Controller Design Based on ISE Minimization and Dominant Pole Retention Method

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***Abstract*— A computer based method to reduce the complexity of the higher order controller, based on the minimization of integral square error (ISE) and Dominant Pole Retention method pertaining to unit step input is presented in this paper. In this order-reduction technique, dominant pole of the higher order plant is retained and reduced order model of the plant is obtained using ISE Minimization Technique. Using this reduced order plant a reduced order controller is obtained. The method has built in stability – preserving feature.**

Keywords- Controller Design, Dominant Pole, ISE

I. INTRODUCTION

The simulation and design of controllers for higher-order systems is a difficult problem. The cost and complexity of the controller increases as the system order goes high. This problem can be overcome if a "good" reduced-order model is available for the original higher-order system and if it is possible to design a controller using a lower-order model, which will stabilize the original higher-order system when placed in the closed loop. Hence, for cost and time saving in design, and for simplifying implementation, reduced-order models are highly desirable for engineers in analysis, synthesis and simulation of complicated higher-order systems [1].

A review of concepts and approaches for controller reduction has been presented by Anderson and Liu [2]. Basically, the approaches can be divided into direct and indirect ones [3]: direct methods seek to obtain a low-order controller directly [4] in which, generally, a quadratic optimization problem is posed with an order constraint and a closed loop stability constraint; indirect methods are two types [5]: (a) a high-order controller can be derived from the assigned high-order plant, by using some LQG or H^{∞} design method, and then an approximation of the controller is obtained, and (b) a low-order plant can be computed from the original one, and then a low-order controller is designed to be used with the original plant.

In [2], indirect strategies of type (a) using the methods of balanced realization [6, 7, 8], Hankel norm optimal approximation [2,9,10], and q-covariance equivalent realization [11, 12] have been discussed. In their usual form, these techniques replace one stable high-order model by a second stable low-order model that usually is not an optimal L_{∞} approximation; further, usually no frequency weighting is employed [2]. However, some frequency weighted versions of the first two methods are available [13].

In the following, the indirect strategy of type (b) is used to design a low-order controller. The plant is first approximated by a low-order model using the Integral Square Error Minimization Technique[14] and Dominant Pole Retention method. Integral Square Error was calculated with the help of Luss-Jakola Algorithm [15, 16]. The controller is then designed for this low-order plant and attached to the original plant.

II. BACKGROUND

Consider the control system [17] as shown in Fig. 1. Given G_n (s) and H(s), the problem is to derive the transfer function of the controller C_f (s) which yields the desired response of the closed loop system. A classical approach to the design of the controller $C_f(s)$ is to specify the desired (also called reference) closed loop transfer function G_{ref} (s), equate it to the closed-loop transfer function, and solve for the controller [17].

The overall closed-loop transfer function in Fig. 1 is

$$G_{ooo}(s) = \frac{C_f(s)G_n(s)}{1 + C_f(s)G_n(s)H(s)} = G_{ref}(s) \tag{1}$$

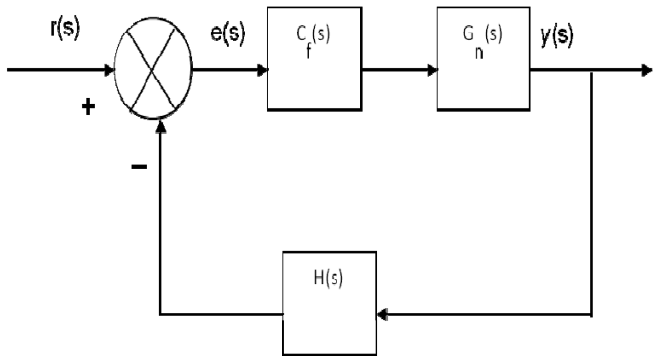


Fig. 1 Controll configuration

On simplification for controller, (1) yields:

$$C_f(s) = \frac{G_{ref}\,(s)}{G_n(s)[1 - G_{ref}(s)H(s)]} \tag{2}$$

By approximating G_n (s) by a reduced-order transfer function $\hat{G}_r$(s), Fig.3 is obtained. In other words, the system of Fig. 2 is approximated by that of Fig. 3, where H(s) is assumed to be same in both these figures.

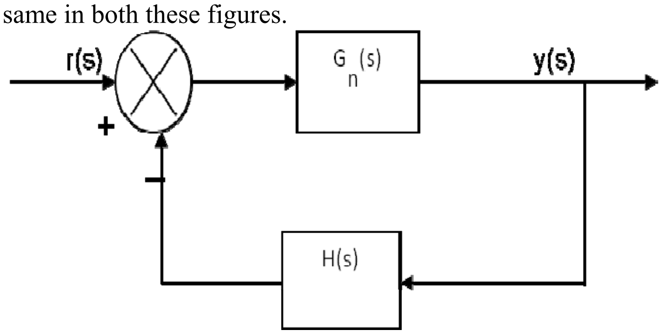


Fig. 2 A closed – loop system

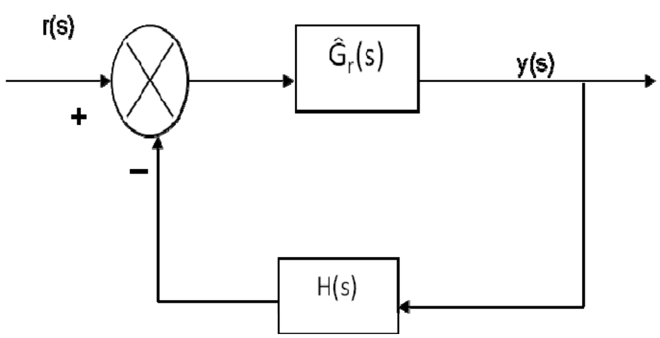


Fig. 3 A reduced-order approximant of the system of Fig. 2

The closed-loop control configuration with reduced-order model and reduced-order controller is shown in Fig. 4.

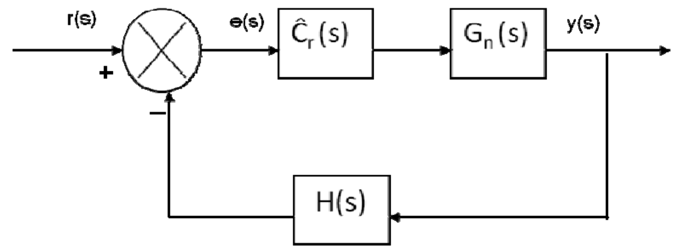


Fig. 4 closed loop control with $\hat{\boldsymbol{G}}_r$(s) and $\hat{\boldsymbol{C}}_r$(s)

The overall closed-loop transfer function in Fig. 4 is

$$G_{orr}(s) = \frac{\hat{C}_r\,(s)\hat{G}_r\,(s)}{1 + \hat{C}_r\,(s)\hat{G}_r\,(s)H(s)]} \tag{3}$$

If original plant is along with reduced order controller, the closed loop system is shown in Fig. 5.

The overall transfer function in Fig. 5 takes the form

$$G_{oro}(s) = \frac{\hat{C}_r\,(s)G_n(s)}{1 + \hat{C}_r\,(s)G_n(s)H(s)]} \tag{4}$$

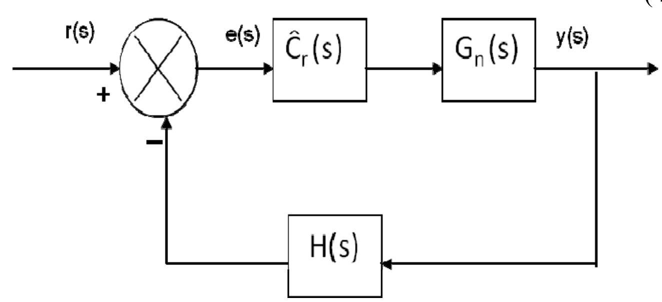


Fig. 5 Closed loop configuration with $\boldsymbol{G}_n$(s) and $\hat{\boldsymbol{C}}_r$(s)

The procedure to obtain a reduced-order controller, $\hat{C}_r$(s) for the system shown in Fig. 5 is explained with the help of following example.

III. REDUCTION PROCEDURE EXAMPLE:

Suppose G_n(s) and H(s), See Fig 1 are given as:

$$G_4(s) = \frac{18.439s^3 + 14.446s^2 + 11.454s + 1.3765}{35.83s^4 + 40.388s^3 + 32.541s^2 + 13.94s + 1} \tag{5}$$

And H(s) =1, The problem is to obtain a reduced order controller $\hat{C}_r$(s) (Fig. 4)

Step 1: Choose a reference model which satisfies the control specification. In this example, a standard second-order transfer function is chosen with damping ratio ε=0.7 and natural frequency w_n=1.5 rad/sec. Thus,

$$G_{ref}(s) = \frac{2.25}{s^2 + 2.1s + 2.25}. \tag{6}$$

Step 2: Derive a second-order model of (5). Using the Integral Square Error Minimization Technique and Luss-Jakola Algorithm, the following second-order approximant is found:

$$G_2(s) = \frac{0.5683s + 0.06965}{s^2 + 0.70038s + 0.506} \tag{7}$$

Also, by retaining the dominant pole of (5), which is at s = -0.088, we have obtained the reduced second order model of (5) as

$$\hat{G}_2(s) = \frac{0.69611s + 0.0984}{(s + 0.088)(s + 0.8123)} = \frac{0.69611s + 0.0984}{s^2 + 0.90035s + 0.0719} \tag{8}$$

Step 3: Derive the reduced-order controller from (3) and (6) together with H(s)=1, one obtains

$$\hat{C}_r(s) = \frac{G_{ref}(s)}{\hat{G}_2(s)[1 - G_{ref}(s)]} \tag{9}$$

Therefore, $\hat{C}_r(s)$ is obtained as

$$\hat{C}_r(s) = \frac{2.25s^2 + 2.02579s + 0.16085}{0.69611s^3 + 1.56023s^2 + 0.20664s} \tag{10}$$

Note that, by contrast, the controller transfer function without reducing $G_4(s)$, Eqn. (2) turns out to be

$$C_f(s) = \frac{80.6175s^4 + 90.873s^3 + 73.2172s^2 + 31.365s + 2.25}{18.439s^5 + 53.1679s^4 + 41.7906s^3 + 25.4294s^2 + 2.8896s} \tag{11}$$

which is of fifth-order.

The overall closed loop transfer function with reduced-order controller $\hat{C}_r(s)$ and reduced-order model $\hat{G}_2(s)$ (Fig. 4) takes the following form:

$$G_{orr}(s) = \frac{1.56635s^3 + 1.63157s^2 + 0.31131s + 0.01583}{0.69611s^5 + 2.186974s^4 + 3.22392s^3 + 1.9298s^2 + 0.32617s + 0.01583} \tag{12}$$

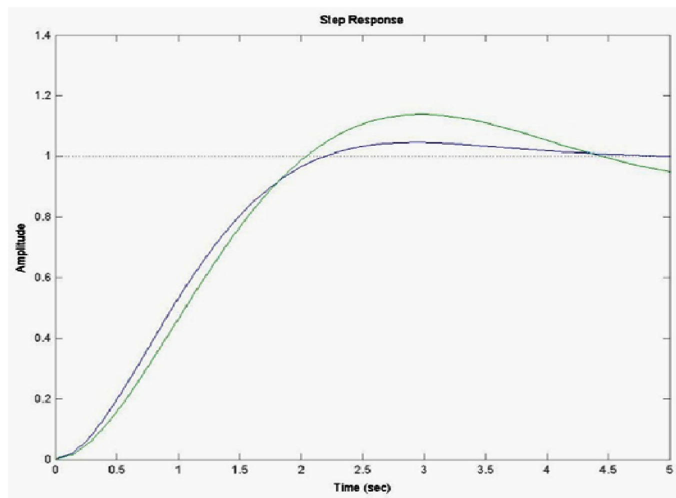
If the original plant $G_4(s)$ along with reduced-order controller $\hat{C}_r(s)$ (Fig. 5), then overall closed loop transfer function turns out to be

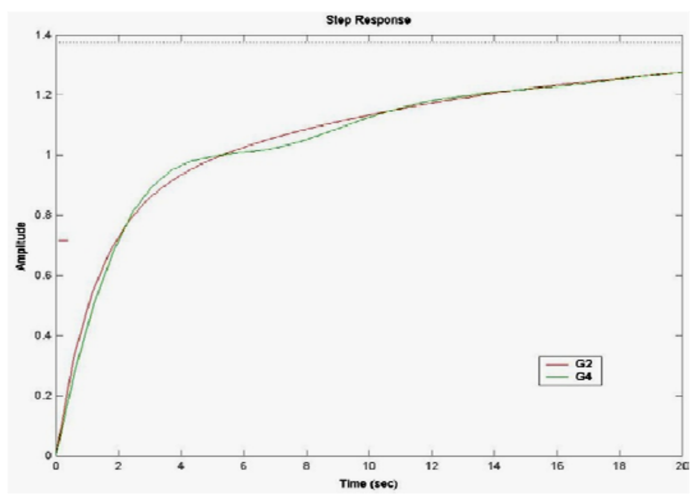
$$G_{oro}(s) = \frac{41.48775s^5 + 69.85704s^4 + 58.00196s^3 + 28.62416s^2 + 4.63087s + 0.22141}{24.94162s^7 + 84.01753s^6 + 134.55834s^5 + 138.67803s^4 + 87.17194s^3 + 33.06495s^2 + 4.83751s + 0.22141} \tag{13}$$

And if the original plant ($G_4(s)$) along with full-order controller ($C_f(s)$) (fig. 1), then overall closed loop transfer function takes the form

$$G_{ooo}(s) = \frac{1486.5061s^7 + 2840.2077s^6 + 3586.1962s^5 + 2787.8642.s^4 + 1458.3030s^3 + 492.5417s^2 + 68.9454s + 3.097125}{660.6694s^9 + 2649.7202s^8 + 5731.2319s^7 + 7426.3581s^6 + 6836.2805s^5 + 4367.7964s^4 + 1948.61s^3 + 558.2521s^2 + 71.8350s + 3.097125} \tag{14}$$

The step responses of (13) and (14) are shown in figure 3.17. It can be observed that the response of the system with reduced order controller is satisfactory.





IV. CONCLUSIONS

A computer based method for controller design based on minimization of integral square error (ISE) and Dominant Pole Retention method pertaining to unit step has been developed. The method retains dominant pole and allow rest of the numerator and denominator coefficients of the controller/plant as free parameter in the process of order reduction. This reduces the high order complexity of the plant/controller to have low order plant/controller. The step response of the original plant with reduced order controller is almost similar to original plant with original controller.

V. ACKNOWLEDGMENT

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VI. REFERENCES

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