Quantum Teleportation circuit using Matlab and Mathematica

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Abstract— **This Paper describes a basic Quantum Teleportation circuit using mat lab Qlib tool. Teleportation is a new and exciting field of future communication. We know that security in data communication is a major concern nowadays. Among the encryption technologies that are available at present, shared key is the most reliable which depends on secure key generation and distribution. Teleportation/ Entanglement is a perfect solution for secure key generation and distribution, as for the no cloning theorem of quantum mechanics any attempt to intercept the key by the eavesdropper will be detectable immediately. A program is simulated with successful simulation which give successful transfer of random qubit to output and which governs perfect communication between Alice and Bob.**

Keywords— **Quantum Teleportation, Computing, Qlib, Bell states, Entanglement**

I. INTRODUCTION

Quantum Teleportation is a Process by which we can transfer the state of a system and can create replica of a state to another system [1].Basic concept behind Teleportation is entanglement as entangled states act like a channel for transformation [2]. Quantum teleportation is the transfer of an unknown quantum state form a sender (Alice) to a receiver (Bob) by means of a shared bipartite entangled state and appropriate classical communication. If the shared entanglement is infinite than Bob recovers an exact copy of the state sent by Alice. The first Quantum Teleportation Protocol was given by Bennett and Brassad [3].The word “teleportation” could be misleading, since there is not a transfer of matter, energy or information, but only the transfer of a quantum state. For this reason, quantum teleportation is not in contradiction with the relativistic principle according to which a signal can not travel with a velocity greater than the speed of light and it is consistent with the no-cloning theorem [4].The idea behind Teleportation is Scanning a person from head to toe in such a way to extract all the information from it and transmitting it to receiving end and used to construct Replica, not necessary from actual material but from atoms of same kinds, arranged in such a way to represent original information. In classical cryptography, we can use public key encryption or shared key encryption. But public key is vulnerable to attack by quantum computer, as quantum computer would be able to factor the prime product very quickly. Though shared key is secured but it requires many shared random numbers that can not be used more than once, hence the problem of distributing random numbers arises. Quantum Teleportation/Entanglement is therefore, to solve the problem. A completely secure quantum key can be generated and distributed (for communication and decoding of encrypted messages) using entangled photons has been demonstrated in [5]-[7].

II Quantum Teleportation

Quantum Entanglement is the root of Quantum Teleportation. It has been Known since 1935 when Einstein Podolsky and Rosen(EPR)[8]and Schrodinger[9] invented a counterintuitive properties of Quantum systems. EPR says that quantum theory allows certain correlations to exist between two physically distant parts of a quantum system. If there are such correlations then it is possible to predict the result of a measurement on one part of a system by looking at the other part. For this reason EPR argued that the predicted quantity should have a definite value even before it is measured, if quantum theory is complete and respects locality. As it disallows such definite values prior to measurement EPR concluded that, from a classical perspective, quantum theory must be incomplete. Schrödinger’s speech on this argument gives the modern view of quantum mechanics; he says that the wave function provides all the information there is about a quantum system. About the nature of entangled quantum states, Schrödinger [10]-[13] stated that, “The whole is in a definite state, the parts taken individually are not.” This statement defines the essence of pure-state entanglement. Bell [14] later solved the EPR dilemma by deriving correlation which is violated in quantum mechanics but is satisfied within every model that is local and complete. Any quantum system such as a particle that possesses a position in space, energy, angular and linear momentum, and spin is completely described by a wave function $|\varphi\rangle$. Anything that we want to know about the particle is mathematically encoded within $|\varphi\rangle$. We never know the wave function completely because there is no measurement that can determine it completely. By measuring one of the properties of a quantum system, we can get a glimpse of the overall quantum state that is encoded within $|\varphi\rangle$. According to the quantum uncertainty principle, the act of doing such a measurement will destroy any ability to subsequently determine the other properties of the quantum system. That is why it is impossible to copy particles and reproduce them elsewhere via quantum teleportation. However one can recreate an unmeasured quantum state in another particle if he is prepared to sacrifice the original particle. This is done by exploiting the EPR process to circumvent the quantum uncertainty principle.EPR discovered if we measure one of the entangled sub-systems that will puts it into a particular quantum state, while instantaneously putting the other sub-system with which it is entangled into a corresponding quantum state, though the two sub-systems are separated by arbitrarily large distances in space-time .

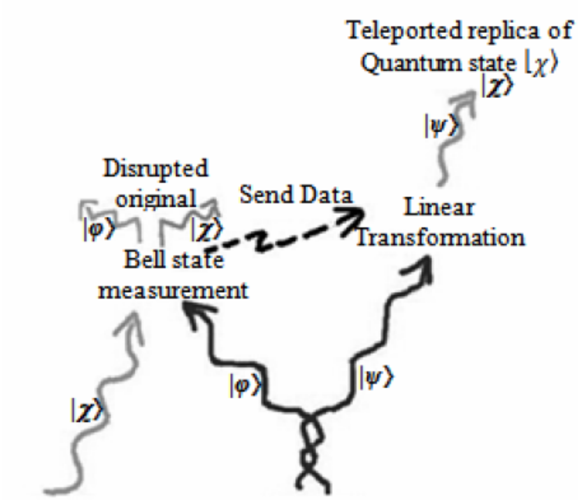


Fig. 1. Quantum Teleportation (Modified IBM Press Image)

III Experimental Process of Quantum Teleportation

Quantum Teleportation Consist of Quantum Logic gates. Quantum teleportation can be implemented with a quantum circuit that is much simpler than that required by any nontrivial quantum computational task. The state of an arbitrary qubit (quantum bit) can be teleported with as few as two quantum exclusive-or (controlled-not) gates. Thus, quantum teleportation is significantly easier to implement than quantum computing if we are concerned only with the complexity of the required circuitry. Short-distance quantum teleportation will play a role in transporting quantum information inside quantum computers. Gates used are controlled not; Hadamdard gate, Pauli gates(X, Y, Z) and respective matrix are as follows

$$H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

IV Idea of Quantum Teleportation

The idea of quantum teleportation is described in fig.1.
The procedure is given below:
1. Make an EPR-pair. Send one of them to Alice and the other to Bob.
2. Alice entangles the received qubit and her own qubit. Thus all qubits are entangled.
3. Alice measures two qubits that belong to her. This measurement also has an effect on the qubit that belongs to Bob.
4. When Alice informs Bob of the measurement result, he can restore the information by a proper operation.

In the first step, an EPR-pair $(|00\rangle + |11\rangle)$ is made. Then, the zeroth (right) qubit is sent to Bob, and the first (left) qubit to Alice. We use the second qubit to represent the qubit that Alice originally had. At this point, the state is described as

$$(a|0\rangle + b|1\rangle) \otimes (|00\rangle + |11\rangle)$$
$$= a|000\rangle + b|100\rangle + a|011\rangle + b|111\rangle$$

After Alice operates CNOT, and H , and measures the first and the second qubits, the state becomes one of the four states, i.e., $|00\rangle(a|0\rangle + b|1\rangle)$, $|01\rangle(a|1\rangle + b|0\rangle)$, $|10\rangle(a|0\rangle - b|1\rangle)$, or $|11\rangle(a|1\rangle - b|0\rangle)$
For example, if she measures 11 and informs Bob of the result, he knows that his own qubit is $(a|1\rangle - b|0\rangle)$Then, he can restore the state $|f\rangle = a|0\rangle + b|1\rangle$ i.e., Alice's original state, by applying the operation$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$When she gets a different result, the same type of operation can be used to restore the original information. Note that Alice's original state is destroyed and that Bob gets the same state. Thus, this is regarded as a teleportation. Alice informs Bob of her measurement result in a classical way (i.e., the speed is less than that of light). Therefore, the speed of this teleportation is slower than the light speed.

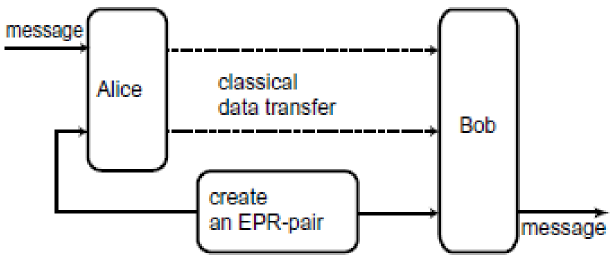
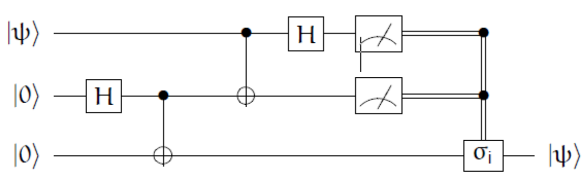


Figure 1: Idea of quantum teleportation.

Programming steps using Matlab Tool Qlib

QLib is an open-source Matlab package intended to provide everybody within the Quantum Information community with a comprehensive toolset, and to allow us to quickly and efficiently frame and explore ideas

Steps: Generate a Random Qubit
Shared Qubit
Over all state is developed by tensor product (KRON)
Action on ALICE side-Controlled not gate (data, Alice) to calculate overall state
Now apply hadmdard gate to data
Measurement on Alice side with probability (1/4 for states 00,01,10,11
Measurement and Alice side and state at bob side with probability (1/4)
Correcting the state at bob side
For 00- act with y=iZ+X (final Bob state)
For 01 act with x (final Bob state)
For 10 acts with z (final Bob state)
For 11 acts with y (final Bob state)



Quantum circuit diagram for the teleportation protocol. The value depends on the two measurement outcomes.

Result

Random Qubit	Probability(1/4)	Bob state	Corrected Bob State (Pauli Operation)
0.4047+0.000i 0.1217-0.9063i	00	0.9415 -0.0539-4011i	0.4047+0.000i 0.1217-0.9063i
	01	0.4047 -0.1217+0.9063i	0.4047+0.000i 0.1217-0.9063i
	10	0.9415 0.0539+0.4011i	0.4047+0.000i 0.1217-0.9063i
	11	0.4047 -0.121-0.9063i	0.4047+0.000i 0.1217-0.9063i

Graph:

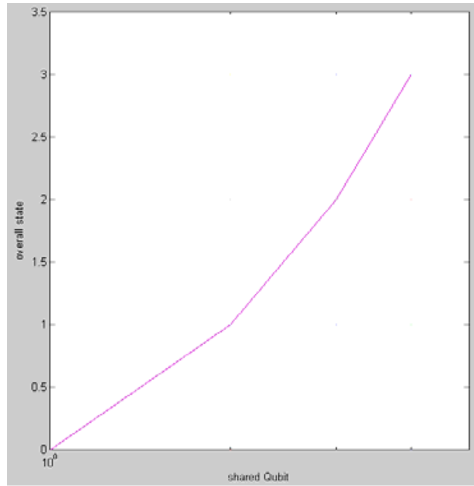


Figure-3

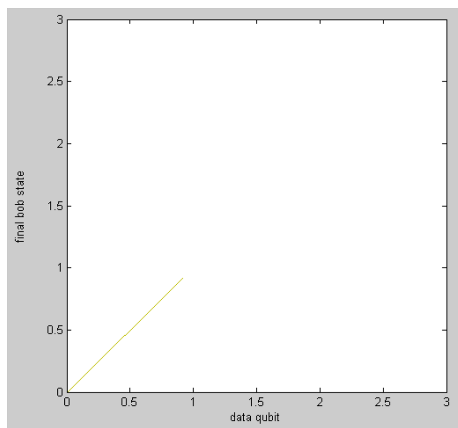


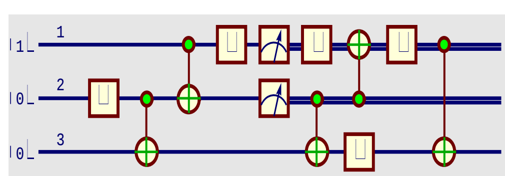
Figure-4

Hence Random Qubit which is to be transmitted by Alice is achieved at the output or Bob Receiver.

Circuit Designed Using Wolfram Mathematica

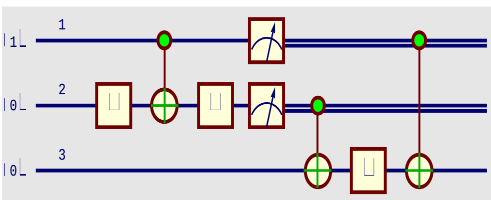
Circuit is designed using Mathematica Quantum Computing, Quantum Notation Package, and Quantum Matrix Package.

Basic Quantum Teleportation Circuit



"Probability"	"Measurement"	"State"
$\frac{1}{4}$	$\{\{0_1, 0_2\}\}$	$\lvert 0_1\rangle \otimes \lvert 0_2\rangle \otimes \frac{a \lvert 0_3\rangle + (-1)^{1/4} b \lvert 1_3\rangle}{\sqrt{aa^* + bb^*}}$
$\frac{1}{4}$	$\{\{0_1, 1_2\}\}$	$\lvert 1_1\rangle \otimes \lvert 1_2\rangle \otimes \frac{(-1)^{3/4} b \lvert 0_3\rangle + i a \lvert 1_3\rangle}{\sqrt{aa^* + bb^*}}$
$\frac{1}{4}$	$\{\{1_1, 0_2\}\}$	$\lvert 1_1\rangle \otimes \lvert 0_2\rangle \otimes - \frac{-(-1)^{1/4} b \lvert 0_3\rangle + a \lvert 1_3\rangle}{\sqrt{aa^* + bb^*}}$
$\frac{1}{4}$	$\{\{1_1, 1_2\}\}$	$\lvert 0_1\rangle \otimes \lvert 1_2\rangle \otimes \frac{i(a \lvert 0_3\rangle - (-1)^{1/4} b \lvert 1_3\rangle)}{\sqrt{aa^* + bb^*}}$
"Probability"	"Measurement"	"State"

Optimized Teleportation Circuit



"Probability"	"Measurement"	"State"
$\frac{aa^*}{aa^* + bb^*}$	$\{\{0_1, 0_2\}\}$	$\lvert 0_1\rangle \otimes \lvert 0_2\rangle \otimes \frac{\lvert 0_3\rangle + \lvert 1_3\rangle}{\sqrt{2}}$
$\frac{bb^*}{aa^* + bb^*}$	$\{\{1_1, 0_2\}\}$	$\lvert 1_1\rangle \otimes \lvert 0_2\rangle \otimes \frac{\lvert 0_3\rangle + \lvert 1_3\rangle}{\sqrt{2}}$
"Probability"	"Measurement"	"State"

Conclusion

By doing programming in Matlab Qlib tool a successful quantum Teleportation is achieved via which one can share the Random Qubit Between Alice and Bob with equal probability of ¼(for 00,01,10,11).Some Pauli gate operation is also applied to get the corrected state. Same Quantum Teleportation circuit is designed in Wolfram Mathematica which gives better probability measurement with easy computations. The field of quantum teleportation has made a remarkable progress ever since it has been initiated. We have presented concept on Quantum Teleportation which is based on the well known concept of Quantum Entanglement that Einstein called "spooky action at a distance" in his EPR paper [14]. We showed that entangle particles can serve as "transporters" that is by introducing a third "message" particle to one of the entangle particles one could transfer its properties to the other one. We have listed the recent developments in Entanglement and Quantum Teleportation Physics and saw that information can be teleported over 40 Km using existing technology (H. Everett, Army Research Office, 2000).

Future work

Optimization can be done by changing the gates .No of gates can be reduced for quantum Teleportation. Experimental Teleportation can be performed .Till now the distance where quantum Teleportation is possible is 200 m .This distance can be increase.

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