Centroid-Point of Ranking Fuzzy Numbers and Its Application to Health Related Quality of Life Indicators

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Abstract— **Ranking of fuzzy numbers is not an easy task as fuzzy numbers are represented by possibility distributions and they can overlap with each other. Since it was introduced, various approaches on ranking fuzzy numbers have been proposed. The recent ranking fuzzy numbers proposed by Wang and Li is claimed to be the improved version in ranking. However, the method was never been simplified and tested in real life application. This paper presents a four-step computation of ranking fuzzy numbers and its application in ranking indicators of health related quality of life. The four steps algorithm was proposed as to establish ranking and followed by a testimony in health related symptoms among elderly. Data in form of linguistic terms were collected from three experts at State of Terengganu Malaysia. Decisions were made based on the centroid-point ($\bar{x}, \bar{y}$), where the degree of representative location ($\bar{x}$) is higher than average height ($\bar{y}$). These points permit to characterize the evaluation behaviour of each indicator. The ranking reflects the problematic level of elderly people. The results show the feasibility of the proposed stepwise computation in real application.**

Keywords- Centroid-points, fuzzy numbers, ranking, health related indicator

I. INTRODUCTION

The concept of fuzzy numbers has been developed in many decision making problems. Ranking fuzzy numbers is one the methods that conceptualize fuzzy numbers to describe preference or rank in decision making. Development of ranking fuzzy numbers started when [1] and [2] introduced the relevant concepts of fuzzy numbers. Since then, many researches proposed the related methods for ranking fuzzy numbers. Reference [3] proposed fuzzy multiple attribute decision making. Reference [4] proposed an index for ordering fuzzy numbers. Reference [5] ranked alternatives by ordering fuzzy numbers while [6] ranked fuzzy numbers with a satisfaction function, reference [7] utilized artificial neural networks for the automatic ranking of fuzzy numbers. After some time, [8] classified the ranking methods into four major classes which are preference relation, fuzzy mean and spread, fuzzy scoring and linguistic expression. Of these classifications, the ranking methods of fuzzy numbers based on centroid index which is a sub-class of fuzzy scoring are the most commonly used technique and thoroughly studied. Reference [9] was the first researcher who contributed the centroid concept in the ranking method. Horizontal coordinate x was used as the ranking index. In 1983, reference [10] presented both the horizontal x and vertical y coordinates of the centroid-point as the ranking index. However, [11] argued that in many cases of Murakami et al.'s method, only the horizontal value x is chosen as the rational index since the vertical value y is the same for all normal triangular fuzzy numbers and all normal rectangular fuzzy numbers. Also, [12] disagree with Murukami et al.'s method due to the value of x can also be an aid index and y becomes the important index especially when the values of x are equal or the left and right spread are the same for all fuzzy numbers.

As an effort to overcome the problems of choosing either x or y as the important index, [12] proposed a distance index which is based on the calculation of using both values of x and y. Reference [13] proposed an area between centroid-point and original points (0,0). In a recent study by [14] the area method by [13] was found to produce counterintuitive results. Besides that, according to [14] multiplying the value of x and y will degrade the importance of the value x whereby the importance of the degree of x should be higher than y. Therefore, based on the concept of the importance of the degree of x, [14] presented a revised method which they claimed can improve [13] area method. Based on the value of centroid-point in [13], the conclusion are made where the larger the value of x, the better the ranking of the fuzzy number. The ultimate ranking is established by considering the horizontal value x. Vertical value y is only considered in case when x produces the same values for fuzzy numbers. Nevertheless the Wang and Lee's method is always not straightforward and difficult to implement due to computational complexity especially when it comes to applications in real case study. Perhaps a ranking index in real case study is not the aim for some researchers. Most of the above researches focused on the proposing correct formulae and supported with merely several hypothetical examples. For example, [15] focused on producing the correct formula of the centroid-point ($\bar{x}$, $\bar{y}$) after discovering that the centroid formulae proposed by [12] are incorrect. A numerical example was demonstrated to show that Cheng's formulae can significantly alter the result of the ranking procedure. Although the novelty of the centroid concept in ranking fuzzy numbers has been known for many years its application in real case study is silent. Against all this background this paper proposes a simplified version of the recent Wang and Lee's ranking fuzzy numbers and test it to health related quality of life indicators. The applications of fuzzy decision making in health related research is not something new and have been applied in many medical decision making (see [16];[17], [18]). Specifically the aim of this paper is to propose a modified version of a centriod based ranking fuzzy numbers and rank the indicators of health related quality of life. The rest of this paper is structured as follows. The next section elucidates the basic notions of fuzzy sets and fuzzy numbers. Section III describes the modified version of Wang and Lee 's method. Section IV presents a case study of establishing a ranking for indicators of health related quality of life. Finally this paper concludes in Section V.

II. PRELIMINARIES

In this section the basic notions of fuzzy sets and fuzzy numbers by [19], [20] and [21] are presented. These notions are expressed as follows.

Definition 2.1 Let U be a universe set. A fuzzy set A of U is defined by a membership function $f_A(x) \rightarrow [0,1]$, where $f_A(x)$, $\forall x \in U$, indicates the degree of x in A.

Definition 2.2 A fuzzy subset A of universe set U is normal if and only if $\sup_{x \in U} f_A(x)=1$, where U is the universe set.

Definition 2.3 A fuzzy subset A of universe set U is convex if and only if $f_A(\lambda x+(1-\lambda)y) \geq (f_A(x) \wedge f_A(y))$, $\forall x,y \in U$, $\forall \lambda \in [0,1]$, where $\wedge$ denotes the minimum operator.

Definition 2.4 A fuzzy set A is a fuzzy number if and only if A is normal and convex on U.

Definition 2.5 A triangular fuzzy number (TFN) A is a fuzzy number with a piecewise linear membership function f_A defined by:

$$f_A = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ \dfrac{a_3-x}{a_3-a_2}, & a_2 \leq x \leq a_3, \\ 0, & otherwise \end{cases}$$

which can be denoted as a triplet (a_1,a_2,a_3).

Definition 2.6 A trapezoidal fuzzy number A is a fuzzy number with a membership function f_A defined by:

$$f_A = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \leq x \leq a_2, \\ 1, & a_1 \leq x \leq a_2, \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise}, \end{cases}$$

which can be denoted as a quartet (a_1,a_2,a_3,a_4).

Definition 2.7 An extended fuzzy number A is described as any fuzzy subset of the universe set U with membership function f_A defined as follows:

(a) f_A is a continuous mapping from U to the closed interval $[0,\omega]$, $0<\omega\leq 1$.

(b) $f_A(x)=0$, for all $x \in (-\infty,a_1]$.

(c) f_A is strictly increasing on $[a_1,a_2]$.

(d) $f_A(x)=\omega$, for all $x \in [a_2,a_3]$, as ω is a constant and $0<\omega\leq 1$.

(e) f_A is strictly decreasing on $[a_3,a_4]$.

(f) $f_A(x)=0$, for all $x \in [a_4,\infty)$.

In these situations, a_1, a_2, a_3 and a_4 are real numbers.

Definition 2.8 The membership function f_A of the extended fuzzy number A is expressed by

$$f_A = \begin{cases} f_A^L(x), & a_1 \leq x \leq a_2, \\ \omega, & a_2 \leq x \leq a_3, \\ f_A^R(x), & a_3 \leq x \leq a_4, \\ 0, & \text{otherwise}, \end{cases}$$

where f_A^L : $[a_1,a_2]\rightarrow[0,\omega]$ and f_A^R : $[a_3, a_4] \rightarrow[0,\omega]$.

Based on the basic theories of fuzzy numbers, A is a normal fuzzy number if ω=1, whereas A is a non-normal fuzzy number if $0<\omega<1$.

III. A CENTROID-POINT METHOD

The proposed steps are based on the ranking fuzzy number with an area between the centroid and the original point of [13] and the revised version of [14]. Reference [14] relinquished the area proposed by Chu and Tsao but instead consider centroid-point. However, the revised version seems escalate in the complexity of computation. Moreover, validation of the revised method was solely banked on several hypothetical examples and far too little attention has been paid to test it into real case study. Therefore the four-step algorithm is proposed. Computational complexity especially in real case study can be relaxed by executing the following steps and ultimately reduced the computational costs.

Step 1: Define triangular fuzzy numbers and its respective linguistic variables

The triangular fuzzy numbers is based on a three-value judgment of a linguistic variable. The minimum possible value is denoted as a_1, the most possible value denoted as a_2 and the maximum possible value denoted as a_3.

Step 2: Delineate Inverse Function

The inverse function of f_A^L exists as f_A^L : $[a_1,a_2]\rightarrow[0,\omega]$ is continuous and strictly increasing, and the inverse function of f_A^R exists as f_A^R : $[a_3, a_4] \rightarrow[0,\omega]$ is continuous and strictly decreasing. The inverse functions I_A^L and I_A^R of f_A^L and f_A^R respectively. Since f_A^L : $[a_1,a_2]\rightarrow[0,\omega]$ is continuous and strictly increasing, I_A^L : $[0, \omega] \rightarrow [0,1]$ is also continuous and strictly increasing. Similarly, f_A^R : $[a_3, a_4] \rightarrow[0,\omega]$ is continuous and strictly decreasing, and thus f_A^R : $[0,\omega]\rightarrow[a_3,a_4]$ is continuous and strictly decreasing as well. In short, I_A^L and I_A^R are continuous on $[0,\omega]$, so I_A^L and I_A^R exist.

Step 3: Establish Centroid-Point ($\bar{x}$, $\bar{y}$).

The centroid-point of a fuzzy number A corresponded to a value $\bar{x}$ on the horizontal axis and a value $\bar{y}$ on the vertical axis. The centroid-point ($\bar{x}$ (A), $\bar{y}$ (A)) of a fuzzy number A was defined as

$$\bar{x}\,(A) = \frac{\int_{a1}^{a2} x f_A^L(x)dx + \int_{a2}^{a3} x dx + \int_{a3}^{a4} x f_A^R(x)dx}{\int_{a1}^{a2} f_A^L(x) + \int_{a2}^{a3} dx + \int_{a3}^{a4} f_A^R dx}$$

and

$$\bar{y}(B) = \frac{\int_0^{\omega} y I_{\tilde{B}}^L(y)dy + \int_0^{\omega} y I_{\tilde{B}}^R(y)dy}{\int_0^{\omega} I_{\tilde{B}}^L(y)dy + \int_0^{\omega} I_{\tilde{B}}^R(y)dy}$$

where f_A^L and f_A^R were left and right membership functions of A respectively, and f_A^L and f_A^R were inverse functions of I_A^L and I_A^R respectively. .

Step 4: Decision Rule

For two fuzzy numbers A and B, they had the following relation:

If $\bar{x}(A) > \bar{x}(B)$, then $A > B$.
If $\bar{x}(A) < \bar{x}(B)$, then $A < B$
If $\bar{x}(A) = \bar{x}(B)$, then
  if $\bar{y}(A) > \bar{y}(B)$, then $A > B$;
  else if $\bar{y}(A) < \bar{y}(B)$, then $A < B$;
  else $\bar{y}(A) = \bar{y}(B)$, then $A = B$.

In short, A and B are ranked based on their $\bar{x}$'s values if they are different. In the case they are equal, they are ranked by comparing their $\bar{y}$'s values. Feasibility of the proposed steps can be seen in a decision making problem of health related case study

IV. A CASE STUDY

An experiment was conducted to elicit linguistic judgment over the health related problematic status of elderly people. Three decision makers comprise a medical officer and two nurses were voluntarily formed a group of experts. All decision makers were attached at a Malaysian government funded hospital. Decision makers were asked to express their opinion in health related problem among the elderly people based on a guided interview. The closed ended questions of interviewing process were developed by the authors based on literature in health related quality of life and also the SF-36 questionnaire proposed by [22]. There were eight indicators of in the questionnaire and decision makers need to make their decisions about the health related problem with respect to eight health related indicators among elder people. The eight indicators used in the case study are Physical-functioning (A), Role-physical (B), Bodily pain (C), Vitality (D), Social (E), Emotional (F), Role-emotional (G), Workplace (H). Decision makers need to respond in five linguistic scales from 'never has a problem' to 'always has a problem' to indicate their views over experiences of health related symptoms of eight indicators among elderly people. Linguistic scales and their respective TFNs are given in Table I.

TABLE I LINGUISTIC VARIABLES AND FUZZY NUMBERS FOR THE IMPORTANCE WEIGHTS

Linguistic variables	TFN
Never (N)	(1,1,2)
Almost Never (AN)	(1,2,3)
Often (O)	(2,3,4)
Sometimes (S)	(3,4,5)
Almost Always (AA)	(4,5,5)

Analogously, fuzziness of judgments among the group of DMs can be translated to TFNs. The flexibility of linguistic judgment and their TFNs for the indicators can be visualized in Figure I.

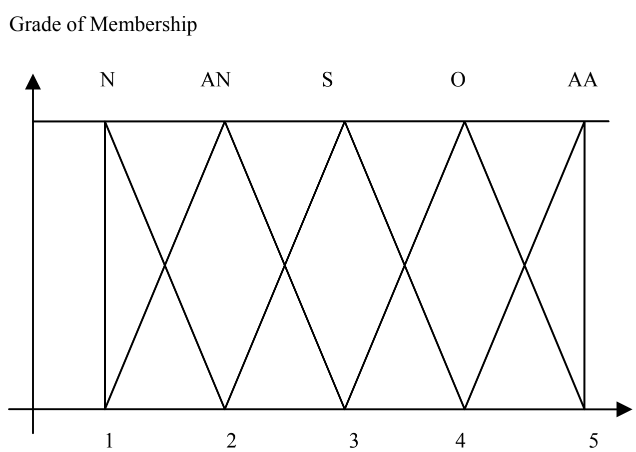

FIGURE I TFN TO REPRESENT JUDGMENT SCORE OF THE INDICATORS

The proposed method in Section III is employed in the following computations. For the sake of clarity, indicator A is used as an example.

Linguistic responses from the experts were compromised using arithmetic mean to obtain average weight score and the score for indicator A is (1.89, 2.67, 3.67). The centroid-point of A was computed using a computer algebra system. Finally the centroid-point for indicator A is obtained as 2.743333334 after executing the proposed steps. The centroid-points for seven other indicators were computed with the same fashion. The indicators are ultimately ranked in ascending order according to the magnitude of the centroid-point. The centroid-points for the eight indicators and their respective ranking are presented in Table II.

TABLE II RANKING OF HEALTH RELATED QUALITY OF LIFE INDICATORS

HRQoL Indicators	Centroid-point $\overline{x}$	Ranking
Physical-functioning (A)	2.743333334	1
Role-physical (B)	3.230000000	4
Bodily pain (C)	3.513333334	7
Vitality (D)	2.776666667	2
Social (E)	3.393333334	6
Emotional(F)	2.776666667	2
Role-emotional (G)	2.780000000	3
Workplace (H)	3.243333334	5

Based on the magnitudes, ranking of the indiactors are obtained as $A \prec D \approx F \prec G \prec B \prec H \prec E \prec C$ where the symbol $\prec$ represents 'has less problem than' and the symbol $\approx$ represents 'has equal problem as'. Physical-functioning, Emotional and Vitality were among the indicators that portrayed less problematic indicator among elderly people. On the other extreme, Bodily pain was the most problematic indicator experienced by elderly people. Notwithstanding the significance of other indicators, the results really show the physical aspects of elderly person and may need extra attentions among medical fraternity. Interestingly, the experts viewed elderly people that the latter were emotionally stable.

V . CONCLUDING REMARKS

This paper has proposed the step-wise computation to ease complexity in ranking fuzzy numbers. The four-step algorithm was developed after considering the potential applications of ranking fuzzy numbers in real case of decision making. The basis for this paper is the theoretical work of Wang and Li, and Chu and Tsao who exploring the ranking fuzzy numbers in decision making. An application to test the algorithm in health related quality of life evaluation has been experimented. The centroid-point ($\overline{x}$, $\overline{y}$) of indicators determined the problematic level of elderly from the experts' point of view. This algorithm was successfully used in ranking of the eight indicators of health related quality of life. This case study offers a new extension in application of ranking fuzzy numbers. Further experimental investigations are needed to amplify the application of ranking fuzzy numbers and hopefully broaden the horizon of computing in real life application.

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