# Dependency Through Axiomatic Approach On Rough Set Theory

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***Abstract*: The idea of rough set consist the approximation of a set by pair of sets called the lower and the upper approximation of the set. In fact, these approximations are interior and closer operations in a certain topology generated by available data about elements of the set. The rough set is based on knowledge of an agent about some reality and his ability to discern some phenomenon processes etc. Thus this approach is based on the ability to classify data obtained from observation, measurement, etc. In this paper we define the dependency of knowledge through the axiomatic approach instead of the traditional (Pawlak) method of rough set.**

***Keywords- Rough set, knowledge base, algebraic or axiomatic approach, dependency***

## I. INTRODUCTION

Rough Set theory was born in early 1980 as a mental child of Professor Zdzisław Pawlak[7]. Rough set theory can be seen as a new mathematical approach to vagueness. The rough set philosophy is found on the assumption that every object in the universe we associate some information (data, knowledge). Objects characterized by the same information are indiscernibility (similar) in view of the available information about them. The indiscernibility relation generated in this way is the mathematical basis of rough set theory. This understanding of indiscarnibility is related to the idea of Leibnitz, the great German mathematician, which is known as Leibnitz's Law of indiscernibility : The Identity of indiscernibles, that is "objects are indiscernible if and only if all available functional take on them identical values". However, in the rough set approach indiscarnibility is defined relative to a given set of functional (attributes). Z. Pawlak [7] introduced rough set theory (1982) which is new mathematical tool dealing with vagueness and uncertainty. Before Z. Pawlak fuzzy set theory was introduced by L. Zadeh [3] in 1965. Both rough sets and fuzzy sets are the theories to handle uncertain, vague, imprecise problems but their view points are different. Many researchers have made much work in combining both of them, the results are rough fuzzy sets and fuzzy rough sets. This paper introduces the idea of rough fuzzy set in a more general setting, named rough intuitionistic fuzzy set.

Axiomatic approach to rough sets was introduced by T. Iwinski [5] in 1987. In a note Y.Yao ([6]) compares constructive and algebraic (axiomatic) approaches in the study of rough sets. In the constructive approach one can define a pair of lower (inner) and upper (outer) approximation operators using the binary relation. In the algebraic approach, one defines a pair of dual approximation operators and states axioms that must be satisfied by the operators. Various classes of rough set algebras are characterized by different sets of axioms.

Rough set is being used as an effective model to deal imprecise knowledge. One of the main goals of the rough set analysis is to synthesize approximation of concepts from the acquired data. According to Pawlak , knowledge about a universe can be considered as one's capability to classify objects of the universe. By classification or partition of a universe U we write ,a set of objects { $Y_i$ , i=1,2,3 …,n} of U for $Y_i \bigcap Y_j = \phi$, $i \neq j$ and $\bigcup Y_i = U$ . Let a relation $R \subseteq U \times U$ be an equivalence relation on U. The equivalence class of an element $x \in U$ with respect to R, denotes $[x]_R$ is the set of elements $x \in U$ such that xRy. The set U/R be the family of all equivalence classes of R called as concepts or categories of R and $[x]_R$ a category in U. We know that the notion of an equivalence relation R and classification $\{Y_i\}$ are mutually interchangeable. Thus R will be the knowledge on U which is an equivalence relation on U.

Any family of concepts in U will be referred to as abstract knowledge about U. A relational system

$\mathcal{K}$= (U, $\mathcal{R}$) is called the knowledge base where $\mathcal{R}$ be a

family of equivalence relations over U.

For any subset $\mathcal{P} \subseteq \mathcal{R}$, IND($\mathcal{P}$) is the intersection of all equivalence relations in $\mathcal{P}$, called as indiscernibility relation over $\mathcal{P}$. By IND ($\mathcal{K}$) we denote the family of all equivalence relations defined in $\mathcal{K}$.

Let U is non empty finite set called the universe of discourse and R be an equivalence relation (knowledge) on U. Given any arbitrary set X⊆U it may not be possible to describe X precisely in the approximation space (U, R).The set X be characterized by a pair of approximation sets. This leads to the concept of rough set. We define

$$\underline{R}X = \bigcup\{Y \in U/R : Y \subseteq X\} = \{x \in U : [x]_R \subseteq X\}$$

and

$$\overline{R}X = \bigcup\{Y \in U/R : Y \bigcap X \neq \phi\} = \{x \in U : [x]_R \bigcap X \neq \phi\}$$

are called R-lower approximation and R -upper approximation of X, respectively with respect to R.

The R -boundary region of X denoted by $BN_R(X)$, be defined by $BN_R(X) = \overline{R}X - \underline{R}X$ .We say that X is rough with respect to knowledge R if and only if $\overline{R}X \neq \underline{R}X$ and X is said to be R -definable if and only if $\overline{R}X = \underline{R}X$ , that is $BN_R(X) = \varphi$, and at that time X becomes a crisp set.

For an element x ∈ U, we say that x is certainly in X under the equivalence relation R (knowledge R) if and only if x ∈ $\underline{R}$(X) and that x is possibly in X under R if and only if $x \in \overline{R}(X)$. The set $BN_R(X)$ is the set of elements which cannot be classified as either belonging to X or belonging to ~ X having the knowledge R.

This becomes Pawlak's definition (constructive method) of rough set ([7]). The system $(2^U, \bigcup, \bigcap, \sim, \overline{R}, \underline{R})$ is called rough set algebra, where $\cup, \cap, \sim$ are set union, intersection and complement respectively. The lower and upper approximations in (U, R) have the following properties, for any subsets X, Y ⊂ U,

1.1 $\underline{R}(X) \subseteq X \subseteq \overline{R}(X)$

1.2 $\underline{R}U = \overline{R}U = U$, $\underline{R}\varphi = \overline{R}\varphi = \varphi$

1.3 $\overline{R}(X \cup Y) = \overline{R}(X) \cup \overline{R}(Y)$ and $\underline{R}(X \cap Y) = \underline{R}(X) \cap \underline{R}(Y)$

1.4 $\overline{R}(X \cap Y) \subseteq \overline{R}(X) \cap \overline{R}(Y)$ and $\underline{R}(X \cup Y) \supseteq \underline{R}(X) \cup \underline{R}(Y)$

1.5 $\overline{R}(\sim X) = \sim \underline{R}(X)$, $\underline{R}(\sim X) = \sim \overline{R}(X)$

1.6 $\underline{R}(\underline{R}(X)) = \overline{R}(\underline{R}(X)) = \underline{R}(X)$
$\overline{R}(\overline{R}(X)) = \underline{R}\left(\overline{R}(X)\right) = \overline{R}(X)$

Algebraic or axiomatic definition to rough set was given by T.Iwinski [5] in 1987. Let *P, Q* be two sets such that $P \subset Q \subset U$ .Then the pair *(P, Q )* forms a rough set for which P be the below (Lower) and Q be the above (Upper) approximation concept. Applying some operational criteria to P, Q it can be converted to Pawlak's Rough set $(\overline{R}X, \underline{R}X)$, $X \subset U$.

Throughout this paper we use the axiomatic definition of rough set.

## 2.Axiomatic Definition of Rough Set

**Definition 2.1:**

Let U be a finite and non empty set, called the universe. Let L, H: $2^U \longrightarrow 2^U$ are two unary operators on the power set $2^U$ of U. These two operators are dual if

2.1 L ~ A = ~HA

2.2 $H \sim A = \sim LA$ for all $A \subseteq U$

**Definition 2.2 :**

Let L, H: $2^U \longrightarrow 2^U$ are dual unary operators which satisfy

2.3 $LU = U$

2.4 $H\phi = \phi$

2.5 $L(A \cap B) = LA \cap LB$

2.6 $H(A \cup B) = HA \cup HB$, where A, B are the subsets of U

Also L and H satisfy the weaker conditions

2.7 $L(A \cup B) \supseteq LA \cup LB$

2.8 $H(A \cap B) \subseteq HA \cap HB$

2.9 $A \subseteq B \Rightarrow LA \subseteq LB$

2.10 $A \subseteq B \Rightarrow HA \subseteq HB$

**Definition 2.3**

Let L , H : $2^U \longrightarrow 2^U$ be a pair of dual unary operators which satisfy (2.3) to (2.6) of the definition 2.2 and let $LA \subset A \subset HA$ for $A \in 2^U$, then the system $(2^U, \cup, \cap, \sim, L, H)$ be called a rough set algebra , where L,H are called lower and upper approximation operators respectively, and $\cup, \cap, \sim$ are set union, intersection and complement respectively . Now the pair (LA, HA) forms a rough set of A on U and is called an axiomatic rough set.

**Definition 2.4**

Let $\mathbb{A}$= (LA, HA), $\mathbb{B}$= (LB, HB) be two rough sets of A and B respectively on U, then the union, denotes $\mathbb{A} \cup \mathbb{B}$ and the intersection, denotes $\mathbb{A} \cap \mathbb{B}$, be defined by

$$\mathbb{A} \cup \mathbb{B} = (L(A \cup B), H(A \cup B))$$

and $\mathbb{A} \cap \mathbb{B} = (L(A \cap B), H(A \cap B))$ respectively.

**Definition: 2.5** Let $\mathbb{A}$ = (LA, HA), $\mathbb{B}$ = (LB, HB) be two rough sets defined on U then the subset, $\mathbb{A} \subset \mathbb{B}$ is defined by $LA \subset LB$ and $HA \subset HB$. Two rough sets $\mathbb{A}$, $\mathbb{B}$ are equal , $\mathbb{A} = \mathbb{B}$ if and only if $\mathbb{A} \subset \mathbb{B}$ and $\mathbb{A} \subset \mathbb{B}$.

If $\mathbb{A}$ = (LA, HA) be a rough set of A on U then the complement of $\mathbb{A}$, denotes $\sim \mathbb{A}$ , be defined by $\sim \mathbb{A} = (\sim HA, \sim LA)$, where $\sim LA$, $\sim HA$ are respective complement of LA and HA in U.

**Proposition 2.1 ( [4] )** For any three rough sets $\mathbb{A}$, $\mathbb{B}$ and $\mathbb{D}$

(i) $\mathbb{A} \cup \mathbb{B} = \mathbb{B} \cup \mathbb{A}$
(ii) $\mathbb{A} \cap \mathbb{B} = \mathbb{B} \cap \mathbb{A}$
(iii) $\mathbb{A} \cup (\mathbb{B} \cup \mathbb{D}) = (\mathbb{A} \cup \mathbb{B}) \cup \mathbb{D}$
(iv) $(\mathbb{A} \cap \mathbb{B}) \cap \mathbb{D} = \mathbb{A} \cap (\mathbb{B} \cap \mathbb{D})$

## 3. Dependency of Knowledge:

According to Pawlak [8], knowledge Q depends on knowledge P denotes P⇒Q, if and only if IND (P) ⊆ IND (Q). It is observed that for any subset X⊂U, the IND (P) boundary region of X is contained in IND (Q) boundary region of X of the universe U where P,Q ⊆ $\mathfrak{R}$ and $\mathcal{K}$ = (U, $\mathfrak{R}$) be the given knowledge base.

Let P and Q be two equivalence relations (Knowledge) on the finite universe U. Let U/P ⊂U/Q that is any equivalence class of U/P is contained by one of the equivalence class of U/Q. Then Q depends on P if and only if for any subset X⊂ U, the P-boundary region of X is contained in Q-boundary region of X, that is, $BN_P(X) \subset BN_Q(X)$.

The above concept generates the definition of dependency of Rough set in the Algebraic method which is given below.

**Definition 3.1:** Let $\mathbb{A}$ = (LA, HA) be the rough set on U. Then the borderline region of $\mathbb{A}$ or the boundary of $\mathbb{A}$ be defined by

BN ($\mathbb{A}$) = HA−LA =HA∩(~LA).

If HA=LA, that is, if BN ($\mathbb{A}$) = ϕ then A is an axiomatically definable set otherwise A is rough with respect to LA and HA on U.

**Definition 3.2:** Let $\mathbb{A}$ = (LA, HA) and $\mathbb{B}$ = (LB, HB) be two rough sets in U. Then $\mathbb{B}$ depends on $\mathbb{A}$ or $\mathbb{B}$ is derivable from $\mathbb{A}$, denoted by A⇒B if and only if LB⊂LA and HA⊂HB.

It is clear that if $\mathbb{A} \Rightarrow \mathbb{B}$ then BN($\mathbb{A}$) ⊂ BN ($\mathbb{B}$), but the converse is not true.

**Definition 3.3:** Two rough sets $\mathbb{A}$ and $\mathbb{B}$ are independent on U if and only if neither $\mathbb{A} \Rightarrow \mathbb{B}$ nor $\mathbb{B} \Rightarrow \mathbb{A}$ hold.

Two axiomatic rough sets $\mathbb{A}$ and $\mathbb{B}$ are equivalent if and only if $\mathbb{A} \Rightarrow \mathbb{B}$ and $\mathbb{B} \Rightarrow \mathbb{A}$ and we denote it by $\mathbb{A} \equiv \mathbb{B}$.

It is clear that $\mathbb{A}$ and $\mathbb{B}$ are equivalent if and only if LA=LB and HA = HB, that is, if and only if BN ($\mathbb{A}$) = BN ($\mathbb{B}$). Also if a rough set $\mathbb{A}$ is independent to a rough set $\mathbb{B}$ then the intersection of BN($\mathbb{A}$) and BN ($\mathbb{B}$) may not be empty.

We find the following properties on dependency.

**Proposition 3.1:** Let $\mathbb{A}$, $\mathbb{B}$ and $\mathbb{D}$ be three axiomatic rough sets on U. If $\mathbb{D} \subset \mathbb{A}$ and $\mathbb{A}$ is independent to $\mathbb{B}$ then $\mathbb{D}$ is also independent to $\mathbb{B}$ provided $\mathbb{D}$ cannot be compared with $\mathbb{B}$ (That is neither $\mathbb{D} = \mathbb{B}$ nor $\mathbb{D} \subset \mathbb{B}$ nor $\mathbb{B} \subset \mathbb{D}$ ).

**Proof:** Starting $\mathbb{A} \not\Rightarrow \mathbb{B}$ and $\mathbb{B} \not\Rightarrow \mathbb{A}$ as $\mathbb{A}$ is independent to $\mathbb{B}$

This implies $LB \not\subset LA$, $HA \not\subset HB$, and $HA \not\subset HB, HB \not\subset HA$

As $\mathbb{D} \subset \mathbb{A}$, and as $\mathbb{D} \not\subset \mathbb{B}$, we have

$LD \not\subset LB$ and $LB \not\subset LD$. In similar manner we can prove $HD \not\subset HB$ and $HB \not\subset HD$. Hence $\mathbb{D}$ is independent to $\mathbb{B}$. This completes the prove.

We find here that if $\mathbb{D} \subset \mathbb{A}$, and BN($\mathbb{D}$) $\not\subset$ BN($\mathbb{A}$) ∩ BN ($\mathbb{B}$) then $\mathbb{A}$ is independent to $\mathbb{B}$ implies $\mathbb{D}$ is independent to $\mathbb{B}$.

## 4. EQUALITIES AND INCLUSIONS

We first define the rough equalities of sets through algebraic method. Let $\mathbb{A}$ = (LA, HA) and $\mathbb{B}$ =

(LB, HB) be two rough sets of A and B respectively, on U. Then

(a) Sets A and B are axiomatically bottom equal, that is, A $\eqsim$ B if and only if LA = LB

(b) Sets A and B are axiomatically top equal, that is , A $\simeq$B if and only if HA = HB

(c) Sets A and B are axiomatically equal if and only if A $\eqsim$ B and A $\simeq$B, that is, if and only if LA = LB and HA = HB.

Also the rough inclusion of sets through algebraic approach be defined as follows

($a_1$) Set A is axiomatically bottom included by B, that is A $\subset_*$ B , if and only if LA $\subseteq$LB

($b_1$) Set A is axiomatically top included by the set B ,that is A $\subset^*$ B if and only if HA $\subseteq$ HB.

($c_1$) Set A is axiomatically included by B, that is A $\subset_*^*$ B, if and only if A $\subset_*$B and A $\subset^*$B.

Let $\mathbb{A}$ = (LA, HA) and $\mathbb{B}$ = (LB, HB) be two rough sets on *U*. Then the difference

$$\mathbb{A} - \mathbb{B} = \mathbb{A} \cap (\sim\mathbb{B}) = (LA, HA) \cap (L{\sim}B, H{\sim}B)$$

$$= (L(A \cap {\sim}B), H(A \cap {\sim}B))$$

$$= (L(A-B), H(A-B))$$ from definition 2.3

**Proposition: 4.1** Let L , H: $2^U \rightarrow 2^U$ be dual unary operators which satisfy condition (2.3) to (2.6) of definition 2.2 for A, B $\subseteq$ U, let $\mathbb{A}$ = (LA, HA) , $\mathbb{B}$ = (LB, HB) then

(i) L (A $-$B) =LA – HB $\subseteq$ LA – LB

(ii) H (A – B) $\subseteq$ HA$-$ LB

(iii) A $\subset_*$ B if and only if ~B $\subset^*$ ~A

(iv) A $\subset^*$ B if and only if ~B$\subset_*$ ~A

(iii) A $\subset_*$ B $\Rightarrow$ A-B $\eqsim$ $\varphi$

**Proof:** We have L (A$-$B) = L (A$\cap$~B) = LA $\cap$ L (~B)

= LA $\cap$ ~ HB = LA –HB $\subseteq$ LA$-$LB.

Hence (i) is proved. Proof of (ii), (iii), & (iv) are similar.

Next suppose that A $\subset_*$ B $\Leftrightarrow$ LA $\subseteq$ LB

$\Rightarrow$ LA $-$LB = $\varphi$

But L(A$-$B)$\subseteq$ LA$-$LB = $\varphi$ which implies L(A$-$B) = $\varphi$ = L$\varphi$.

Hence $A-B \approx \varphi$ .This completes the proof of (v).

We claim that $A \simeq B \Leftrightarrow \sim A \approx \sim B$.

We have $A \approx B \Leftrightarrow HA = HB \Leftrightarrow \sim L\sim A = \sim L\sim B \Leftrightarrow L\sim A = L\sim B \Leftrightarrow \sim A \approx \sim B$.

**Theorem:4.1** Let $A,B,A_1,B_1 \subset U$, and let $\mathbb{A} = (LA, HA)$ and $\mathbb{B} = (LB, HB)$ , $\mathbb{A}_1 = (LA_1, HA_1)$ and $\mathbb{B}_1 = (LB_1, HB_1)$ be the axiomatic rough sets of $A,B,A_1,B_1$ , respectively .If $A \approx A_1$ and $B \simeq B_1$ then $A-B \approx A_1-B_1$.

**Proof:** Now $L(A-B) = L(A \cap \sim B) = LA \cap L(\sim B) = LA \cap \sim HB$

$= LA_1 \cap \sim HB_1 = LA_1 - HB_1 = L(A_1-B_1)$

Hence $A-B \approx A_1 - B_1$ and hence the theorem.

**Corollary 4.1** Let U be the universe, R be an equivalence relation on U, $R \in IND(\mathcal{K})$. Then for any set $X, Y \subseteq U$, we have

(i) $\underline{R}(X-Y) = \underline{R}X - \overline{R}Y \subset \underline{R}X - \underline{R}Y$

(ii) $\overline{R}(X-Y) \subseteq \overline{R}X - \underline{R}Y$

(iii) X is bottom included by Y if and only if $\overline{R}(\sim Y) \subseteq \overline{R}(\sim X)$

(iv) X is top included by Y if and only if $\underline{R}(\sim Y) \subseteq \underline{R}(\sim X)$

(iv) X is both included Y if and only if $\underline{R}(X-Y) = \varphi$

**Proof:** Taking $\underline{R}$ instead of L and $\overline{R}$ instead of H the corollary follows directly. We prove this theorem for clarity. Now $\underline{R}(X-Y) = \underline{R}(X \cap \sim Y)$

$= \underline{R}X \cap \sim \overline{R}Y$

$= \underline{R}X - \overline{R}Y \subseteq \underline{R}X - \underline{R}Y$ .

Hence (i) is proved.

Next we have $\underline{R}X \subseteq \underline{R}Y$ , then $\underline{R}X \subseteq \overline{R}Y \Rightarrow$

$\underline{R}X - \overline{R}Y = \varphi \Rightarrow \underline{R}(X-Y) = \varphi$.

## 5. Rough Intuitionistic Fuzzy Set (RIFS)

**Definition 5.1 ([1], [2]):** An intuitionistic fuzzy set A ⊆ U is characterized by two functions $\mu_A$ and $\nu_A$, called the membership and non-membership function of A such that $\mu_A : U \to [0,1], \mu_B : U \to [0,1]$, where $0 \le \mu_A(x) + \nu_A(x) \le 1$ for all x Є *U*.

The set $A = \{< x, \mu_A(x), \nu_A(x) >: x \in U\}$ is called intuitionistic fuzzy set (IFS).

The function $\pi_A$, called the hesitation function of A be defined by $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ for all x ∈ U and when $\pi_A(x) = 0$, the IFS A reduces to a fuzzy set.

**Definition 5.2:** Let A be an IFS defined on *U*. The complement of A denoted by ~A and defined by the function $\mu_{\sim A} = 1 \quad \mu_A(x)$ *and* $\nu_{\sim A}(x) = 1 \quad \nu_A$ for all x Є *U*.

We find the algebraic definition for RIFS as

**Definition 5.3** Let *U* be any non empty set and $\mathcal{A}$ be the set of all IFS in *U*. The pair (*U*, $\mathcal{A}$) is the rough intuitionistic fuzzy universe.

For any element A of $\mathcal{A}$ we find two elements LA and HA of $\mathcal{A}$ such that LA ⊂A⊂ HA. Then the pair $\mathbb{A}$ = (LA, HA) is called a rough intuitionistic fuzzy set (RIFS) of A on U, where the dual unary operators L , H satisfy the conditions of definition 2.2.

**Definition: 5.4** Let $\mathbb{A}$ = (LA, HA) and $\mathbb{B}$ = (LB, HB) be two RIFS on U. Then the subset, $\mathbb{A} \subseteq \mathbb{B}$ be defined by LA ⊆ LB and HA ⊆ HB which is equivalent to $\mu_{LA} \le \mu_{LB}$, $\nu_{LA} \ge \nu_{LB}$ and $\mu_{HA} \le \mu_{HB}$, $\nu_{HA} \ge \nu_{HB}$.

**Proposition 5.1** Let $\mathbb{A}$= (LA, HA), $\mathbb{B}$ = (LB, HB) be two RIFS on U. Then

(i) $\mathbb{A} \subset_* \mathbb{B}$ if and only if $\sim\mathbb{B} \subset^* \sim\mathbb{A}$

(ii) $\mathbb{A} \subset^* \mathbb{B}$ if and only if $\sim\mathbb{B} \subset_* \sim\mathbb{A}$.

**Proof:** Suppose that $\mathbb{A} \subset_* \mathbb{B}$ .This is equivalent to LA ⊆ LB so that $\mu_{LA}(x) \le \mu_{LB}(x)$ and $\nu_{LA}(x) \ge \nu_{LB}(x)$ for all xЄU. This implies and implied by $1 - \mu_{LA}(x) \ge 1 - \mu_{LB}(x)$ and $1 - \nu_{LA}(x) \le 1 - \nu_{LB}(x)$

for all x Є U ⇔ ~LA ⊇~LB ⇔ H~A ⊇ H~B

⇔ $\sim\mathbb{A} \subset^* \sim\mathbb{B}$.

**Proposition 5.2** If $\mathbb{A}$ = (LA, HA), and $\mathbb{B}$= (LB, HB) are two RIFSs on U and are independent to each other then, ~$\mathbb{A}$ and ~$\mathbb{B}$ are independent.

**Proof:** From hypothesis $\mathbb{A} \not\Rightarrow \mathbb{B}$ and $\mathbb{B} \not\Rightarrow \mathbb{A}$

That is $LB \not\subset LA$ *and* $LA \not\subset LB$

Hence $\sim LA \not\subset \sim LB$ *and* $\sim LB \not\subset \sim LA.$ in similar way we can get $\sim HA \not\subset \sim HB$ *and* $\sim HB \not\subset \sim HA$

That is $\sim\mathbb{A} \Rightarrow \sim\mathbb{B}$ and $\sim\mathbb{B} \Rightarrow \sim\mathbb{A}$ is not true.

Hence the proposition.

**Example:** Let $\mathbb{A}$ = (LA, HA) and $\mathbb{B}$ = (LB, HB) be two RIFS on U. Let U = {u,v,w,x} be the universe;
LA = { <u,.5,.5>,<v,.6,.2>,<w,.7, .3>, <x, .7,.2 >}
HA = {<u, .6, .3>, <v, .8, .2 >, <w, .7, .2>, <x, .8,.1>}
and
LB = {<u, .7, .2>, <v, .7, .1>, <w, .8, .2>, <x, .7, .2>}
HB = {<u, .8, .1>, <v, .7, .1>,<w, .9, .1>, <x, .8, .2>}

Clearly LA ⊆LB, that is $\mathbb{A} \subset_* \mathbb{B}$ . Then

H ~A = ~LA = {<u, .5, .5 >, <v, .4, .8 >, <w, .3, .7 >, <x, .3,.8>} = {<u, .5, .5 >, <x, .3, .7 >}
Since $\mu_{\sim LA}$(v) + $\nu_{\sim LA}$(v) = .4 + .8 > 1, the element v is not included in ~LA.
Also H~B=~LB = {<u, .3, .8 >, <v, .3, .9 >, <w, .2, .8 >, <x, .3, .8 >} = {<w, .2, .8 >}

Hence H ~ A ⊃ H~B; that is, ~ $\mathbb{A} \subset^* \sim\mathbb{B}$.

## 6. Conclusion:

The constructive approach is more suitable for practical applications of rough sets, while the algebraic or axiomatic approach to rough set is appropriate for studying the structure of rough set algebra. The axiomatic approach deals with axioms that must be situated by approximation operators without explicitly referring to a binary relation. Here we define dependency of knowledge through axiomatic approach and some properties are studied and at the end a new notion called rough intuitionistic fuzzy set is defied through axiomatic approach.

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