Output Regulation of the Arneodo Chaotic System

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***Abstract*—This paper solves the problem of regulating the output of the Arneodo chaotic system (1981), which is one of the paradigms of chaotic dynamical systems. Explicitly, using the state feedback control laws, the output of the Arneodo chaotic system is regulated so as to track constant reference signals as well as to track periodic reference signals. The control laws are derived using the regulator equations of Byrnes and Isidori (1990), which provide the solution of the output regulation problem for nonlinear control systems involving neutrally stable exosystem dynamics. Numerical results are shown to verify the results.**

Keywords-Output regulation; nonlinear control systems; chaos; Arneodo system.

I. INTRODUCTION

Output regulation of control systems is one of the very important problems in control systems theory. Basically, the output regulation problem is to control a fixed linear or nonlinear plant in order to have its output tracking reference signals produced by some external generator (the exosystem). For linear control systems, the output regulation problem was solved by Francis and Wonham ([1], 1975). For nonlinear control systems, the output regulation problem was solved by Byrnes and Isidori ([2], 1990), who generalized the *internal model principle* obtained by Francis and Wonham [1]. In their work [2], Byrnes and Isidori made an important assumption which demands that the exosystem dynamics generating reference and disturbance signals is a *neutrally stable* system (Lyapunov stable in both forward and backward time). The class of neutrally stable exosystem signals includes the important special cases of constant reference signals as well as sinusoidal reference signals. Using Centre Manifold Theory [3], Byrnes and Isidori derived regulator equations in their work [2], which completely characterize the solution of the output regulation problem for nonlinear control systems.

The output regulation problem for linear and nonlinear control systems has been the focus of many important studies in recent years. Mahmoud and Khalil ([4], 1996) obtained results on the asymptotic regulation of minimum phase nonlinear systems using output feedback. Serrani *et al*. ([5], 2000) derived results on the semi-global and global output regulation problem for minimum-phase nonlinear systems. Fridman ([6], 2003) solved the output regulation problem for nonlinear control systems with delay. Marconi et al. ([7], 2004) derived non-resonance conditions for uniform observability in the problem of nonlinear output regulation. J. Huang and Z. Chen ([8], 2004) established a general framework that systematically converts the robust output regulation problem for a general nonlinear system into a robust stabilization problem for an appropriately augmented system. Chen and Huang ([9], 2005) derived results on the robust output regulation for output feedback systems with nonlinear exosystem. Immonen (10], 2007) derived results on the practical output regulation for bounded linear infinite-dimensional state space systems. Xi and Ding ([11], 2007) obtained results on the global adaptive output regulation of a class of nonlinear systems with exosystem. Pavolv *et al.* ([12], 2007) derived results on the global nonlinear output regulation using convergence based controller design. Liu and Huang ([13], 2008) derived results on the global robust output regulation problem for lower triangular nonlinear systems with unknown control direction.

This paper solves the output regulation problem for the Arneodo chaotic system ([14], 1981), which is a classical example of a nonlinear dynamical system exhibiting chaotic motion. In chaos theory for nonlinear dynamical systems, the Arneodo system [14] is one of the paradigms of three-dimensional chaotic systems. This paper uses the regulator equations obtained by Byrnes and Isidori [2] to derive the state feedback control laws for regulating the output of the Arneodo chaotic system for the important cases of constant and periodic reference signals.

This paper is organized as follows. Section 2 reviews the output regulation problem and the solution derived by Byrnes and Isidori [2]. Section 3 contains the main results of this paper, viz. the output regulation of the Arneodo chaotic system. Section 4 contains the numerical results illustrating the output regulator design for the Arneodo chaotic system. Finally, Section 5 provides the conclusions of this paper.

II. REVIEW OF THE OUTPUT REGULATION OF NONLINEAR CONTROL SYSTEMS

In this section, we consider a multivariable nonlinear control system modelled by equations of the form

$$\dot{x} = f(x) + g(x)\, u + p(x)\, \omega \tag{1a}$$

$$\dot{\omega} = s(\omega) \tag{1b}$$

$$e = h(x) - q(\omega) \tag{2}$$

Here, the differential equation (1a) describes a *plant dynamics* with state x defined in a neighbourhood X of the origin of, and the input u takes values in R^n, subject to the effect of a disturbance represented by the vector field $p(x)\ \omega$. The differential equation (1b) describes an autonomous system, known as the *exosystem*, defined in a neighbourhood W of the origin of R^k, which models the class of disturbance and reference signals taken into consideration. The equation (2) defines the *error* between the actual plant output $h(x) \in R^p$ and a reference signal $q(\omega)$, which models the class of disturbance and reference signals taken into consideration.

We also assume that all the constituent mappings of the system (1) and the error equation (2), namely f, g, p, s, h and q are C^1 mappings vanishing at the origin, *i.e.* $f(0)=0,\ g(0)=0,\ p(0)=0,\ s(0)=0,\ h(0)=0$ and $q(0)=0$. Thus, for $u=0$, the composite system (1) has an equilibrium state $(x,\omega)=(0,0)$ with zero error (2).

A state feedback controller for the system (1) has the form

$$u = \alpha(x,\omega) \tag{3}$$

where $\alpha(x,\omega)$ is a C^1 mapping defined on $X \times W$ such that $\alpha(0,0)=0$. Composing the plant dynamics (1) with (3), we get the closed-loop system given by

$$\begin{aligned} \dot{x} &= f(x) + g(x)\ \alpha(x,\omega) + p(x)\ \omega \\ \dot{\omega} &= s(\omega) \end{aligned} \tag{4}$$

The purpose of control action is to achieve internal stability and output regulation, which are explained as follows.

Internal stability means that when the input is disconnected (*i.e.* when $\omega = 0$), the closed-loop system (4) has an exponentially stable state equilibrium at $x=0$.

Output regulation means that for the closed-loop system (4), for all initial states $(x(0),\omega(0))$ sufficiently close to the origin, $e(t)\to 0$ asymptotically as $t\to\infty$.

Formally, we can summarize the requirements as follows.

State Feedback Regulator Problem [2]:

Find, if possible, a state feedback control law $u=\alpha(x,\omega)$ such that

(OR1) [Internal Stability] The dynamics

$$\dot{x} = f(x) + g(x)\ \alpha(x,0)$$

is locally exponentially stable at $x=0$.

(OR2) [Output Regulation] There exists a neighbourhood $U \subset X \times W$ of $(x,\omega)=(0,0)$ such that for each initial condition $(x(0),\omega(0)) \in U$, the solution $(x(t),\omega(t))$ of (4) satisfies

$$\lim_{t\to\infty}\left[h(x(t)) - q(\omega(t))\right] = 0. \ \blacksquare$$

Byrnes and Isidori [2] have completely solved this problem under the following assumptions:

(H1) The exosystem dynamics is *neutrally stable* at $\omega=0$, *i.e.* the system is Lyapunov stable in both forward and backward time at $\omega=0$.

(H2) The pair $(f(x), g(x))$ has a stabilizable linear approximation at $x=0$, *i.e.* if

$$A = \left[\frac{\partial f}{\partial x}\right]_{x=0} \quad \text{and} \quad B = \left[\frac{\partial g}{\partial x}\right]_{x=0},$$

then (A,B) is *stabilizable*, which means that we can find a gain matrix K so that $A+BK$ is Hurwitz. ■

Next, we recall the solution of the output regulation problem derived by Byrnes and Isidori [2].

Theorem 1. [2] *Under the hypotheses (H1)-(H2), the state feedback regulator problem is solvable if, and only if, there exist* C^1 *mappings* $x=\pi(\omega)$ *with* $\pi(0)=0$ *and* $u=\varphi(\omega)$ *with* $\varphi(0)=0$, *both defined in a neighbourhood* $W^0 \subset W$ *of* $\omega=0$ *such that the following equations (called the* ***Byrnes-Isidori regulator equations****) are satisfied:*

(1) $$\frac{\partial \pi}{\partial \omega} s(\omega) = f(\pi(\omega)) + g(\pi(\omega))\varphi(\omega) + p(\pi(\omega))\omega$$

(2) $$h(\pi(\omega)) - q(\omega) = 0.$$

When the Byrnes-Isidori regulator equations (1) and (2) are satisfied, a control law solving the state feedback regulator problem is given by

$$u = \varphi(\omega) + K(x - \pi(\omega))$$

where K *is any gain matrix such that* $A+BK$ *is Hurwitz.* ■

III. Output Regulation of the Arneodo System

The Arneodo chaotic system ([14], 1981) is one of the paradigms of the three dimensional chaotic systems and is described by the dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 + u \end{aligned} \tag{7}$$

where the parameters a and b are positive real constants and u is the controller to be designed.

The unforced Arneodo chaotic system (i.e. Eq. (7) with $u = 0$) undergoes a chaotic behaviour when $a = 7.5$ and $b = 3.8$, which is illustrated in Figure 1.

This paper considers two cases of output regulation for the Arneodo chaotic system, viz.

(A) Tracking of constant reference signals;

(B) Tracking of periodic reference signals.

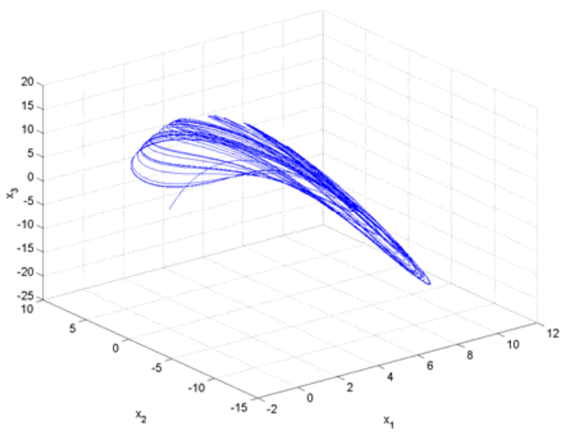

Figure 1. Chaotic Portrait of Unforced Arneodo System

A. Tracking of Constant Reference Signals

In this case, it is assumed that the exosystem is given by the scalar dynamics

$$\dot{\omega} = 0 \tag{8}$$

It is important to observe in this case that the exosystem given by (8) is neutrally stable. Thus, the assumption (H1) of Theorem 1 holds.

Linearization of the dynamics of the Arneodo system (7) at $x = 0$ yields the system matrices

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & -b & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Using Kalman's rank test for controllability [15, p.738], it can be easily shown that the pair (A, B) is controllable. Since the linearized system is in Bush canonical form, it is easy to see that the feedback gain matrix

$$K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$$

will render the matrix $A + BK$ Hurwitz if and only if the following inequalities are satisfied:

$$k_1 < -a, k_2 < b, k_3 < 1, (1-k_3)(b-k_2) + (k_1 + a) > 0. \tag{9}$$

(This follows simply by applying Routh's stability criterion [15, p.234] to the characteristic polynomial of $A + BK$.)

It is assumed that the gain matrix K satisfies the inequalities (9) so that the hypothesis (H2) of Theorem 1 also holds.

A.1 Constant Tracking Problem for x_1

Here, the tracking problem for Arneodo chaotic system is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 + u \\ \dot{\omega} &= 0 \\ e &= x_1 - \omega \end{aligned} \tag{10}$$

The Byrnes-Isidori regulator equations for the system (10) are given by Theorem 1 as

$$\begin{aligned} 0 &= \pi_2(\omega) \\ 0 &= \pi_3(\omega) \\ 0 &= a\pi_1(\omega) - b\pi_2(\omega) - \pi_3(\omega) - \pi_1^2(\omega) + \varphi(\omega) \\ 0 &= \pi_1(\omega) - \omega \end{aligned} \tag{11}$$

Solving the Byrnes-Isidori regulator equations (11), the unique solution is obtained as

$$\begin{aligned} \pi_1(\omega) &= \omega \\ \pi_2(\omega) &= 0 \\ \pi_3(\omega) &= 0 \\ \varphi(\omega) &= -a\omega + \omega^2 \end{aligned} \tag{12}$$

Using Theorem 1 and the solution (12) of the Byrnes-Isidori regulator equations (11), the following result is obtained which gives a state feedback control law solving the output regulation problem for (10).

Theorem 2. *A state feedback control law solving the output regulation problem for (10) is given by*

$$u = -a\omega + \omega^2 + k_1(x_1 - \omega) + k_2 x_2 + k_3 x_3$$

where k_1, k_2 and k_3 satisfy the inequalities given in (9). ■

A.2 Constant Tracking Problem for x_2

Here, the tracking problem for Arneodo chaotic system is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 + u \\ \dot{\omega} &= 0 \\ e &= x_2 - \omega \end{aligned} \tag{13}$$

The Byrnes-Isidori regulator equations for the system (13) are given by Theorem 1 as

$$\begin{aligned} 0 &= \pi_2(\omega) \\ 0 &= \pi_3(\omega) \\ 0 &= a\pi_1(\omega) - b\pi_2(\omega) - \pi_3(\omega) - \pi_1^2(\omega) + \varphi(\omega) \\ 0 &= \pi_2(\omega) - \omega \end{aligned} \tag{14}$$

Since the first and last equations of (14) contradict each other, the regulator equations (14) are not solvable. Thus, by Theorem 1, it follows that the output regulation problem is not solvable for this case.

A.3 Constant Tracking Problem for x_3

Here, the tracking problem for Arneodo chaotic system is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 + u \\ \dot{\omega} &= 0 \\ e &= x_3 - \omega \end{aligned} \tag{15}$$

The Byrnes-Isidori regulator equations for the system (15) are given by Theorem 1 as

$$\begin{aligned} 0 &= \pi_2(\omega) \\ 0 &= \pi_3(\omega) \\ 0 &= a\pi_1(\omega) - b\pi_2(\omega) - \pi_3(\omega) - \pi_1^2(\omega) + \varphi(\omega) \\ 0 &= \pi_3(\omega) - \omega \end{aligned} \tag{16}$$

Since the second and last equations of (16) contradict each other, the regulator equations (16) are not solvable. Thus, by Theorem 1, it follows that the output regulation problem is not solvable for this case.

B. Tracking of Periodic Reference Signals

In this case, it is assumed that the exosystem is given by the two-dimensional dynamics

$$\begin{aligned} \dot{\omega}_1 &= \nu\omega_2 \\ \dot{\omega}_2 &= -\nu\omega_1 \end{aligned} \tag{17}$$

It is important to observe in this case that the exosystem given by (17) is neutrally stable. Thus, the assumption (H1) of Theorem 1 holds.

As noted earlier, the system linearization pair (A, B) is controllable. It is assumed that the gain matrix K satisfies the inequalities (9) so that the hypothesis (H2) of Theorem 1 also holds.

B.1 Periodic Tracking Problem for x_1

Here, the tracking problem for the Arneodo chaotic system is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 + u \\ \dot{\omega}_1 &= \nu\omega_2 \\ \dot{\omega}_2 &= -\nu\omega_1 \\ e &= x_1 - \omega_1 \end{aligned} \tag{18}$$

The Byrnes-Isidori regulator equations for the system (18) are given by Theorem 1 as

$$\begin{aligned} \begin{bmatrix} \frac{\partial \pi_1}{\partial \omega_1} & \frac{\partial \pi_1}{\partial \omega_2} \end{bmatrix} \begin{bmatrix} \nu\omega_2 \\ -\nu\omega_1 \end{bmatrix} &= \pi_2(\omega) \\ \begin{bmatrix} \frac{\partial \pi_2}{\partial \omega_1} & \frac{\partial \pi_2}{\partial \omega_2} \end{bmatrix} \begin{bmatrix} \nu\omega_2 \\ -\nu\omega_1 \end{bmatrix} &= \pi_3(\omega) \\ \begin{bmatrix} \frac{\partial \pi_3}{\partial \omega_1} & \frac{\partial \pi_3}{\partial \omega_2} \end{bmatrix} \begin{bmatrix} \nu\omega_2 \\ -\nu\omega_1 \end{bmatrix} &= a\pi_1(\omega) - b\pi_2(\omega) - \pi_3(\omega) \\ &\quad - \pi_1^2(\omega) + \varphi(\omega) \\ 0 &= \pi_1(\omega) - \omega_1 \end{aligned} \tag{19}$$

Solving the Byrnes-Isidori regulator equations (19), the unique solution is obtained as

$$\begin{aligned} \pi_1(\omega) &= \omega_1 \\ \pi_2(\omega) &= \nu\omega_2 \\ \pi_3(\omega) &= -\nu^2\omega_1 \\ \varphi(\omega) &= -(a + \nu^2)\omega_1 + \omega_1^2 + \nu(b - \nu^2)\omega_2 \end{aligned} \tag{20}$$

Using Theorem 1 and the solution (20) of the Byrnes-Isidori regulator equations (19), the following result is obtained which gives a state feedback control law solving the output regulation problem for (18).

***Theorem 3**. A state feedback control law solving the output regulation problem for (19) is given by*

$$u = \varphi(\omega) + k_1(x_1 - \omega_1) + k_2(x_2 - \nu\omega_2) + k_3(x_3 + \nu^2\omega_1)$$

where $\varphi(\omega)$ is given by (20), and k_1, k_2 and k_3 satisfy the inequalities given in (9). ■

B.2 Periodic Tracking Problem for x_2

Here, the tracking problem for the Arneodo chaotic system is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 + u \\ \dot{\omega}_1 &= \nu\omega_2 \\ \dot{\omega}_2 &= -\nu\omega_1 \\ e &= x_2 - \omega_1 \end{aligned} \tag{21}$$

The Byrnes-Isidori regulator equations for the system (21) are given by Theorem 1 as

$$\begin{aligned} \begin{bmatrix} \dfrac{\partial\pi_1}{\partial\omega_1} & \dfrac{\partial\pi_1}{\partial\omega_2} \end{bmatrix} \begin{bmatrix} \nu\omega_2 \\ -\nu\omega_1 \end{bmatrix} &= \pi_2(\omega) \\ \begin{bmatrix} \dfrac{\partial\pi_2}{\partial\omega_1} & \dfrac{\partial\pi_2}{\partial\omega_2} \end{bmatrix} \begin{bmatrix} \nu\omega_2 \\ -\nu\omega_1 \end{bmatrix} &= \pi_3(\omega) \\ \begin{bmatrix} \dfrac{\partial\pi_3}{\partial\omega_1} & \dfrac{\partial\pi_3}{\partial\omega_2} \end{bmatrix} \begin{bmatrix} \nu\omega_2 \\ -\nu\omega_1 \end{bmatrix} &= a\pi_1(\omega) - b\pi_2(\omega) - \pi_3(\omega) \\ &\quad - \pi_1^2(\omega) + \varphi(\omega) \\ 0 &= \pi_2(\omega) - \omega_1 \end{aligned} \tag{22}$$

Solving the Byrnes-Isidori regulator equations (22), the unique solution is obtained as

$$\begin{aligned} \pi_1(\omega) &= -\nu^{-1}\omega_2 \\ \pi_2(\omega) &= \omega_1 \\ \pi_3(\omega) &= \nu\omega_2 \\ \varphi(\omega) &= (b - \nu^2)\omega_1 - (\nu + a\nu^{-1})\omega_2 + \nu^{-2}\omega_2^2 \end{aligned} \tag{23}$$

Using Theorem 1 and the solution (23) of the Byrnes-Isidori regulator equations (22), the following result is obtained which gives a state feedback control law solving the output regulation problem for (21).

***Theorem 4**. A state feedback control law solving the output regulation problem for (19) is given by*

$$u = \varphi(\omega) + k_1(x_1 + \nu^{-1}\omega_2) + k_2(x_2 - \omega_1) + k_3(x_3 - \nu\omega_2)$$

where $\varphi(\omega)$ is given by (23), and k_1, k_2 and k_3 satisfy the inequalities given in (9). ■

B.3 Periodic Tracking Problem for x_3

Here, the tracking problem for the Arneodo chaotic system is given by

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 + u \\ \dot{\omega}_1 &= \nu\omega_2 \\ \dot{\omega}_2 &= -\nu\omega_1 \\ e &= x_3 - \omega_1 \end{aligned} \tag{24}$$

The Byrnes-Isidori regulator equations for the system (24) are given by Theorem 1 as

$$\begin{aligned} \begin{bmatrix} \dfrac{\partial\pi_1}{\partial\omega_1} & \dfrac{\partial\pi_1}{\partial\omega_2} \end{bmatrix} \begin{bmatrix} \nu\omega_2 \\ -\nu\omega_1 \end{bmatrix} &= \pi_2(\omega) \\ \begin{bmatrix} \dfrac{\partial\pi_2}{\partial\omega_1} & \dfrac{\partial\pi_2}{\partial\omega_2} \end{bmatrix} \begin{bmatrix} \nu\omega_2 \\ -\nu\omega_1 \end{bmatrix} &= \pi_3(\omega) \\ \begin{bmatrix} \dfrac{\partial\pi_3}{\partial\omega_1} & \dfrac{\partial\pi_3}{\partial\omega_2} \end{bmatrix} \begin{bmatrix} \nu\omega_2 \\ -\nu\omega_1 \end{bmatrix} &= a\pi_1(\omega) - b\pi_2(\omega) - \pi_3(\omega) \\ &\quad - \pi_1^2(\omega) + \varphi(\omega) \\ 0 &= \pi_3(\omega) - \omega_1 \end{aligned} \tag{25}$$

Solving the Byrnes-Isidori regulator equations (25), the unique solution is obtained as

$$\begin{aligned} \pi_1(\omega) &= -\nu^{-2}\omega_1 \\ \pi_2(\omega) &= -\nu^{-1}\omega_2 \\ \pi_3(\omega) &= \omega_1 \\ \varphi(\omega) &= (1 + a\nu^{-2})\omega_1 - (\nu - b\nu^{-1})\omega_2 + \nu^{-4}\omega_1^2 \end{aligned} \tag{26}$$

Using Theorem 1 and the solution (26) of the Byrnes-Isidori regulator equations (25), the following result is obtained which gives a state feedback control law solving the output regulation problem for (24).

***Theorem 5**. A state feedback control law solving the output regulation problem for (19) is given by*

$$u = \varphi(\omega) + k_1(x_1 + \nu^{-2}\omega_1) + k_2(x_2 + \nu^{-1}\omega_2) + k_3(x_3 - \omega_1)$$

where $\varphi(\omega)$ is given by (26), and k_1, k_2 and k_3 satisfy the inequalities given in (9). ■

IV. Numerical Results

For simulation, the parameters are chosen as in the chaotic case of Arneodo's system (7), *viz.* $a = 7.5$ and $b = 3.8.$

For achieving internal stability of the state feedback regulator problem, a gain matrix K must be chosen so that the inequalities given in (9) are satisfied. The gain matrix K is chosen so that $A + BK$ has stable eigenvalues $\{-5, -5, -5\}$.

A simple calculation using MATLAB yields

$$K = [k_1 \quad k_2 \quad k_3] = [-132.5 \quad -71.2 \quad -14].$$

In the periodic tracking output regulation problem, the value $\nu = 1$ is taken in the exosystem dynamics (17). For the simulations, the fourth order Runge-Kutta method is used to solve the system using MATLAB. The simulation results are discussed as follows.

A. Tracking of Constant Reference Signals

A.1 Constant Tracking Problem for x_1

Here, the initial conditions are taken as

$$x_1(0) = 8,\ x_2(0) = 5,\ x_3(0) = 9 \text{ and } \omega = 2.$$

The simulation graph is depicted in Figure 2 from which it is clear that the state trajectory $x_1(t)$ tracks the constant reference signal $\omega = 2$ in about 2 seconds.

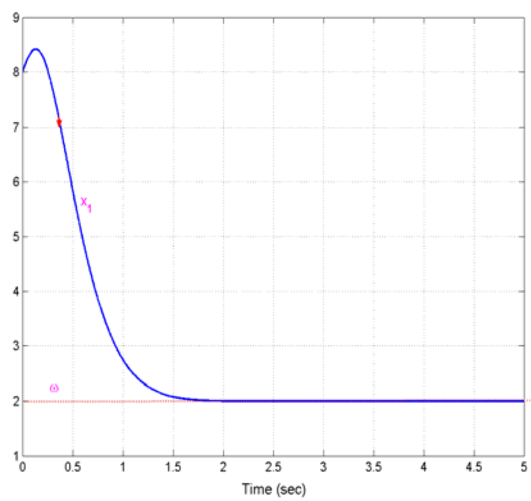

Figure 2. Constant Tracking Problem for x_1

A.2 Constant Tracking Problem for x_2

As pointed out in Section III, the output regulation problem is not solvable for this case, because Byrnes-Isidori regulator equations (14) do not admit any solution.

A.3 Constant Tracking Problem for x_3

As pointed out in Section III, the output regulation problem is not solvable for this case, because Byrnes-Isidori regulator equations (16) do not admit any solution.

B. Tracking of Constant Reference Signals

Here, it is assumed that $\nu = 1.$

B.1 Periodic Tracking Problem for x_1

Here, the initial conditions are taken as

$$x_1(0) = 5,\ x_2(0) = 3, x_3(0) = -4, \omega_1(0) = 0, \omega_2(0) = 1.$$

The simulation graph for this case is depicted in Figure 3 from which it is clear that the state trajectory $x_1(t)$ tracks the periodic reference signal $\omega_1(t) = \sin t$ in about 2 seconds.

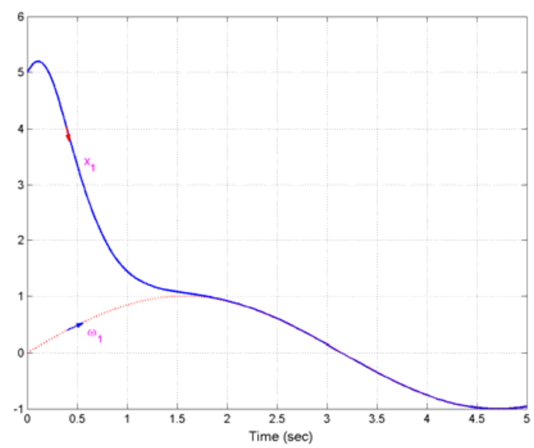

Figure 3. Periodic Tracking Problem for x_1

B.2 Periodic Tracking Problem for x_2

Here, the initial conditions are taken as

$$x_1(0) = 6,\ x_2(0) = -5, x_3(0) = 4, \omega_1(0) = 0, \omega_2(0) = 1.$$

The simulation graph for this case is depicted in Figure 4 from which it is clear that the state trajectory $x_2(t)$ tracks the periodic reference signal $\omega_1(t) = \sin t$ in about 2 seconds.

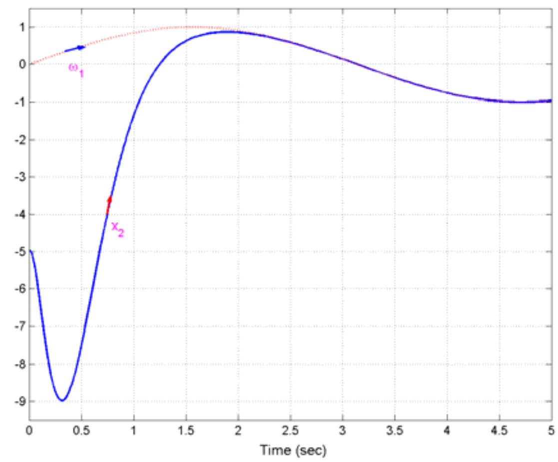

Figure 4. Periodic Tracking Problem for x_2

B.3 Periodic Tracking Problem for x_3

Here, the initial conditions are taken as

$$x_1(0) = 6,\ x_2(0) = 4,\ x_3(0) = -3, \omega_1(0) = 0, \omega_2(0) = 1.$$

The simulation graph for this case is depicted in Figure 5 from which it is clear that the state trajectory $x_3(t)$ tracks the periodic reference signal $\omega_1(t) = \sin t$ in about 2 seconds.

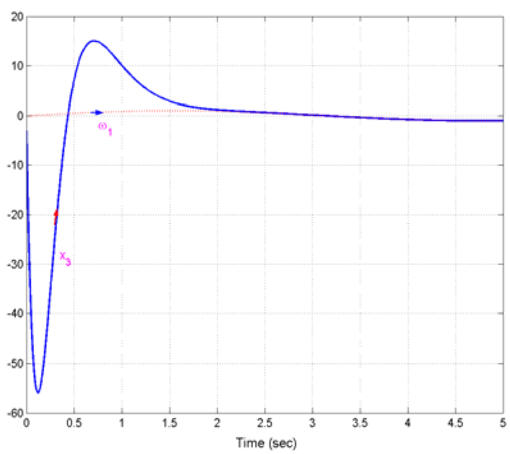

Figure 5. Periodic Tracking Problem for x_3

V. CONCLUSIONS

The output regulation problem for the Arneodo chaotic system (1981) has been studied in detail and solved in this paper. Explicitly, using the Byrnes-Isidori regulator equations (1990), new state feedback control laws have been derived for regulating the output of Arneodo chaotic system. As tracking reference signals, constant input and sinusoidal reference signals have been considered and in each case, feedback control laws regulating the output of the Arneodo chaotic system have been derived. As tracking reference signals, constant and periodic reference signals have been considered and in each case, feedback control laws regulating the output of the Arneodo chaotic system have been derived. Numerical simulations are shown to verify the results.

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