Fuzzy Group Decision Making Using Surrogate Worth Trade-Off Method

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Abstract— In this paper, fuzzy theory is used in group decision making problem. Surrogate worth trade-off method is discussed and according to fuzzy aspect of human decisions, linguistic variables are applied to state the Decision makers' opinions. Applying linguistic variables enables proposed algorithm to deal with group decision making. Fundamentals of ε -constraint method, surrogate worth trade-off (SWT) method and linguistic variables are surveyed as predecessors of proposed algorithm. Proposed algorithm is explained step by step and finally, computability of proposed algorithm is demonstrated with a numerical example. This study shows that it is possible to use linguistic variables in order to group decision making applying SWT.

Keywords-Decision making, Multi objective optimization, Surrogate worth trade-off (SWT), Linguistic variables

I. Introduction

Optimization models have been studied widely during past decades. In former decision making problems, optimization models were generated and solved as a minimization or maximization scalar function, but in recent years multi objective models have been studied to solve complicated decision making problems. More explicitly, optimization models and their analytic goals are more realistic considering several objects. Usually for optimizing a model we should consider various objectives and in real world problems, some of these objectives may conflict with each other such as maximizing system reliability and minimizing system costs; therefore, developing models and solving procedures to deal with multi objective problems is necessary and this study is performed to present a new method in solving multi objective decision making problems.

Generally, multi objective problems are solved to find non-dominated solutions. A non-dominated solution is defined as a solution that using it imposes a condition where improving one objective is not possible unless by worsening values for other objectives. There are many studies related to obtaining non-dominated solutions [1]. Multi objective simplex, weighted method [2] and constraint method [3] are widely applied in order to estimate non-inferior sets. Andersson [4] comprehensively surveyed multi objective optimization problems from engineering viewpoint. Multi objective problems solving methods are different within two aspects:

A. Applying different methods to generate non-inferior solutions.
B. The way of interacting with decision maker (DM) and the type of information being provided to him.

Almost, in all of decision making problems there are several criteria to analyze possible alternatives. The main decision maker's problem is to satisfy the conflicting objectives considering system constraints. There are two approaches for solving such problems: 1- Assume there is one utility function for a specific problem and this function is used to find the best alternative. 2- There is no assumption about objective function but DM is able to use a simple but efficient tool to find the best solution [5].

Using multi objective optimization techniques enables the DM to: i) manage different objectives, ii) make decision simpler and iii) make oriented decisions due to objective function sequences. Therefore, multi objective methodologies' output is different from that of standard optimization methods [6]: traditional optimization procedures introduce a point as the solution but multi objective methods generate a set of optimized solutions named Pareto set; consequently in order to solve a multi objective problem, the following steps must be considered:

A. Defining problem objectives
B. Obtaining a Pareto set
C. Choosing a solution from the Pareto set

Three main approaches can be used in order to choose a specific solution among the non-dominated solutions which is known as adaptive optimized solution:

A. Profit and utility approach
B. Goal programming
C. Interactive approach

First two approaches assume that DM can specify his preferred function according to weighted combination of objective functions or distance functions (e.g. distance from the optimal point). Totally, these two approaches assume that weighted combination of objective functions appeared in adaptive solution is obtained from linear combinations (e.g. weighted summation of objective functions). And the third approach only applies the local information to obtain a coincidence acceptable solution.

The best adaptive solution can be obtained by the Pareto set which can satisfy the objective functions according to system constraints but it is notable that quantitative trade-off measuring among conflicting objectives is an essential issue in such problems [7]. SWT is a method which can interact with DM in an efficient manner.

In order to generate non-inferior solutions in SWT method, constraint method which is shown as ε-constraint is applied. This method enables DM to trade-off between objective levels of each Pareto solution [8].

There are several algorithms to obtain the amount of trade-off between objectives but taking an absolute value of trade-off is a basic problem for DMs, while making a decision is simpler through having the trade-off relative values.

SWT method has two steps. Effective solutions in solving a problem are obtained in the first step and trade-off functions λ_{li} are given to DMs due to effective solutions. Second step involves searching a preferred solution among effective solutions. This preferred solution must be obtained from the indifferent range in turn of worth functions set.

Brizzi et.al. [9] applied SWT to increase security and certainty factors in power plants in short term. In this study, many multi objective solving methods are offered to DM and finally, the optimal operation point is obtained based on worth trade-off between objective functions. Also, SWT is used in power plants scheduling in some articles [10,11,12]. Babic and Peric [13] applied revised SWT to determine optimal production plan and studied multi criterion planning methods to determine optimal production plan in production plants.

SWT method is placed in multi criterion interactive decision making methods and as mentioned, preferred solution is obtained from decision makers' indifference range in exchange of a surrogate worth functions set; however, the method's weakness returns to choosing the preferred solution. DM has to state his preference by a number from -10 to +10 but human brain does not have a numerical logic and it works in fuzzy logic. Moreover, in all studies where SWT is used to choose the preferred solution, only one DM is engaged in decision making process who expresses his opinion with a number from -10 to +10. In this study we try to apply linguistic variables to determine the preferred solution which enables more than one DM to participate in the decision making process; therefore, proposed algorithm is improved in two aspects comparing to conventional SWT which was used in decision making processes:

A. Applying linguistic variables enables DMs to express their opinions by fuzzy logic which is tangible to human brain.
B. Proposed algorithm enables us to apply a group of DMs instead of one DM.

Remainder of this paper is as follows. In section II, needed foundations for performing fuzzy group decision making in SWT Method are explained. Section III is dedicated to explain the proposed algorithm step by step and in section IV a simple numerical example is presented to better the understanding of proposed algorithm. Finally, conclusions are drawn and some remarks regarding the future research are made.

II. ALGORITHM FOUNDATIONS

Decision making methods and developed algorithms to obtain the preferred solution are explained in last section but in order to figure out SWT method and developing it, realizing some contents such as ε-constraint method, SWT function and linguistic variable is necessary; therefore, these contents are explained in this section and sequentially the proposed approach in developing SWT method is presented in next section.

A. ε-constraint Method

Applying SWT method, ε-constraint method is used in order to generate Pareto solutions [14]. In this method one of the objective functions is considered as the main objective and the other ones are considered as constraints. Model (1) is showing ε-constraint method formulation:

$$\begin{aligned} &\max f_1(x) \\ &s.t: f_i(x) \geq \varepsilon_i; i = 2,\dots,k \\ &g_j(x) \leq 0; j = 1,2,\dots,m \\ &x \in E^n \end{aligned} \tag{1}$$

ε_i is the lowest variability level of objective function i. ε_i is choused in such a way that $g_j(x) \leq 0; j = 1,2,\dots,m$ be satisfied. ε_i's value varies parametrically due to measuring its effect on objective function $f_1(x)$. In order to solve model 1, the lagrangian function (2) must be solved:

$$\begin{aligned} &L(x,u,\lambda) = f_1(x) + \sum_{i=2}^{k} \lambda_{1i}(f_i(x) - \varepsilon_i) - \sum_{j=1}^{m} u_j g_j \\ &u_j, \lambda_{1i} \geq 0 \end{aligned} \tag{2}$$

λ_1 and μ are lagrangian multipliers. Index $1i$ shows that λ is a lagrangian multiplier related to constraint *i* and $f_1(x)$ is the main objective. The optimal solution must satisfy Kuhn-Tucker solution (3) due to dual relations [15]:

$$\begin{aligned} &\lambda_{1i}(f_i(x) - \varepsilon_i) = 0 \\ &\lambda_{1i} \geq 0; i = 1,2,\dots,k \end{aligned} \tag{3}$$

Model constraints can be zero or non-zero, so Lagrangian multipliers must be related to objective function. Non-zero lagrangian multipliers set is related to Pareto solutions set. ε_i initial values are choused in such a way that $\varepsilon_i > f_i^{\min}$ and $\varepsilon_i < f_i^{\max}$. In these models some objectives may be maximized while the others are in their lowest values due to possible objective functions confliction (and vice versa).

B. SWT Function

SWT function assigns a numeric value (ordinal scale) to each Pareto solution. Trade-off functions could be used in order to obtain Pareto solution. There is a close relation among SWT function, W_{1i} and relative derivatives of utility functions. In multi objective analysis, it is assumed that DM maximizes his utility function which is a univocal descending function of objective functions. Having the decision vector *P* and related sequence F_i , utility function of DM is obtained as relation (4):

$$U = U[f_1(P), f_2(P),..., f_k(P)] \qquad (4)$$

Relation (5) is obtained by linear the utility function for a small change in main objective f_1 [16]:

$$\Delta U_{1i} = \left(\frac{\partial U}{\partial f_i} - \frac{\partial U}{\partial f_1} \lambda_{1i} \right) \qquad (5)$$

Relation (6) is resulted by replacing $\delta_1 = \dfrac{\partial U}{\partial f_1}$ to (5):

$$\Delta U_{1i} = (\delta_i - \delta_1 \lambda_{1i}) \qquad (6)$$

SWT function W_{1i} is a univocal function of ΔU_{1i} with characteristic $W_{1i} = 0 \leftrightarrow \Delta U_{1i}$ and is obtained by relation (7):

$$W_{1i} = h_i \Delta U_{1i}; \quad i = 2,...,k \qquad (7)$$

h_i is an univocal ascending function of its argument in range -10 to +10 with characteristic $h_i(0) = 0$. It can be assumed that W_{1i} is only depending to λ_{1i} if δ_i be a constant value or varies a little towards $f_i, i = 1,...,k$.

The problem is to determine the decision maker's indifferent range with a number from -10 to +10 in a way that -10 equals to completely no preference of trading λ_{li} units of f_l to one unit of f_i while the other objectives are fixed in their goal levels. +10 equals to completely preference of trading λ_{li} units of f_l to one unit of f_i while the other objectives are fixed in their goal levels and zero equals to indifference of trading λ_{li} units of f_l to one unit of f_i while the other objectives are fixed in their goal levels. This numerical decision making process is not appropriate for human mind nature; therefore, the necessity of using linguistic variables is felt more.

C. Linguistic Variables

Linguistic variables are the words are formed in human language and are applied to explain characteristics of complex systems which are not well-defined. In sets theory definitions, each set has a well-defined characteristic such that if a subject has the assumed characteristic is a member of the related set and if it doesn't have this characteristic, is not the set member. Each linguistic variable is defined by variable x and linguistic variables set $S(x)$ where each linguistic variable is a defined fuzzy number on *x*. For example, if an index status is a linguistic variable, the linguistic variable set is {very low, low, medium, high, and very high} where each linguistic value is defined by a fuzzy number.

There are several tables and procedures in applying linguistic variables but in this paper we have three (odd) concepts to explain the DMs opinion (preferred, indifferent, and not preferred), thus, we have to apply linguistic variables which are oddly categorized and fit to explained concepts. According to above statement, in this paper we apply the linguistic variables which are introduced by Chen [17] and shown in Table 1. This proposed approach makes the algorithm able to apply more than one DM in order to choose indifference range.

TABLE1. LINGUISTIC VARIABLES APPLIED TO SORT OBJECTIVE FUNCTIONS WORTH PREFERENCES

Linguistic variable	Related fuzzy number
Completely Not Preferred (CNP)	(0,0,0/1)
Partly Pot Preferred (PPP)	(0,0/1,0/3)
Weakly Not Preferred (WNP)	(0/1,0/3,0/5)
Indifferent (I)	(0/3,0/5,0/7)
Weakly Preferred (WP)	(0/5,0/7,0/9)
Partly Preferred (PP)	(0/7,0/9,1)
Completely Preferred(CP)	(0/9,1,1)

According to the explained foundations in this section, SWT method and proposed algorithm steps in group decision making can be explained in Section III.

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[1] DECISION MAKING ALGORITHM

Performing SWT method has two aspects, as mentioned in introduction. Effective solutions to solve a problem with SWT method are generated in the first stage and choosing a preferred solution among effective solutions is done in the second stage. Also, in proposed method of this article the first step is same as the original SWT method but in the second step, fuzzy priority is applied and the algorithm becomes an appropriate one for group decision making. The proposed SWT algorithm is formed as the following steps:

Step 1- Consider the primary multi objective decision making problem (8):

$$\begin{aligned} &\max : F = \{f_1(x),..., f_k(x)\} \\ &s.t : g_j(x) \le 0;\ j = 1,2,...,m \\ &x \in E^n \end{aligned} \qquad (8)$$

Step 2- Obtain the ideal solutions for each $f_i^*(x)$ by solving *k* single objective problems and choose one of the objectives such as f_l by decision makers' opinions.

Step 3- Create a set of effective solutions for model (1) by varying ε_i parametrically. Effective solutions have a non-zero and positive trade-off function ($\lambda_{li} = -\partial f_l(0)/\partial f_i(0)$). It is notable that $\varepsilon_i = f_i^*(x) - \varepsilon_i^*, \varepsilon_i^* > 0, i = 1,2,...,k$ and ε_i^* are parametrically denoting deviation from the ideal solution of objective i, so ε_i will remain parametric in optimal model (1).

Step 4- Obtain the dual Lagrange function for optimal model (1) and write it like equation (2). According to Kuhn- Tucker [15] condition, in Lagrange optimal point, equation (9) must be active:

$$\begin{aligned} &\lambda_{1i}(f_i(x) - \varepsilon_i) = 0 \\ &\lambda_{1i} \geq 0; i = 1,2,...,k \end{aligned} \tag{9}$$

Step 5- After determining F vector and λ_{li} values for DMs, obtain the solutions preference for each DM (suppose there are d DMs) by linguistic variables. If a DM opinion is more important than the others, apply w_i to determine related weight for each decision maker's opinion.

Step 6- Calculate mean fuzzy preference of each solution using (10):

$$\widetilde{A}_t = \frac{\sum_{i=1}^{d} w_i \widetilde{A}_{ti}}{\sum_{i=1}^{d} w_i} \tag{10}$$

$\widetilde{A}_{ti}$ refers to fuzzy preference of solution *t* for decision maker *i* and $\widetilde{A}_t$ is mean fuzzy preference of solution *t*.

Step7- Calculate the difference between mean fuzzy preference of each solution and indifferent fuzzy number (0/3, 0/5, 0/7).

Step 8- Defuzzify the differences which are obtained in step 7.

Step 9.a- Choose the solution which has no difference with indifferent fuzzy number as the optimal solution for F^*. This solution has zero difference in step 8.

Step 9.b- If there is no solution with step 9.a's condition, generate more solutions between two solutions which have the least difference with indifferent fuzzy number until achieving the indifferent number by decision makers' opinions.

Optimal values of this model x^* and f_l^* are the final solutions for multi objective problem. Proposed model effectiveness is illustrated by a numerical example in Section IV.

IV. NUMERICAL EXAMPLE

In this section a simple example is presented to show the computability of proposed model. There is an optimization problem for a production plant with two conflicted objective functions: a profit function $x_1.x_2$ which must be maximized and a cost function $(x_1-4)^2 + x_2^2$ which must be minimized. This plant has a resource constraint $x_1 + x_2 \leq 25$ which must be satisfied. According to problem assumptions, the optimization model is formulated as follows:

$$\begin{aligned} &\max: f_1(x) = x_1.x_2 \\ &\min: f_2(x) = (x_1-4)^2 + x_2^2 \\ &s.t: x_1 + x_2 \leq 25 : g_1 \\ &x_1, x_2 \geq 0 \end{aligned}$$

Since all objective functions in SWT method must be written in maximization form in SWT method, second function changes to $\max: f_2(x) = -(x_1-4)^2 - x_2^2$. As it is said in algorithm steps, ideal solutions for each objective must be obtained:

$$\begin{aligned} &f_1^* = 156.25 \qquad , \qquad f_2^* = 0 \\ &x_1^* = x_2^* = 12.5 \qquad x_1^* = 2, x_2^* = 0 \end{aligned}$$

Effective solutions can be computed by solving following model:

$$\begin{aligned} &\max: f_1 = x_1.x_2 \\ &s.t: -(x_1-4)^2 - x_2^2 \geq \varepsilon_2 = -C \\ &x_1 + x_2 \leq 25 : g_1 \\ &x_1, x_2 \geq 0 \end{aligned}$$

In order to calculate ε_2, a constant value C can be considered as the inventory costs, so $\varepsilon_2 = f_2^* - \varepsilon_2^* = 0 - \varepsilon_2^* = -C$. Lagrangian function of this equation is as follows:

$$L = x_1.x_2 + \lambda_{12}\{-(x_1-4)^2 - x_2^2 + C\} - u_1\{x_1 + x_2 - 25\}$$

The Kuhn-Tucker condition must be written for this function:

$$\begin{aligned} &x_2 - 2\lambda_{12}(x_1 - 4) - u_1 = 0 \\ &x_1 - 2\lambda_{12}x_2 - u_1 = 0 \\ &u_1.(x_1 + x_2 - 25) = 0 \\ &\lambda_{12}(-(x_1-4)^2 - x_2^2 + C) = 0 \end{aligned}$$

It is assumed that the optimal point in decision makers' viewpoints is an inner point relative to g_1. And $x_1, x_2 \geq 0$ is confirmed due to fact in order to offer the trade-off function λ_{12}, constraint $f_2(x) \geq \varepsilon_2$ must be satisfied. Therefore, $u_1 = 0$ must be applied in order to make the problem feasible and also dual variables must be zero due to $x_1, x_2 \geq 0$. According to these explanations, by solving upper relations we attain the following equation:

$$\lambda_{12} = \frac{x_2}{2(x_1-4)} = \frac{x_1}{2x_2}$$

It is obvious that to have a positive λ_{12}, inequalities $x_1 \geq 4, x_2 > 0$ must be satisfied. Briefly, we achieve two key equations by solving the relations:

$$x_2 = \sqrt{x_1(x_1 - 4)}$$

$$(x_1 - 4)^2 + x_2^2 = C$$

An effective solution set is obtained by initializing *C* and solving two equations mentioned above. According to the model constraints, *C* can take value from zero to 222 but due to preventing the increased amount of calculations, just multipliers of ten are surveyed:

TABLE 2. SOLVING PROBLEM FOR DIFFERENT C VALUES

C	x_1	x_2	λ_{12}	f_1	f_2
0	4	0.0008	2387.742	0.0032	0.00000064
10	5.449	2.81	0.969	15.311	9.995
20	6.316	3.825	0.825	24.158	20.201
30	7	4.582	0.763	32.074	29.994
40	7.582	5.212	0.727	39.517	39.995
50	8.099	5.761	0.702	46.65	49.99
60	8.567	6.255	0.684	53.586	59.982
70	9	6.708	0.67	60.372	69.997
80	9.403	7.127	0.659	67.015	79.986
90	9.782	7.52	0.65	73.56	89.981
100	10.141	7.891	0.642	80.022	99.979
110	10.483	8.244	0.635	86.421	109.992
120	10.81	8.58	0.629	92.749	119.992
130	11.124	8.902	0.624	99.025	129.997
140	11.426	9.211	0.62	105.244	139.988
150	11.717	9.509	0.616	111.417	149.973
160	12	9.797	0.612	117.56	159.981
170	12.273	10.077	0.608	123.675	169.988
180	12.539	10.347	0.605	129.741	179.974
190	12.797	10.611	0.603	135.789	189.98
200	13.049	10.867	0.6	141.803	199.976
210	13.295	11.117	0.597	147.8	209.984
220	13.535	11.36	0.595	153.757	219.965
222	13.583	11.409	0.595	154.968	221.999

Now, DMs must be interacted and their opinions must contribute to decision making process. Linguistic variables in this process must have odd levels due to mentioned reasons.

In order to solve this problem a group of 3 DMs is applied. In this group we can consider different weights for each decision maker's opinion but in this problem we assume that all decision makers' opinions have the same importance. The solutions which are obtained in Table 2 must be given to DMs in order to receive their opinions about each solution. Table 3 shows decision makers' opinions by linguistic variables.

TABLE 3. DECISION MAKERS' OPINIONS ABOUT OBJECTIVE FUNCTIONS TRADE-OFF BY LINGUISTIC VARIABLES

C	First DM	Second DM	Third DM
0	CP	CP	CP
10	CP	PP	CP
20	PP	PP	CP
30	PP	PP	PP
40	PP	WP	PP
50	WP	WP	PP
60	WP	I	PP
70	WP	I	PP
80	WP	I	WP
90	I	I	WP
100	I	WNP	I

110	I	PPP	I
120	WNP	PPP	I
130	WNP	PPP	WNP
140	PPP	PPP	WNP
150	PPP	PPP	WNP
160	PPP	PPP	WNP
170	PPP	PPP	PPP
180	PPP	CNP	PPP
190	PPP	CNP	PPP
200	CNP	CNP	PPP
210	CNP	CNP	PPP
220	CNP	CNP	CNP
222	CNP	CNP	CNP

Now, steps 6 to 9 can be performed according to Table 3. Final values which are used to obtain the optimal solution are shown in Table 4.

TABLE 4. PERFORMING STEPS 6 TO 9 IN NUMERICAL EXAMPLE

C	Mean fuzzy preferences	Difference between mean fuzzy preferences and indifferent fuzzy number	Defuzzified absolute value of Difference between mean fuzzy preferences and indifferent fuzzy number
0	(0.9,1,1)	(0.2,0.5,0.7)	0.4667
10	(0.83,0.96,1)	(0.13,0.46,0.7)	0.426
20	(0.76,0.93,1)	(0.06,0.43,0.7)	0.3922
30	(0.7,0.9,1)	(0,0.4,0.7)	0.3667
40	(0.63,0.83,0.96)	(-0.07,0.33,0.66)	0.3069
50	(0.56,0.76,0.93)	(-0.14,0.26,0.63)	0.2498
60	(0.5,0.7,0.86)	(-0.2,0.2,0.56)	0.1899
70	(0.5,0.7,0.86)	(-0.2,0.2,0.56)	0.1899
80	(0.43,0.63,0.83)	(-0.27,0.13,0.53)	0.13
90	(0.36,0.56,0.76)	(-0.34,0.06,0.43)	0.0432
100	(0.23,0.43,0.63)	(-0.47,-0.07,0.33)	0.07
110	(0.2,0.36,0.56)	(-0.5,-0.14,0.26)	0.1233
120	(0.13,0.3,0.5)	(-0.57,-0.2,0.2)	0.1927
130	(0.06,0.23,0.43)	(-0.64,-0.27,0.13)	0.2606
140	(0.03,0.16,0.36)	(-0.67,-0.34,0.06)	0.316
150	(0.03,0.16,0.36)	(-0.67,-0.34,0.06)	0.316
160	(0.03,0.16,0.36)	(-0.67,-0.34,0.06)	0.316
170	(0,0.1,0.3)	(-0.7,-0.4,0)	0.3667
180	(0,0.06,0.23)	(-0.7,-0.44,-0.07)	0.3991
190	(0,0.06,0.23)	(-0.7,-0.44,-0.07)	0.3991
200	(0,0.03,0.16)	(-0.7,-0.47,-0.14)	0.4323
210	(0,0.03,0.16)	(-0.7,-0.47,-0.14)	0.4323
220	(0,0,0.1)	(-0.7,-0.5,-0.2)	0.4667
222	(0,0,0.1)	(-0.7,-0.5,-0.2)	0.4667

There is no difference in defuzzification method due to the fact that all fuzzy numbers are triangular and just their comparisons are important. In this example the center of area method is used to defuzzificating the fuzzy numbers.

As seen in Table 4, the least difference from the indifferent number is related to a number between $C = 90$ and $C = 100$; therefore, generating new solutions in this range and interacting with DMs are performed. After generating new solutions, all DMs have consensus on indifferent solution $C = 95$ and select this solution as the optimal solution F^* with following values:

$$x_1 = 9.9641, x_2 = 7.7089, \lambda_{12} = 0.6462, f_1 = 76.8122,$$
$$f_2 = 94.9976$$

$\lambda_{12} = 0.6462$ expresses that DMs are incurious to increasing 0.6462 units of profit in exchange of increasing one unit of cost while first and second functions are fixed in values 76.8122 and 94.9976, respectively. This incuriosity denotes the SWT method optimal solution is obtained and numeric values of two objective functions are optimal.

V. CONCLUSIONS

Early optimization models used to focus on single objective problems, but due to growth of evaluating criteria, multi objective problems are developed and many solution methods are applied to solve them. Interactive methods are efficient approaches to deal with DMs during the problem solving procedure. Although SWT is one of the interactive methods, it uses numerical values in dealing with DM, and moreover it is not capable to transact with a group of DMs. In this paper fuzzy approach was applied to enable the method in group decision making and let DMs to explain their opinions by linguistic variables. Although proposed SWT method has more steps than the original one, it is more compatible with human brain decisions.

Proposed algorithm in this article can be expanded in some ways. Using fuzzy multipliers and fuzzy right hand side values in optimization models is an appropriate way to model practical uncertainties. Future research can be performed by considering a probabilistic condition in formulating the model.

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