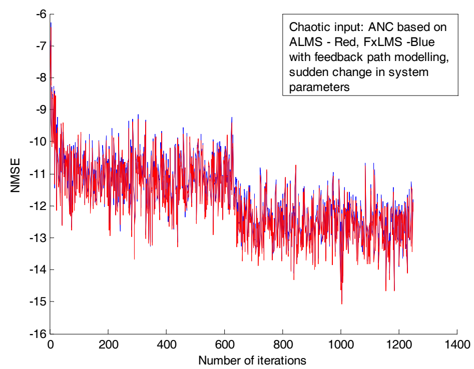

Fig. 6: ALMS (Red) and FxLMS (Blue) based ANC system with feedback path modeling with variable plant parameters for chaotic input.

6 Conclusions

It is known that the convergence characteristics of the ALMS based ANC are similar to those based on FxLMS when feedback path effect is not considered. By the proposed method it is observed that the same is valid in the case of ANC where the feedback effect is considered. Further it is observed that the number of computations remain unchanged for the proposed system based on ALMS with respect to its FxLMS counterpart for the single input single output (SISO) system. Computational savings are anticipated in the multichannel version based on ALMS in comparison to the FxLMS counterpart. FxLMS algorithm is based on the assumption that the order of the weighing filter and secondary path can be interchanged and it is only then the number of computations gets reduced in FxLMS. Otherwise the ALMS will have less computations than in FxLMS based system. Moreover the change of order of blocks is not always possible in many ANC applications. Proposed algorithm based on ALMS is free from such assumptions and hence is more robust.

7 References

[1] S. SYDNER: 'Active Noise Control Primer' (Springer Publications, 2000)
[2] S. M. KUO, D. R. MORGAN: 'Active Noise Control Systems-Algorithms and DSP Implementations' (Wiley, 1996)
[3] S. C.DOUGLAS: 'Fast Implementations of the Filtered-x LMS and LMS Algorithms for Multi-channel Active Noise Control', *IEEE Trans. on Speech Audio Process.*, vol. 7. pp. 454-465, Jul. 1999.
[4] P. Vikram Kumar, K. M. M. Prabhu and D. P. Das, 'Block Filtered-s Least Mean Square Algorithm for Active Control of Nonlinear Noise Systems', *IET Journal of Signal Processing*, vol. 4, issue 2, pp.168-180, Jan. 2010.
[5] E. Prashanth, D. P. Das and K. M. M. Prabhu, 'Fast Adaptive Algorithms for Active Control of Nonlinear Noise Processes,' *IEEE Trans. on Signal Proc.*, vol. 56, issue 9, pp. 4530-4536, 2008.
[6] M. K. AKHTAR, M. ABE, M. KAWAMATA: 'On Active Noise Control Systems with Online Acoustic Feedback Path Modeling', *IEEE Trans. on Audio, Speech and Language Process.*, vol.15, no. 2, pp. 593-600, Feb. 2007.
[7] M. K. AKHTAR, M. Tufail, M. ABE, M. KAWAMATA: 'Acoustic Feedback Neutralization in Active Noise Control Systems', *IEICE Electronic Express.*, vol.4, no. 7, pp. 221-226, Apr. 2007.
[8] E. A. Wan, 'Adjoint LMS: an efficient alternative to the filtered-x LMS and multiple error LMS algorithms', ICASSP-1996, pp(s): 1842 – 1845, vol. 3, DOI: 10.1109/ICASSP.1996.544227
[9] V. E. De Brunner, D. Zhou, 'Hybrid Filtered Error LMS Algorithm: Another Alternative to Filtered-x LMS', *IEEE Trans. on Circuits and Systems.*, vol.53, no. 3, pp. 653-661, Mar. 2006.